dc Electric Invisibility Cloak

Fan Yang, Zhong Lei Mei,* and Tian Yu Jin
School of Information Science and Engineering, Lanzhou University, Lanzhou 730000, People's Republic of China

Tie Jun Cui†
Department of Radio Engineering, State Key Laboratory of Millimeter Waves, Southeast University, Nanjing 210096, People's Republic of China
(Received 20 April 2012; published 2 August 2012)

We present the first experimental demonstration of a dc electric cloak for steady current fields. Using the analogy between electrically conducting materials and resistor networks, a dc invisibility cloak is designed, fabricated, and tested using the circuit theory. We show that the dc cloak can guide electric currents around the cloaked region smoothly and keep perturbations only inside the cloak. Outside the cloak, the current lines return to their original directions as if nothing happens. The measurement data agree exceptionally well with the theoretical prediction and simulation result, with nearly perfect cloaking performance. The proposed method can be directly used to realize other dc electric devices with anisotropic conductivities designed by the transformation optics. Manipulation of steady currents with the control of anisotropic conductivities has a lot of potential applications, such as electric impedance tomography, graphene, natural resource exploration, and military camouflage.

DOI: 10.1103/PhysRevLett.109.053902 PACS numbers: 41.20.Cv, 02.40.−k

In the past few years, the concept of an electromagnetic invisibility cloak has generated tremendous attention [1–11]. Invisibility in the time domain has also been proposed and demonstrated very recently [12,13]. Readers are kindly referred to a recent review paper for more information [14]. However, most of the literature on invisibility cloaks is focused on the time-varying electromagnetic field [1–13], and only a few papers have dealt with the invisibility for static fields [15–24]. Like their time-varying counterpart, the static fields also play an important role in various sectors. For example, they are involved in photocopy machines, electrostatic spraying systems, and electric impedance tomography (EIT). They also help detect land mines and torpedoes in military applications.

In dealing with the anisotropic conductivities in EIT technology, Greenleaf *et al.* constructed anisotropic conductivities in three dimensions that give rise to the same voltage and current measurements on the boundary as those of a homogeneous and isotropic conductivity. This work is also considered one of the early works on invisibility [15]. Then in 2007, Wood and Pendry proposed a dc metamaterial design, which is based on superconducting materials, and the cloaking for static magnetic field is also suggested [16]. After that, the dc metamaterial is further experimentally validated and theoretically investigated by other groups [17,18]. Recently, the concept of antimagnet has been presented, which is actually a cloaking device for the static magnetic field [19]. However, the experimental verification of the dc magnetic cloak has not appeared until very recently (during the preparation of the current Letter). Two groups independently designed and fabricated dc cloaks for static magnetic fields, both of which were implemented using superconducting materials [20,21]. Cloaking in conducting materials was also studied by Chen, Kohn, and Li and their co-workers [22–24]. However, the above contributions are still limited to theoretical study and numerical simulations.

Here, we experimentally explore the cloaking phenomenon for the steady current fields and realize the first dc electric invisibility cloak. Using the transformation optics theory (TO) and exploiting the connection between electric conductivity in conducting materials and resistors in the circuit theory, we fabricate such a dc cloak using the resistor network. Experimental results firmly verify the correctness and effectiveness of the design. The proposed dc cloak could be useful in cloaking or detecting land mines and torpedoes, EIT imaging, and other static-field manipulations. We remark that electromagnetic (EM) invisibility cloaks made of inductor and capacitor networks have been demonstrated experimentally in the past [25], which work for the time-varying EM fields.

The working principle is shown in Fig. 1, in which Fig. 1(a) illustrates a bundle of electric currents flowing in a homogeneous and isotropic conducting material (for the sake of comparison, however, the imaginary cloak and cloaked object are given) while Fig. 1(b) demonstrates the same currents when we put an object inside the material. It is clear that the change of conductivity profile inside the material leads to a large deviation of currents, and the same bundle of lines have significantly different distributions from their original paths, which can be used to detect the object. When the dc cloak is wrapped on the object, as shown in Fig. 1(c), however, it can smoothly guide the electric currents around the object and keep the

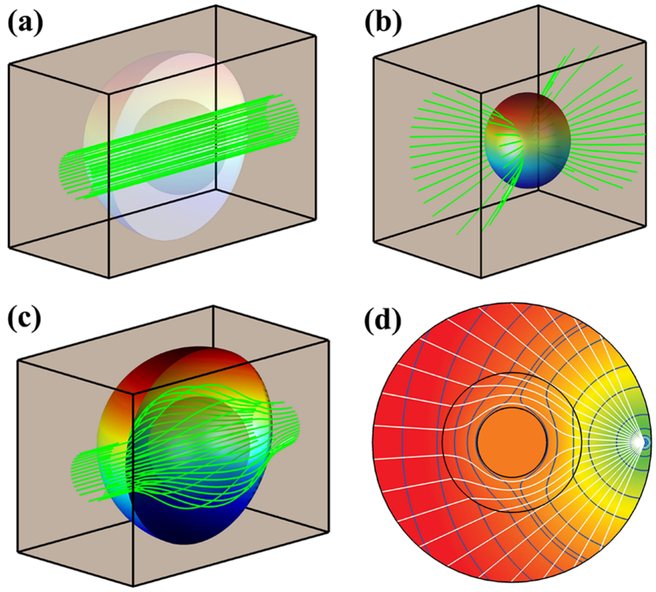

FIG. 1 (color online). The principle of dc invisibility cloak. (a) The currents in an isotropic and homogeneous conducting material (the cloak does not exist). (b) The currents distribution when a perturbation (object) exists. (c) The currents distribution when the object is covered by the cloak. (d) The equipotential lines and current density vectors on a cross section when the cloaked object is illuminated by a point source, in which the white curves (those originating from the point source) denote the current density vectors and the blue curves (those intersecting with the current density vectors) represent the equipotential lines.

perturbations only inside the cloak. Outside the cloak, the current lines return to their original direction as if nothing happens. Figure 1(d) gives a cross-section view of the current and potential distributions near the cloak excited by a point source. The same arguments apply.

As has been proved, the TO theory applies equally well to the dc field [19,21,22,24]. Using the TO theory, the above dc cloak can be easily designed by transforming the center of a sphere into another sphere while keeping the surface unchanged. For the two-dimensional case, we adopt the linear transformation between the physical and virtual spaces [1,26], which is

$$\rho' = f(\rho) = \frac{b-a}{b}\rho + a, \quad \varphi' = \varphi, \quad z' = z, \qquad (1)$$

where the cylindrical coordinate system is adopted, and a and b represent the inner and outer radii of the cloak. Then the transformed conductivity for the invisibility cloak is written as

$$\bar{\bar{\sigma}}' = \Lambda\left[\frac{\rho'-a}{\rho'}, \frac{\rho'}{\rho'-a}, \frac{\rho'-a}{\rho'}\left(\frac{b}{b-a}\right)^2\right]\bar{\bar{\sigma}}, \qquad (2)$$

in which $\Lambda(\cdot)$ represents a diagonalized tensor, the primed variables belong to the physical space, and the unprimed ones belong to the virtual space.

Equation (2) clearly shows that the realization of dc cloak requires anisotropic and inhomogeneous conductivities, which are difficult to obtain. That also explains why experimental verifications to dc electric cloak have not been reported by now. Figure 2 demonstrates the required radial and tangential components of the conductivity tensor for the dc cloak with inner and outer radii of 6 and 10 cm, respectively. Apparently, the required conductivities are difficult to be realized in nature, but we will show that they can be easily emulated using the circuit theory.

Figure 3(a) illustrates a continuous conducting material plate with the conductivity σ and thickness h, which may extend to infinity in the radial direction. The material may be inhomogeneous and anisotropic. To make an

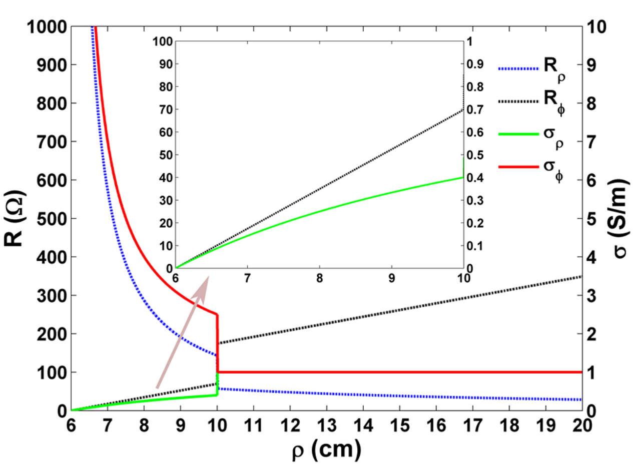

FIG. 2 (color online). Components of anisotropic conductivity tensor required for the dc invisibility cloak and their corresponding resistors. The inset gives an enlarged view for details.

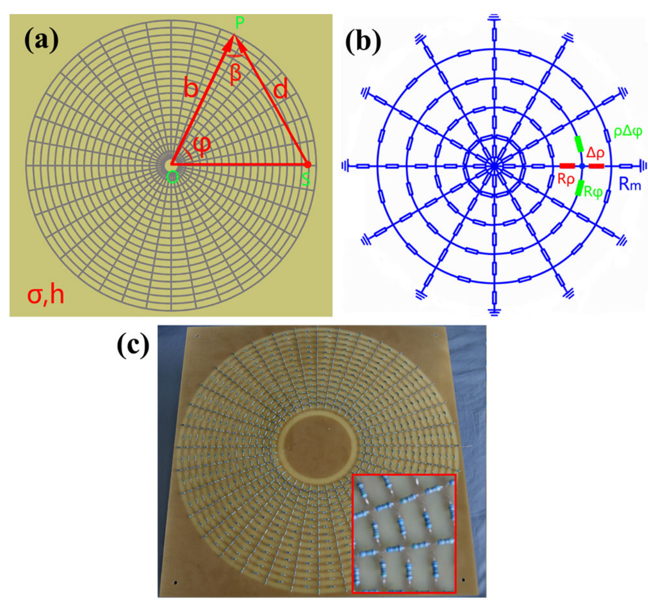

FIG. 3 (color online). A conducting material plate, its equivalent resistor network, and the fabricated device. (a) A conducting material plate with thickness h and its polar grids. (b) The equivalent resistor network of the continuous material. (c) The fabricated resistor network with an enlarged view for details. The matching resistors on the outer boundary make the network similar to an infinite material.

equivalence of the material to a resistor network, the continuous material is first discretized using the polar grids, as shown in Fig. 3(a). According to Ohm's law, each elementary cell in the grid can be implemented by two resistors

$$R_\rho = \frac{\Delta\rho}{\sigma_\rho \rho \Delta\varphi h}, \qquad R_\varphi = \frac{\rho\Delta\varphi}{\sigma_\varphi \Delta\rho h}, \tag{3}$$

where $\Delta\rho$ and $\Delta\varphi$ are step lengths in the radial and tangential directions, respectively. Thus, the anisotropic conductivity tensor can be implemented easily using different resistors in different directions, as illustrated in Fig. 3(b). To make simulations and measurements, the infinitely large material should be tailored to have a suitable size. Like the perfect matching layers in the time-varying problems, matching resistors are added in the outer ring to emulate an infinite material. Using the theorem of uniqueness, the matching resistors can be easily obtained as

$$R_m = \frac{d(\ln r_0 - \ln d)}{\sigma b h \cos\beta \Delta\varphi}, \tag{4}$$

in which r_0 is the distance between the ground and the source point S and all other variables are demonstrated in Fig. 3(a). The detailed derivation of matching resistors is given in Ref. [27]. In Fig. 2, we also give the required resistors for a dc cloak. It is clearly shown that all resistors have moderate values and can be easily obtained in electric stores.

In experimental setup, the background material has a conductivity of 1 S/m, which is cut into a circle of radius 20 cm. The cloaked region is a circle located at the center with radius $a = 6$ cm. The dc cloak has an annual shape with inner and outer radii of 6 and 10 cm, respectively. Following the above-mentioned design process, the background material together with the dc cloak is discretized into 20×36 cells using the polar grid. The fabricated resistor network is illustrated in Fig. 3(c). The network is

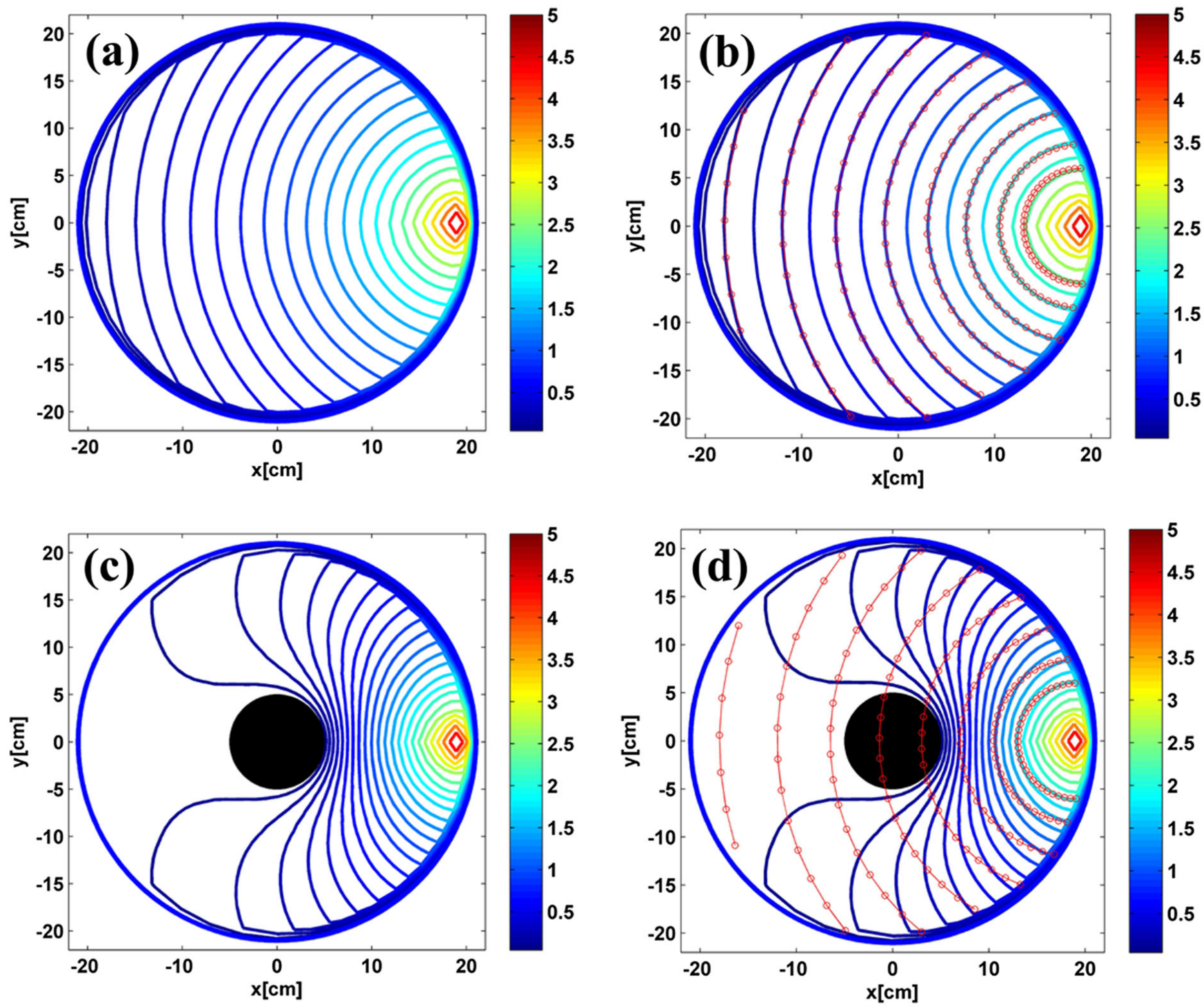

FIG. 4 (color online). The simulated potential (voltage) distributions using ADS in different scenarios. (a),(b) The isotropic and homogeneous background material, in which the concentric circles represent the ideal equipotential lines. (c),(d) The central region is connected to ground, in which the concentric circles denote the original equipotential lines. The pseudocolor represents the potential and curves denote the equipotential lines.

built on a printed circuit board (PCB) with thickness of 2 mm, which contains 20 concentric layers in the radial direction, and 36 nodes in the tangential direction. The cloaked region covers an area of 5 layers, and the cloak occupies another 5 layers. The remaining 10 layers are used for the background material. All resistors are commercial metal film resistors with an accuracy of 1%. See Ref. [27] for all resistors used in the design. A dc power supply with 5 V magnitude is connected to the network at the 19th layer. The voltage at each node is measured using a 4.5-digit multimeter.

Figures 4 and 5 demonstrate the simulation and measurement results of the resistor networks. In experiments, the potential is much easier to measure than the current density. Hence we simulated and measured the potential distributions. All simulations are performed using the commercial software, Agilent Advanced Design System (ADS). The simulation results of potential distributions in the background material under the point source excitation are shown in Fig. 4(a). As can be predicted from electrostatics, the equipotential lines are concentric circles, which confirm the correctness of the matching resistor design. This observation is quantitatively verified in Fig. 4(b), in which the accurate circles for equipotential lines are given. Clearly, they agree excellently with each other.

Then we introduce three kinds of defects or objects (changes of conductivities) in the background material: (1) the central region is connected to the ground; (2) the central area is punched out (i.e., the conductivity becomes zero); and (3) the central area is a perfect conductor (i.e., the conductivity approaches infinity). The simulation results of potential distributions for the first case are illustrated in Figs. 4(c) and 4(d), and the other two cases are given in the Supplemental Material [27]. As expected, the objects do affect the potential profiles, in which the original equipotential lines are significantly distorted, making the object visible. To shield the central region from the outside world, we put a dc invisibility cloak around the region, which is realized using the proposed equivalent resistor network. Figures 5(a) and 5(b) demonstrate the

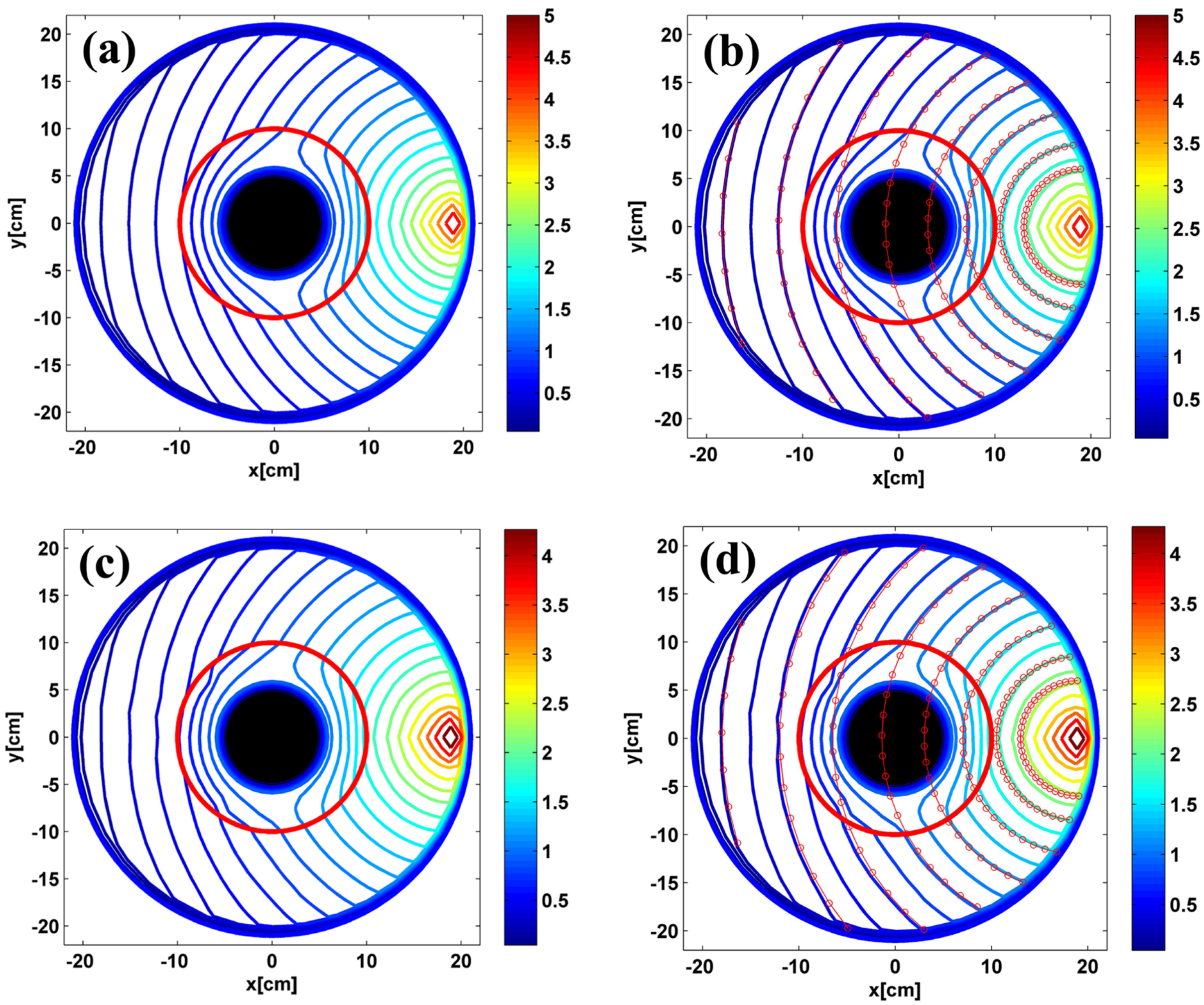

FIG. 5 (color online). The simulated and measured potential (voltage) distributions when the dc cloak exists. (a),(b) The simulated results when the central region is cloaked, in which the concentric circles denote the ideal equipotential lines. (c),(d) The measured results when the central region is cloaked, in which the concentric circles denote the ideal equipotential lines. Nearly perfect cloaking effect is observed.

simulated potential distributions of the dc cloak. It is clearly shown that the potential distributions outside the cloak restore exactly to the original equipotential lines (concentric circles), which renders the central part invisible to the outside observer.

The measurement data for the fabricated dc cloak are presented in Figs. 5(c) and 5(d), demonstrating excellent cloaking performance. A careful examination of Figs. 5(a)–5(d) shows excellent agreements between experiments and simulations, both inside the cloaking shell and outside the cloak. The very small discrepancy is mainly attributed to the following two factors: (1) the deviations of real resistors from their calculated and nominal values; and (2) the manual soldering process, which can also distort the ideal resistor distributions.

According to Eq. (4), the conductivity becomes singular in the inner boundary of the dc cloak, i.e., $\sigma_\rho = 0$ and $\sigma_\varphi = \infty$. The inner singularity has been a big problem for the time-varying cloak, in which reduced constitutive parameters have to be used [1,3]. The reduced parameters result in nearly perfect cloaking performance [3]. However, such a singularity does not pose any difficulties in the steady current field. Actually, the corresponding resistors to the singular conductivity are given by $R_\rho = \infty$ and $R_\varphi = 0$, respectively, which can be easily realized using the short and open circuits in the resistor network. In this scenario, it is clearly seen that the cloaked region is actually disconnected from the whole network, whose content does not affect the potential distributions completely. Hence the fabricated dc cloak has nearly perfect cloaking performance, as shown in Figs. 5(c) and 5(d).

In summary, we present the first experimental result of dc electric cloak for the steady current fields. We demonstrate that the manipulation of steady currents is possible with the control of inhomogeneous and anisotropic conductivities based on the TO theory. It is more important that we propose an accurate analogy between the conducting materials and resistor networks, which can be directly used to realize other dc electric devices (e.g., dc illusions and carpet cloaks) with anisotropic conductivities designed by the TO theory. Based on the modern integrated circuit technologies, it is also possible to extend the proposed method to nanoscale. Hence the proposed result has potential applications in EIT technology, graphene, natural resource exploration, and underground archaeology. The idea can also be extended to the three-dimensional case [27] and other static field manipulations.

This work was supported in part by a Major Project of the National Science Foundation of China under Grant Nos. 60990320 and 60990324, in part by the 111 Project under Grant No. 111-2-05, and in part by National High Tech (863) Projects under Grant Nos. 2011AA010202 and 2012AA030702. Z. L. Mei acknowledges the Open Research Program Funds from the State Key Laboratory of Millimeter Waves (No. K201115), Natural Science Foundation of Gansu Province (No. 1107RJZA181), the Chunhui Project (No. Z2010081), and the Fundamental Research Funds for the Central Universities (No. LZUJBKY-2012-49).

*meizl@lzu.edu.cn
†tjcui@seu.edu.cn

[1] J. B. Pendry, D. Schurig, and D. R. Smith, Science **312**, 1780 (2006).
[2] U. Leonhardt, Science **312**, 1777 (2006).
[3] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, Science **314**, 977 (2006).
[4] J. Li and J. B. Pendry, Phys. Rev. Lett. **101**, 203901 (2008).
[5] R. Liu, C. Ji, J. J. Mock, J. Y. Chin, T. J. Cui, and D. R. Smith, Science **323**, 366 (2009).
[6] H. F. Ma and T. J. Cui, Nature Commun. **1**, 21 (2010).
[7] X. Chen, Y. Luo, J. Zhang, K. Jiang, J. B. Pendry, and S. Zhang, Nature Commun. **2**, 176 (2011).
[8] J. Valentine, J. Li, T. Zentgraf, G. Bartal, and X. Zhang, Nature Mater. **8**, 568 (2009).
[9] B. Zhang, Y. Luo, X. Liu, and G. Barbastathis, Phys. Rev. Lett. **106**, 033901 (2011).
[10] L. H. Gabrielli, J. Cardenas, C. B. Poitras, and M. Lipson, Nature Photon. **3**, 461 (2009).
[11] T. Ergin, N. Stenger, P. Brenner, J. B. Pendry, and M. Wegener, Science **328**, 337 (2010).
[12] M. W. McCall, A. Favaro, P. Kinsler, and A. Boardman, J. Opt. **13**, 024003 (2011).
[13] M. Fridman, A. Farsi, Y. Okawachi, and A. L. Gaeta, Nature (London) **481**, 62 (2012).
[14] H. Chen, C. T. Chan, and P. Sheng, Nature Mater. **9**, 387 (2010).
[15] A. Greenleaf, M. Lassas, and G. Uhlmann, Physiol. Meas. **24**, 413 (2003).
[16] B. Wood and J. B. Pendry, J. Phys. Condens. Matter **19**, 076208 (2007).
[17] F. Magnus, B. Wood, J. Moore, K. Morrison, G. Perkins, J. Fyson, M. C. K. Wiltshire, D. Caplin, L. F. Cohen, and J. B. Pendry, Nature Mater. **7**, 295 (2008).
[18] C. Navau, D. X. Chen, A. Sanchez, and N. Del-Valle, Appl. Phys. Lett. **94**, 242501 (2009).
[19] A. Sanchez, C. Navau, J. Prat-Camps, and D. Chen, New J. Phys. **13**, 093034 (2011).
[20] F. Gömöry, M. Solovyov, J. Šouc, C. Navau, J. Prat-Camps, and A. Sanchez, Science **335**, 1466 (2012).
[21] S. Narayana and Y. Sato, Adv. Mater. **24**, 71 (2012).
[22] T. Chen, C. N. Weng, and J. S. Chen, Appl. Phys. Lett. **93**, 114103 (2008).
[23] R. V. Kohn, H. Shen, M. S. Vogelius, and M. I. Weinstein, Inverse Probl. **24**, 015016 (2008).
[24] J. Y. Li, Y. Gao, and J. P. Huang, J. Appl. Phys. **108**, 074504 (2010).
[25] C. Li, X. Liu, and F. Li, Phys. Rev. B **81**, 115133 (2010).
[26] U. Leonhardt and T. G. Philbin, New J. Phys. **8**, 247 (2006).
[27] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.109.053902 for a detailed deduction process, resistors in the design, and supplementary figures.