Quantum Discord Bounds the Amount of Distributed Entanglement

T. K. Chuan,[1] J. Maillard,[2] K. Modi,[3,1] T. Paterek,[1,4,*] M. Paternostro,[5] and M. Piani[6,†]

[1]*Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543 Singapore*
[2]*Blackett Laboratory, Imperial College London, Prince Consort Road, London SW7 2BZ, United Kingdom*
[3]*Department of Physics, University of Oxford, Clarendon Laboratory, Oxford, OX1 3PU, United Kingdom*
[4]*Division of Physics and Applied Physics, School of Physical and Mathematical Sciences,*
Nanyang Technological University, Singapore
[5]*Centre for Theoretical Atomic, Molecular, and Optical Physics, School of Mathematics and Physics,*
Queen's University, Belfast BT7 1NN, United Kingdom
[6]*Institute for Quantum Computing and Department of Physics and Astronomy, University of Waterloo,*
200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada
(Received 1 April 2012; published 16 August 2012)

The ability to distribute quantum entanglement is a prerequisite for many fundamental tests of quantum theory and numerous quantum information protocols. Two distant parties can increase the amount of entanglement between them by means of quantum communication encoded in a carrier that is sent from one party to the other. Intriguingly, entanglement can be increased even when the exchanged carrier is not entangled with the parties. However, in light of the defining property of entanglement stating that it cannot increase under classical communication, the carrier must be quantum. Here we show that, in general, the increase of relative entropy of entanglement between two remote parties is bounded by the amount of nonclassical correlations of the carrier with the parties as quantified by the relative entropy of discord. We study implications of this bound, provide new examples of entanglement distribution via unentangled states, and put further limits on this phenomenon.

Introduction.—Entanglement is a trademark of quantum physics and a powerful resource enabling faster-than-classical computation, efficient quantum communication, and secure cryptography [1]. For these reasons, the design of efficient methods to distribute entanglement is one of the key goals of mainstream quantum information science. Of particular relevance for tasks of long-haul quantum communication is the distribution of entanglement among the remote noninteracting nodes of a quantum network [2]. In this case, two general architectures able to accomplish this task have been identified: the first relies on the availability of a resource whose entanglement is *transferred* to chosen nodes of the network [3]; the second is a quantum communication scenario based on the exchange of a *carrier* quantum system between two of such distant nodes [4], which might be referred to as the *sender* and *receiver* laboratory, respectively.

Remarkably, Cubitt *et al.* [5] reported a scheme where the carrier exchanged by sender and receiver remains unentangled from them at all times. This result, which was later extended to the continuous-variable scenario in [6], intriguingly implies that the amount of distributed entanglement is not bounded by the entanglement initially shared by the carrier and the sender, given that in these cases they are unentangled at all times. These observations pave the way to some interesting considerations. First, quite clearly, the carrier must display some quantum features, otherwise the protocol would simply consist of the exchange of classical communication aided by local

node-carrier operations, which cannot increase entanglement [7]. Second, in Refs. [8] a link has been suggested between the distribution of entanglement by separable states and the presence of more general forms of quantum correlations, as captured for example by quantum discord [9,10], between nodes of the network and the carrier.

In light of such considerations, here we address the following fundamental questions: How much can the entanglement between sender and receiver laboratories increase under the exchange of a carrier? Is there a quantitative relation between such increase and the nonclassical correlations between the carrier and the parties?

Our key finding is a general bound on the entanglement gain between distant laboratories under local operations and quantum communication, which is given by the quantum discord between them and the carrier. In turn, this result provides an operational interpretation of quantum discord as the truly necessary prerequisite for the success of entanglement distribution as opposed to entanglement itself. We show that the relation thus formulated generalizes the subadditivity of entropy and can be quite naturally linked to the possibility that quantum conditional entropy attains negative values [11]. Finally, we study in detail the resources required for entanglement creation and increase via the use of a separable carrier, and illustrate our findings with some new concrete examples of such a phenomenon.

Definitions.—In order to treat entanglement and discord on the same footing, throughout this Letter we consider the former as measured by the relative entropy of

entanglement [12] and the latter as quantified by the one-way quantum deficit [13], also known as relative entropy of discord [14]. The quantum relative entropy between two states ρ and σ is defined as $S(\rho \| \sigma) := -S(\rho) - \mathrm{tr}(\rho \log \sigma)$, where $S(\rho) = -\mathrm{tr}(\rho \log \rho)$ is the von Neumann entropy of ρ. The relative entropy is monotonic under any completely positive trace-preserving map $\mathcal{M}$, that is $S(\rho \| \sigma) \geq S[\mathcal{M}(\rho) \| \mathcal{M}(\sigma)]$. The relative entropy of entanglement in the bipartition X versus Y is defined as the minimum relative entropy $\mathcal{E}_{X:Y}(\rho) := \min_{\rho_{X:Y}} S(\rho \| \rho_{X:Y})$ between the joint state ρ of X and Y and the set of separable states $\rho_{X:Y} = \sum_i p_i \rho_X^i \otimes \rho_Y^i$ [12]. Similarly, the relative entropy of discord is defined as the minimum relative entropy $\mathcal{D}_{X|Y}(\rho) := \min_{\chi_{X|Y}} S(\rho \| \chi_{X|Y})$ between ρ and the set of quantum-classical states $\chi_{X|Y} = \sum_j p_j \chi_X^j \otimes |j\rangle\langle j|_Y$, with $\{|j\rangle\}$ an orthonormal basis for Y. It can be shown that $\mathcal{D}_{X|Y}(\rho)$ corresponds to the minimal entropic increase resulting from the performance of a complete projective measurement Π_Y over Y: $\mathcal{D}_{X|Y}(\rho) = \min_{\Pi_Y} S[\Pi_Y(\rho)] - S(\rho)$ where $\Pi_Y(\rho)$ describes the state after the measurement Π_Y [14]. Finally, mutual information between X and Y is defined as $\mathcal{I}_{X:Y}(\rho) := S(\rho_{XY} \| \rho_X \otimes \rho_Y)$, with ρ_X and ρ_Y the reduced states of X and Y. Mutual information quantifies the total amount of correlations present between X and Y [15]. It holds $\mathcal{I}_{X:Y}(\rho) \geq \mathcal{D}_{X|Y}(\rho) \geq \mathcal{E}_{X:Y}(\rho)$.

Entanglement distribution.—Consider two remote agents, Alice and Bob, having access to local quantum systems A and B, respectively. Their aim is to increase the entanglement that they share by sending an auxiliary quantum system—the carrier C—with which they interact locally (see Fig. 1). The key step of any communication scheme is the transfer of a carrier from one laboratory to the other. The difference in entanglement across the bipartitions $A:CB$ and $AC:B$, corresponding to the situation after and before the transfer of the carrier, can be bound thanks to the following (see the Appendix).

Theorem 1.—For any tripartite state $\rho = \rho_{ABC}$ it holds

$$|\mathcal{E}_{A:CB}(\rho) - \mathcal{E}_{AC:B}(\rho)| \leq \mathcal{D}_{AB|C}(\rho). \qquad (1)$$

We apply this relation to the scenario of Fig. 1. Let us call α the initial state of A, B, and C, and $\beta = \mathcal{M}_{AC}(\alpha)$ the state obtained from it by means of a local *encoding* operation $\mathcal{M}_{AC}$. A local operation on AC cannot increase entanglement in the $AC:B$ cut, i.e., $\mathcal{E}_{AC:B}(\beta) \leq \mathcal{E}_{AC:B}(\alpha)$. System C is then sent to Bob's site, where it interacts with B via a *decoding* operation meant to localize on B the entanglement between the laboratories [16]. Combining the above description with Eq. (1) for β we get

$$\mathcal{E}_{A:CB}(\beta) \leq \mathcal{E}_{AC:B}(\alpha) + \mathcal{D}_{AB|C}(\beta). \qquad (2)$$

This shows that the entanglement gain between distant laboratories is bounded by the amount of quantum discord as measured on the communicated system. In what follows
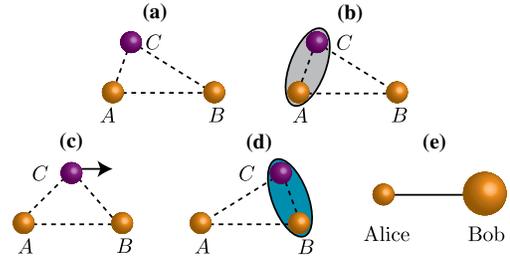
FIG. 1 (color online).   Entanglement distribution. (a) The distribution protocol begins with systems A and C in Alice's lab and system B in Bob's. (b) In the next step, Alice applies an encoding operation to systems A and C. (c) System C is then sent to Bob's site. (d) The carrier C interacts with B via a decoding operation meant to localize on B the entanglement between A and BC. (e) Systems A and B are more entangled than in panel (a).

we discuss the meaning and the implications of the bounds given in Eqs. (1) and (2).

Impossibility of entanglement distribution by local operations and classical communication.—Let us first address the case of $\mathcal{D}_{AB|C}(\beta) = 0$. This corresponds to classical communication from Alice to Bob as it implies that β has the quantum-classical structure $\beta = \sum_i p_i \rho_{AB}^i \otimes |i\rangle\langle i|_C$. The index i embodies classical information that Alice may copy locally before sending C to Bob. After C is transferred from Alice to Bob, both have access to this information. Bob can then perform a local transformation that depends on the index i originally held only by Alice. The process just described is one communication step of a general protocol based on the use of local operations and classical communication (LOCC). The protocol may include several rounds of classical communication with C that is sent back and forth between Alice and Bob; local classical registers can be kept or erased at any stage of the protocol. In this case, Eq. (2) reduces to the statement that entanglement does not increase at any step of a protocol based on LOCC [7]. If $\mathcal{D}_{AB|C}(\beta)$ does not vanish, the transfer of C cannot be interpreted as classical communication, revealing the role of discord in general quantum communication. Hence, Eq. (2) constitutes a nontrivial relaxation of the condition of monotonicity of entanglement under LOCC, bounding the increase of entanglement under local operations and *quantum communication*.

Pure state case.—We now apply Eqs. (1) and (2) to a tripartite pure state $\rho = |\phi\rangle\langle\phi|_{ABC}$. For any bipartite pure state, the relative entropy of entanglement and the relative entropy of discord coincide with the entropy of the reduced states of the parts. Thus, Eq. (1) becomes

$$|S(\rho_A) - S(\rho_B)| \leq S(\rho_{AB}), \qquad (3)$$

which is the Araki-Lieb inequality [17] and is equivalent to the subadditivity of entropy for subsystems AC and BC. Accordingly, Eq. (1) can be seen as a generalization of the subadditivity of entropy valid for tripartite mixed states.

When the carrier is sent from Alice's lab to Bob's, the change in entanglement given in Eq. (2), becomes

$$\mathcal{E}_{A:CB}(\rho) - \mathcal{E}_{AC:B}(\rho) = S_{C|B}(\rho) = -S_{C|A}(\rho), \quad (4)$$

where $S_{C|A}(\rho) := S(\rho_{AC}) - S(\rho_A)$ is the conditional entropy of ρ_{AC}. This gives an operational interpretation of the negative conditional entropy as the increase of entanglement between distant laboratories caused by the transfer of C [11].

Entanglement distribution via a separable system.—The bound in Eq. (1) is tight in some cases; in particular, we have verified that this happens for the three-qubit state of the example of entanglement creation with an unentangled carrier introduced in Ref. [5]. Motivated by this, and in order to emphasize the significance of the appearance of discord rather than entanglement on the right-hand side of Eq. (1), we focus on the general conditions for the success of entanglement creation by means of a separable carrier. This corresponds to requiring

$$\mathcal{E}_{B:AC}(\alpha) = 0 \quad [\Rightarrow \mathcal{E}_{B:AC}(\beta) = 0], \quad (5a)$$

$$\mathcal{E}_{C:AB}(\beta) = 0, \quad (5b)$$

$$\mathcal{E}_{A:BC}(\beta) > 0. \quad (5c)$$

Equation (5a) says that no initial entanglement between the distant sites is present. The implication is due to the local nature of the encoding operation $\mathcal{M}_{AC}$. Equation (5b) encompasses our prescription that the carrier must be separable from A and B. Finally, Eq. (5c) ensures that nonvanishing entanglement is established by exchanging the carrier. Nonvanishing $A:BC$ entanglement does not necessarily imply the possibility of creating $A:B$ entanglement via the local decoding operation on BC. Indeed, if this was always possible, bound entanglement [18] would not exist, as one could always map entanglement into two-qubit entanglement, which is known to be distillable [19]. However, in many relevant cases, including all our examples, entanglement can be localized as shown by Theorem 2 in the Supplemental Material [20].

In order to satisfy the conditions (5), besides the discord present in β, there must be discord on the receiver side already in the initial state α. This is seen by applying Eq. (1) again, but with the roles of B and C interchanged, and using the fact that discord does not increase under operations on the unmeasured systems [21], arriving to

$$\mathcal{E}_{A:CB}(\beta) \leq \mathcal{E}_{AB:C}(\beta) + \mathcal{D}_{AC|B}(\alpha). \quad (6)$$

If Eq. (5b) holds, we obtain the relation $\mathcal{E}_{A:BC}(\beta) \leq \mathcal{D}_{AC|B}(\alpha)$. Note that if C is initially not correlated with AB, the latter further simplifies to $\mathcal{E}_{A:BC}(\beta) \leq \mathcal{D}_{A|B}(\alpha)$. Another interesting limiting case of Eq. (6) is when $\mathcal{D}_{AC|B}(\alpha) = 0$. Then B is classical initially and therefore also in the state β after the encoding: $\beta = \sum_i p_i \beta^i_{AC} \otimes |i\rangle\langle i|_B$. In this case entanglement between Alice and Bob can only be created if the carrier is entangled with the sites and, in particular, only if at least one β^i_{AC} is entangled. Indeed, such β simply describes a situation in which Bob,

upon reading the index i encoded in B, knows which of many states β^i_{AC} he will end up sharing with Alice.

On the other hand, entanglement creation with a separable carrier is possible starting from a state with $\mathcal{D}_{BC|A}(\alpha) = 0$. For instance, it is enough to consider the three-qubit example given in Ref. [5], but starting with A and C interchanged and using a step in the encoding operation $\mathcal{M}_{AC}$ to undo the change before proceeding with the original protocol. However, under further restrictions, the classicality of A may prevent entanglement creation with a separable carrier, as shown for instance in Theorem 3 in the Supplemental Material [20].

Furthermore, we note that when the encoding operation is restricted to be unitary, the presence of discord (on either party) is not a sufficient precondition to make entanglement creation with a separable carrier possible. This follows by combining the fact that any bipartite state that is sufficiently mixed is separable [22] and the existence of discordant states infinitesimally close to any nondiscordant one [23]. As unitary operations do not change mixedness, discord of sufficiently mixed states cannot be converted into entanglement.

Finally, for a fixed dimension of the carrier, it is more efficient to use an entangled carrier rather than a separable one. On one hand, by sending a d-dimensional system that is maximally entangled with a similar one that remains with the sender, we can increase the shared entanglement by $\log_2 d$. On the other hand, Theorem 4 of the Supplemental Material [20] shows that using separable states, the entanglement increase is strictly smaller than $\log_2 d$.

Examples.—In order to make our result more concrete, in the Supplemental Material [20] we provide new examples of both the creation and the increase of entanglement between distant parties by the exchange of an unentangled carrier. The examples are based on the fact that the state of a bipartite system of total dimensions d_{tot} having the form $\rho_p = p|\psi\rangle\langle\psi| + (1-p)\mathbb{1}/d_{\text{tot}}$ is separable if and only if $p \leq p_{\text{cr}} = (1 + a_1 a_2 d_{\text{tot}})^{-1}$, where a_1 and a_2 are the two largest Schmidt coefficients of the bipartite state $|\psi\rangle$, and $\mathbb{1}/d_{\text{tot}}$ is the maximally mixed state of the total system [24]. Consider now a tripartite pure state $|\psi\rangle = |\psi\rangle_{ABC}$. This state admits three Schmidt decompositions corresponding to the three bipartitions $A:BC$, $B:AC$, and $C:AB$. One can choose $|\psi\rangle$ such that p_{cr} is the lowest across the $A:BC$ bipartition, so that there is a finite range for p such that ρ_p is $A:BC$ entangled but separable in the remaining two splittings. Such a ρ_p is meant to play the role of β in our scenario. We remark that the three-qubit example of Ref. [5] uses a carrier system C that is initially classically correlated with A and B. However, a scenario where C initially shares no correlation with the remote nodes is more relevant from a practical point of view, as one can imagine that the carrier is an independent system to be used to distribute entanglement. Even with such a restriction, entanglement can be established via a separable system, as proven explicitly by our examples in the Supplemental Material [20].

Conclusions.—It is the very act of physical transmission of a carrier system that changes the amount of correlations between the remote laboratories. To illustrate this consider total correlations, as captured by mutual information. One expects from the principle of no-signaling that the increase of mutual information is bounded by the amount of communicated correlations. Indeed, applying the chain rule for mutual information and its monotonicity under local operations [25] one finds

$$I_{A:CB} - I_{AC:B} \leq I_{A:C} \leq I_{AB:C}. \tag{7}$$

Both in classical and quantum information theory, the increase of total correlations between the labs is bounded by the correlations between the systems that are kept stored in the labs and the carrier.

However, whereas there is only one kind of correlation between classical random variables, quantum systems can share different kinds of correlations [10]. In this Letter we proved a relation analogous to Eq. (7) for the increase of quantum entanglement between remote elements of a quantum network. We showed that such increase is bounded from above by the amount of nonclassical correlations between the exchanged carrier and the distant nodes as measured by quantum discord, a quantifier for a more general type of nonclassical correlations than entanglement. It follows that, in contrast with what one would expect extrapolating from Eq. (7), our bound for the entanglement increase is in general larger than the entanglement between the carrier and the nodes; in particular, it can be nonzero even when the latter vanishes. Indeed, this has to be the case, as implied by the seminal example of entanglement distribution using a separable carrier of Ref. [5].

Besides providing a natural operational interpretation of quantum discord as the necessary prerequisite for the success of entanglement distribution, our work identifies the conditions for its occurrence with a separable carrier. The scenario tackled by our study fits well with a few experimental settings, including cavity or circuit-QED

and trapped-ion technology and we thus hope that our results will find a prompt experimental demonstration.

Note added.—During the completion of this Letter we became aware of the closely related independent work by Streltsov, Kampermann, and Bruß [29].

APPENDIX

We prove here Theorem 1 of the main text. It is a consequence of the following Lemma.

Lemma 1.—Given $\rho = \rho_{ABC}$, consider Π_C^*, the optimal projective measurement on C for the sake of $\mathcal{D}_{AB|C}(\rho)$. Let p_i be the probability of outcome i for such a measurement, and ρ_{AB}^i be the corresponding conditional states of AB; i.e., $\Pi_C^*(\rho_{ABC}) = \sum_i p_i \rho_{AB}^i \otimes |i\rangle\langle i|_C$. Then

$$\mathcal{E}_{A:CB}(\rho) \leq \mathcal{D}_{AB|C}(\rho) + \sum_i p_i \mathcal{E}_{A:B}(\rho_{AB}^i)$$

$$= \mathcal{D}_{AB|C}(\rho) + \mathcal{E}_{A:CB}[\Pi_C^*(\rho)]$$

$$= \mathcal{D}_{AB|C}(\rho) + \mathcal{E}_{AC:B}[\Pi_C^*(\rho)]. \tag{A1}$$

Proof.—Let $\rho_{A:B}^{i*}$ be the optimal separable state for the sake of $\mathcal{E}_{A:B}(\rho_{AB}^i)$. The state $\sum_i p_i \rho_{A:B}^{i*} \otimes |i\rangle\langle i|_C$ is fully separable and *a fortiori* A:CB-separable; moreover it is invariant under the action of Π_C^*. Then the inequality (A1) is obtained as follows:

$$\mathcal{E}_{A:CB}(\rho) \leq S\left(\rho \,\|\, \sum_i p_i \rho_{A:B}^{i*} \otimes |i\rangle\langle i|_C\right) = -S(\rho) - \mathrm{Tr}\left[\rho \log\left(\sum_i p_i \rho_{A:B}^{i*} \otimes |i\rangle\langle i|_C\right)\right] \tag{A2a}$$

$$= -S(\rho) - \mathrm{Tr}\left[\Pi_C^*(\rho) \log\left(\sum_i p_i \rho_{A:B}^{i*} \otimes |i\rangle\langle i|_C\right)\right] \tag{A2b}$$

$$= \{S[\Pi_C^*(\rho)] - S(\rho)\} + \left\{-S[\Pi_C^*(\rho)] - \mathrm{Tr}\left[\Pi_C^*(\rho) \log\left(\sum_i p_i \rho_{A:B}^{i*} \otimes |i\rangle\langle i|_C\right)\right]\right\}$$

$$= \mathcal{D}_{AB|C}(\rho) + S\left(\sum_i p_i \rho_{AB}^i \otimes |i\rangle\langle i|_C \,\|\, \sum_i p_i \rho_{A:B}^{i*} \otimes |i\rangle\langle i|_C\right) \tag{A2c}$$

$$= \mathcal{D}_{AB|C}(\rho) + \sum_i p_i S(\rho_{AB}^i \,\|\, \rho_{A:B}^{i*}) = \mathcal{D}_{AB|C}(\rho) + \sum_i p_i \mathcal{E}_{A:B}(\rho_{AB}^i), \tag{A2d}$$

where the steps are justified as follows: for Eq. (A2a), the fully separable state $\sum_i p_i \rho_{A:B}^{i*} \otimes |i\rangle\langle i|_C$ cannot be better than optimal for the sake of $\mathcal{E}_{A:CB}(\rho)$; for Eq. (A2b), $\mathrm{Tr}(\sigma \log\Pi(\tau)) = \mathrm{Tr}(\Pi(\sigma) \log\Pi(\tau))$ for all (complete or noncomplete) projective measurements Π, and for all σ

and all τ [25]; for Eq. (A2c), by the optimality of Π_C^* for the sake of $\mathcal{D}_{AB|C}(\rho)$; for the first equality of Eq. (A2d), by the chain rule for relative entropy [26]; for the second equality of Eq. (A2d), by the optimality of each $\rho_{A:B}^{i*}$ for the sake of $\mathcal{E}_{A:B}(\rho_{AB}^i)$. Finally, the two last lines of

Eq. (A1) are due to the fact that relative entropy of entanglement satisfies the "flags" condition of Ref. [27], i.e., $\mathcal{E}_{FX:Y}(\sum_i p_i |i\rangle\langle i|_F \otimes \rho_{XY}^i) = \sum_i p_i \mathcal{E}_{X:Y}(\rho_{XY}^i) = \mathcal{E}_{X:YF}$ $(\sum_i p_i \rho_{XY}^i \otimes |i\rangle\langle i|_F)$. ∎

The statement of the above Lemma regards entanglement redistribution. Nonetheless it is related to—and can be seen as a generalization of—the results of Ref. [28], where it was proven that the relative entropy of entanglement is not lockable by dephasing any single qubit held by one of the parties. In our context, it is further worth recalling that the variation of a generic relative entropy-based measure of correlations—not necessarily entanglement—under the complete dephasing of one of the two parties was considered in Ref. [13]. We notice that the total dephasing of one of the two parties would simply destroy all entanglement. The bound given in Eq. (A1) is based on the consideration of a hypothetical optimal complete von Neumann measurement performed only on the subsystem that is to be transferred from one party to the other.

Proof of Theorem 1.—Applications of Lemma 1 and the monotonicity of the relative entropy of entanglement under LOCC gives

$$\mathcal{E}_{A:CB}(\rho) \leq \mathcal{D}_{AB|C}(\rho) + \mathcal{E}_{AC:B}[\Pi_C^*(\rho)]$$
$$\leq \mathcal{D}_{AB|C}(\rho) + \mathcal{E}_{AC:B}(\rho). \qquad (A3)$$

By inverting the roles of A and B, we obtain Eq. (1). ∎

We remark that Lemma 1, although less amenable to a clear operational interpretation, is in general strictly stronger than Theorem 1. Consider for example the case of a pure tripartite state symmetric under the exchange of A, B and C. For such a case, Eq. (3) is clearly not tight as soon as $S(A) = S(B) = S(C) = S(AB) = S(AC) = S(BC) > 0$, since the left-hand side of Eq. (3) would vanish but its right-hand side would not. On the other hand, in the same case, provided that Π_C^* [i.e., the measurement that is optimal for the sake of $\mathcal{D}_{AB|C}(\rho)$] is such that all conditional states ρ_{AB}^i are separable, Eq. (A1) is tight. This happens, for example, for the tripartite Greenberger-Horne-Zeilinger state $\rho = |GHZ\rangle\langle GHZ|$, with $|GHZ\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$.

---

*tomasz.paterek@ntu.edu.sg
†mpiani@iqc.ca

[1] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[2] H. J. Kimble, Nature (London) **453**, 1023 (2008).
[3] B. Kraus and J. I. Cirac, Phys. Rev. Lett. **92**, 013602 (2004); M. Paternostro, W. Son, and M. S. Kim, Phys. Rev. Lett. **92**, 197901 (2004); M. Żukowski, A. Zeilinger, M. A. Horne, and A. K. Ekert, Phys. Rev. Lett. **71**, 4287 (1993).
[4] F. Ciccarello, M. Paternostro, M. S. Kim, and G. M. Palma, Phys. Rev. Lett. **100**, 150501 (2008).
[5] T. S. Cubitt, F. Verstraete, W. Dür, and J. I. Cirac, Phys. Rev. Lett. **91**, 037902 (2003).
[6] L. Mišta, Jr. and N. Korolkova, Phys. Rev. A **77**, 050302 (R) (2008); **80**, 032310 (2009).
[7] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A **54**, 3824 (1996).
[8] A. Datta, Ph.D. thesis, University of New Mexico, 2008, arXiv:0807.4490; N. Li and S. Luo, Phys. Rev. A **78**, 024303 (2008).
[9] L. Henderson and V. Vedral, J. Phys. A **34**, 6899 (2001); H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001); J. Oppenheim, M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **89**, 180402 (2002).
[10] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, arXiv:1112.6238.
[11] M. Horodecki, J. Oppenheim, and A. Winter, Nature (London) **436**, 673 (2005); L. del Rio, J. Aberg, R. Renner, O. Dahlsten, and V. Vedral, Nature (London) **474**, 61 (2011).
[12] V. Vedral, M. B. Plenio, M. A. Rippin, and P. L. Knight, Phys. Rev. Lett. **78**, 2275 (1997); V. Vedral and M. B. Plenio, Phys. Rev. A **57**, 1619 (1998).
[13] M. Horodecki, P. Horodecki, R. Horodecki, J. Oppenheim, A. Sen(De), U. Sen, and B. Synak-Radtke, Phys. Rev. A **71**, 062307 (2005).
[14] K. Modi, T. Paterek, W. Son, V. Vedral, and M. Williamson, Phys. Rev. Lett. **104**, 080501 (2010).
[15] B. Groisman, S. Popescu, and A. Winter, Phys. Rev. A **72**, 032317 (2005).
[16] F. Verstraete, M. Popp, and J. I. Cirac, Phys. Rev. Lett. **92**, 027901 (2004).
[17] H. Araki and E. H. Lieb, Commun. Math. Phys. **18**, 160 (1970).
[18] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **80**, 5239 (1998).
[19] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **78**, 574 (1997).
[20] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.109.070501 for proofs of the theorems mentioned in the main text and additional examples.
[21] A. Streltsov, H. Kampermann, and D. Bruß, Phys. Rev. Lett. **106**, 160401 (2011).
[22] L. Gurvits and H. Barnum, Phys. Rev. A **66**, 062311 (2002).
[23] A. Ferraro, L. Aolita, D. Cavalcanti, F. M. Cucchietti, and A. Acin, Phys. Rev. A **81**, 052318 (2010).
[24] G. Vidal and R. Tarrach, Phys. Rev. A **59**, 141 (1999).
[25] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).
[26] M. Piani, Phys. Rev. Lett. **103**, 160504 (2009).
[27] M. Horodecki, Open Syst. Inf. Dyn. **12**, 231 (2005).
[28] K. Horodecki, M. Horodecki, P. Horodecki, and J. Oppenheim, Phys. Rev. Lett. **94**, 200501 (2005).
[29] A. Streltsov, H. Kampermann, and D. Bruß, Phys. Rev. Lett. **108**, 250501 (2012).