Anisotropic Optical Response of Dense Quark Matter under Rotation: Compact Stars as Cosmic Polarizers

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Quantum vortices in the color-flavor locked phase of QCD have bosonic degrees of freedom, called the orientational zero modes, localized on them. We show that the orientational zero modes are electromagnetically charged. As a result, a vortex in the color-flavor locked phase nontrivially interacts with photons. We show that a lattice of vortices acts as a polarizer of photons with wavelengths larger than some critical length.

Introduction.—The strong interaction, which is one of the fundamental forces in nature, is fully described by quantum chromodynamics (QCD). QCD matter shows a rich variety of phenomena at finite temperatures and/or baryon densities [1], and the determination of the phase diagram has been a topic of considerable interest in high-energy physics. Quark matter is expected to exhibit color superconductivity, triggered by quark-quark pairings, at high baryon densities and low temperatures [2,3]. It has been reported in Ref. [2] that the ground state is the color-flavor locked (CFL) phase at very high densities, in which the three light flavors (up, down, and strange) of quarks contribute to the pairing symmetrically. The CFL matter is both a superfluid and a color superconductor because of the spontaneous breaking of the global $U(1)_B$ baryon number symmetry and the local $SU(3)_C$ color symmetry, respectively. It is expected to exist in the cores of dense stars, although observational evidence has been elusive.

The purpose of this Letter is to propose a possible observational signal of the CFL matter. The key ingredients are the topological vortices. These vortices are created under rotation owing to the superfluidity of the CFL matter [4,5]. If the CFL phase is realized in the cores of dense stars, the creation of vortices is inevitable since the stars rotate rapidly. The superfluid vortices discussed in Refs. [4,5] were found to be dynamically unstable, decaying into sets of constituent vortices [6]. The stable ones are the so-called non-Abelian vortices, which are superfluid vortices as well as color magnetic flux tubes [7]. Their properties have been studied using the Ginzburg-Landau theory [6,8–13] or the Bogoliubov–de Gennes equation [14]. Interestingly, there are fermionic and bosonic degrees of freedom localized on a vortex. Non-Abelian vortices are endowed with a novel kind of non-Abelian statistics because of the multiple fermion zero modes trapped inside them [15]. On the other hand, the bosonic degrees of freedom are called the orientational zero modes [6,11,13],

which are the Nambu-Goldstone (NG) bosons that are associated with the symmetry breaking inside vortices.

In this Letter, we investigate the electromagnetic properties of non-Abelian vortices in the CFL phase. Although the CFL matter itself is electromagnetically neutral, the orientational zero modes are naturally charged, as is discussed later. The electromagnetic property of vortices can be phenomenologically important as it may lead to some observable effects. As an illustration of such an effect, we show that a lattice of vortices in the CFL phase acts as a polarizer of photons. The rotating CFL matter should be threaded with quantum vortices along the axis of rotation, which results in the formation of a vortex lattice [6,9,10], as in the case of rotating atomic superfluids. Suppose that a linearly polarized photon is incident on a vortex lattice (see Fig. 1). When the electric field of the photon is parallel to the vortices, it induces currents along the vortices, resulting in the attenuation of the photon; on the other hand, waves with electric fields perpendicular to the vortices are not affected. This is exactly what a polarizer does. A lattice passes electromagnetic waves of a specific polarization and blocks waves of other polarizations. This phenomenon, resulting from the electromagnetic interaction of vortices, may be useful in finding observational evidence for the existence of the CFL matter.

In the present analysis, we neglect the mixing of photons and gluons. The gauge field, A'_μ, which remains massless in the CFL phase, is a mixture of the photon A_μ and a part of gluons A^8_μ, $A'_\mu = -\sin\zeta A_\mu + \cos\zeta A^8_\mu$. Here, the mixing angle ζ is given by $\tan\zeta = \sqrt{3}g/2e$ [2], where g and e are the strong and electromagnetic coupling constants. At accessible densities ($\mu \sim 1$ GeV), the fraction of the photon is given by $\sin\zeta \sim 0.999$, and so, the massless field A'_μ consists mostly of the ordinary photon and includes a small amount of the gluon. As a first approximation, we neglect the mixing of the gluon to the massless field.
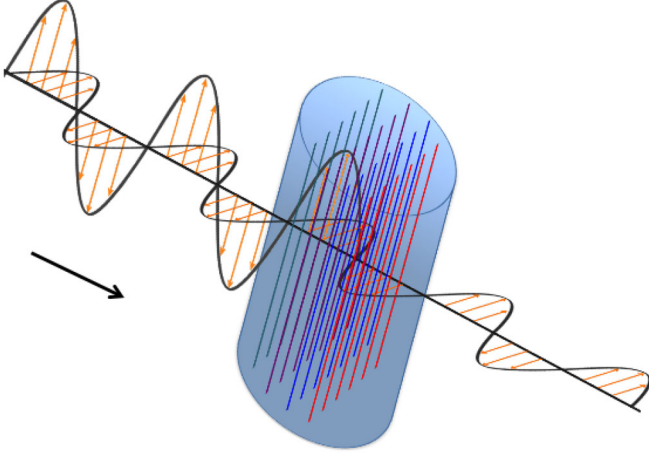
FIG. 1 (color online). Schematic figure of two linearly polarized photons entering a vortex lattice. The big arrow represents the propagating direction. The small arrows indicate the electric field vector. The waves whose electric fields are parallel to the vortices are attenuated, while the ones with perpendicular electric fields are not affected.

Orientational zero modes.—The color superconductivity is brought about by the condensation of diquarks. At very high densities, the ground state is believed to be the CFL phase, which is characterized by the spinless and positive parity condensates of the form

$$\Phi_i^a = \epsilon_{abc}\epsilon_{ijk}\langle(q^T)_b^j C\gamma_5(q)_c^k\rangle = \Delta\delta_i^a, \quad (1)$$

where q is the quark field, $i, j, k = u, d, s$ ($a, b, c = r, g, b$) are the flavor (color) indices, C is the charge conjugation matrix, Δ is a BCS gap function, and the transpose is employed with respect to the spinor index. The symmetry breaking pattern is, apart from discrete symmetry,

$$SU(3)_C \times SU(3)_R \times SU(3)_L \times U(1)_B$$
$$\rightarrow SU(3)_{C+R+L} \equiv SU(3)_{C+F}, \quad (2)$$

where $SU(3)_C$ is the color symmetry, $SU(3)_{R(L)}$ is right (left) flavor symmetry, and $U(1)_B$ is the symmetry associated with the baryon number conservation. The ground state is invariant under the simultaneous rotation of color and flavor; thus, it is called the color-flavor locked phase. In the presence of a vortex, the color-flavor locked symmetry, $SU(3)_{C+F}$, is further broken down to $SU(2)_{C+F} \times U(1)_{C+F}$ around the core of the vortex. Consequently, there appear NG modes confined in the core of the vortex, which parametrize the coset space known as the two-dimensional complex projective space [6,11],

$$\frac{SU(3)_{C+F}}{SU(2) \times U(1)} \simeq \mathbb{C}P^2. \quad (3)$$

There exist classically degenerate vortex solutions, characterized by the value of $\mathbb{C}P^2$ orientational moduli. We denote the NG modes by a complex three-component

vector $\phi \in \mathbb{C}P^2$, which satisfies $\phi^\dagger\phi = 1$. When we neglect the electromagnetic interaction, the low energy effective theory on the vortex which is placed along the z axis is shown to be described by the following $\mathbb{C}P^2$ nonlinear sigma model [11],

$$\mathcal{L}_{\mathbb{C}P^2} = C\sum_{\alpha=0,3} K_\alpha[\partial^\alpha\phi^\dagger\partial_\alpha\phi + (\phi^\dagger\partial^\alpha\phi)(\phi^\dagger\partial_\alpha\phi)], \quad (4)$$

where the orientational moduli ϕ are promoted to dynamical fields, and C and K_α are numerical constants. Under the color-flavor locked transformation, the $\mathbb{C}P^2$ fields ϕ transform as

$$\phi \rightarrow U\phi, \quad (5)$$

with $U \in SU(3)_{C+F}$.

Now, let us consider the electromagnetic fields. The electromagnetic $U(1)_{EM}$ group is a subgroup of the flavor group $SU(3)_F$, which is generated by $T_8 = \frac{1}{\sqrt{6}}\mathrm{diag}(-2, 1, 1)$ in our choice basis. The electromagnetic interaction is incorporated by gauging the corresponding symmetry. Therefore, the low-energy effective action on the vortex should be modified to the gauged $\mathbb{C}P^2$ model,

$$\mathcal{L}_{g\mathbb{C}P^2} = C\sum_{\alpha=0,3} K_\alpha[\mathcal{D}^\alpha\phi^\dagger\mathcal{D}_\alpha\phi + (\phi^\dagger\mathcal{D}^\alpha\phi)(\phi^\dagger\mathcal{D}_\alpha\phi)], \quad (6)$$

where the covariant derivative is defined by

$$\mathcal{D}_\alpha\phi = (\partial_\alpha - ie\sqrt{6}A_\alpha T_8)\phi. \quad (7)$$

Photon-vortex scattering.—Here, we investigate the consequence of the charged degrees of freedom on the vortex. The low-energy behavior is described by photons propagating in three-dimensional space and the $\mathbb{C}P^2$ model localized on the vortex. Hence, the effective action is given by

$$S = -\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}d^4x + \int \mathcal{L}_{g\mathbb{C}P^2}dzdt. \quad (8)$$

Let us consider the scattering of photons by a vortex. The equation of motion of the gauge fields derived from the effective action is given as

$$\partial^\mu F_{\mu\nu} = -CK_\nu ie\sqrt{6}\delta(x_\perp)(\delta_{0\nu} + \delta_{3\nu})\{\phi^\dagger T_8\mathcal{D}_\nu\phi$$
$$- (\mathcal{D}_\nu\phi)^\dagger T_8\phi - 2\phi^\dagger\mathcal{D}_\nu\phi\phi^\dagger T_8\phi\}, \quad (9)$$

where $\delta(x_\perp) \equiv \delta(x)\delta(y)$ is the transverse delta function. We consider the situation where a linearly polarized photon is normally incident on the vortex and assume that the electric field of the photon is parallel to the vortex. Then, the problem is z-independent and we can set $\theta = \theta(t)$, $A_t = A_x = A_y = 0$, and $A_z = A_z(t, x, y)$. The equation of motion can be rewritten as

$$(\partial_t^2 - \partial_x^2 - \partial_y^2)A_z(t, x, y)$$
$$= 12CK_3e^2\{\phi^\dagger(T_8)^2\phi + (\phi^\dagger T_8\phi)^2\}A_z(t, x, y)\delta(x_\perp)$$
$$\equiv 12CK_3e^2f(\phi)A_z(t, x, y)\delta(x_\perp), \tag{10}$$

where we have defined

$$f(\phi) \equiv \phi^\dagger(T_8)^2\phi + (\phi^\dagger T_8\phi)^2. \tag{11}$$

Equation (10) is the same as the one treated by Witten in the context of superconducting strings [16] except for the orientation-dependent factor, $f(\phi)$. The cross section per unit length, $d\sigma/dz$, can be calculated by solving the scattering problem, as in Ref. [16],

$$\frac{d\sigma}{dz} = \frac{[12CK_3e^2f(\phi)]^2\eta^2}{8\pi}\lambda = 288\pi[CK_3\alpha\eta f(\phi)]^2\lambda, \tag{12}$$

where λ is the wavelength of the incident photon, η is a numerical factor of order unity, and α, the fine structure constant. On the other hand, if the electric field of the wave is perpendicular to the vortex, the photon is not scattered since current can flow only along the vortex.

Vortex lattice as a polarizer.—Now let us consider the case where electromagnetic waves of some intensity normally enter the vortex lattice. We consider the electric fields of the waves to be parallel to the vortices. These waves are scattered by the vortices and lose intensity. The fraction of the loss of intensity when the wave passes through the lattice for distance dx is

$$\left\langle\frac{d\sigma}{dz}\right\rangle n_v dx \equiv \frac{dx}{L}, \tag{13}$$

where n_v is the number of vortices per unit area. Here, L is defined by

$$L \equiv 1\Big/\Big(n_v\Big\langle\frac{d\sigma}{dz}\Big\rangle\Big) = \ell^2\Big/\Big\langle\frac{d\sigma}{dz}\Big\rangle, \tag{14}$$

with the inter-vortex spacing ℓ. As the cross section depends on the internal state (value of φ) of the vortex, we have introduced the averaged scattering cross section $\langle d\sigma/dz\rangle$ over the ensemble of the vortices. Let us denote the intensity of waves at distance x from the surface of the lattice as $I(x)$. $I(x)$ satisfies

$$\frac{I(x + dx)}{I(x)} = 1 - \frac{dx}{L}. \tag{15}$$

Therefore, the x dependence of $I(x)$ is characterized by the following differential equation

$$\frac{I'(x)}{I(x)} = -\frac{1}{L}. \tag{16}$$

This equation is immediately solved as $I(x) = I_0e^{-x/L}$, where I_0 is the initial intensity. Hence, the waves are attenuated with the characteristic length L.

We can obtain a rough estimate of the attenuation length. The total number of vortices can be estimated, as in Ref. [5], as

$$N_v \simeq 1.9 \times 10^{19}\Big(\frac{1\text{ ms}}{P_{\text{rot}}}\Big)\Big(\frac{\mu/3}{300\text{ MeV}}\Big)\Big(\frac{R}{10\text{ km}}\Big)^2, \tag{17}$$

where P_{rot} is the rotation period; μ, the baryon chemical potential; and R, the radius of the CFL matter inside dense stars. These quantities are normalized by typical values. The intervortex spacing is given by

$$\ell \equiv \Big(\frac{\pi R^2}{N_v}\Big)^{1/2} \simeq 4.0 \times 10^{-6}\text{ m}\Big(\frac{P_{\text{rot}}}{1\text{ ms}}\Big)^{1/2}\Big(\frac{300\text{ MeV}}{\mu/3}\Big)^{1/2}. \tag{18}$$

Therefore, the characteristic decay length of the electromagnetic waves is roughly estimated as

$$L = \frac{\ell^2}{288\pi(CK_3\alpha\eta)^2\langle f(\phi)^2\rangle\lambda} \simeq \frac{6.5 \times 10^{-12}\text{ m}^2}{\lambda}, \tag{19}$$

where, we have assumed that the variable ϕ is randomly distributed in the $\mathbb{C}P^2$ space. This assumption is natural as there is no particularly favored direction in the $\mathbb{C}P^2$ space for the case with three massless flavors [17,19]. We have also taken $\eta = 1$, $\mu = 900$ MeV and $\Delta = 100$ MeV, from which the values of C and K_3 are determined accordingly [12]. If we adopt the value of $R \sim 1$ km for the radius of the CFL core, the condition that the intensity is significantly decreased within the core is written as $L \lesssim 1$ km. This condition can be rewritten in terms of the wavelength of the photon as

$$\lambda \gtrsim 6.5 \times 10^{-15}\text{ m} \equiv \lambda_c. \tag{20}$$

Therefore, a lattice of vortices serves as a wavelength-dependent filter of photons. It filters out the waves with electric fields parallel to the vortices if the wavelength λ is larger than λ_c. The waves that pass through the lattice are the linearly polarized ones with the direction of their electric fields perpendicular to the vortices, as schematically shown in Fig. 1.

One may wonder why a vortex lattice with mean vortex distance ℓ can serve as a polarizer for photons with wavelength many-orders smaller than ℓ. It is true the probability that a photon is scattered during its propagation for a small distance ($\sim \ell$, for example) is small. However, while the photon travel through the lattice, the scattering probability is accumulated and the probability that a photon remains unscattered decreases exponentially. Namely, the small scattering probability is compensated by the large number of vortices through which a photon passes. This is why the vortex mean distance and the wavelength of the attenuated photons can be different.

Conclusion.—We have shown that a quantum vortex in the CFL phase interacts with photons because of the $\mathbb{C}P^2$ mode on the vortex. We have demonstrated that, as a

consequence, photons with electric fields parallel to the vortices are attenuated in a vortex lattice. This effect would be observable if there exist quark stars in which the CFL phase continues from the core to the surface. However, if the CFL core is covered with a nuclear mantle, it would be difficult for optical probes to penetrate the surface of the star. Even in that case, we expect that the electromagnetic properties of vortices could be useful in finding observational evidence of CFL matter, for example through the electromagnetic interaction of vortices with strong magnetic fields in neutron stars.

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[19] The presence of a finite strange quark mass does not change the qualitative feature of the polarizing phenomenon. The strange quark mass gives rise to a potential in the effective model, as discussed in Ref. [12]. When m_s is larger than the typical kinetic energy of the $\mathbb{C}P^2$ modes, which is given by the temperature $T \lesssim T_c \sim 10^1$ MeV, and is small enough so that the description by the Ginzburg-Landau theory based on the chiral symmetry is still valid, the orientation of vortices falls into $\phi_0^T = (0, 1, 0)$. This assumption is valid for the realistic value of $m_s \sim 10^2$ MeV. The orientation dependence of the cross section is encapsulated in the function $f(\phi)$ defined in Eq. (11). Since $f(\phi_0) = 1/3 \neq 0$, photons still interact with the vortex in the presence of a finite strange quark mass. Assuming that all the vortices are with the orientation ϕ_0, we can redo the numerical estimates as follows. The decay length of the photon intensity is recalculated to be $L \sim (1.2 \times 10^{-11} \text{ m}^2)/\lambda$, instead of Eq. (19), and the condition that the intensity of photons is significantly decreased within the CFL core of order 1 km is given by $\lambda \gtrsim 1.2 \times 10^{-14}$ m, instead of Eq. (20).