Mass Flux Characteristics in Solid ^{4}He for $T > 100$ mK: Evidence for Bosonic Luttinger-Liquid Behavior

Ye. Vekhov and R. B. Hallock

Laboratory for Low Temperature Physics, Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003, USA

(Received 25 May 2012; published 27 July 2012)

At a pressure of ~25.7 bar, the flux F carried by solid ^{4}He for $T > 100$ mK depends on the net chemical potential difference between two reservoirs in series with the solid, $\Delta\mu$, and obeys $F \sim (\Delta\mu)^b$, where $b \approx 0.3$ is independent of temperature. At fixed $\Delta\mu$ the temperature dependence of the flux F can be adequately represented by $F \sim -\ln(T/\tau)$, $\tau \approx 0.6$ K, for $0.1 \leq T \leq 0.5$ K. A single function $F = F_0(\Delta\mu)^b \ln(T/\tau)$ fits all of the available data sets in the range 25.6–25.8 bar reasonably well. We suggest that the mass flux in solid ^{4}He for $T > 100$ mK may have a Luttinger liquidlike behavior in this bosonic system.

DOI: 10.1103/PhysRevLett.109.045303 PACS numbers: 67.80.bd, 71.10.Pm

Following the measurements of Kim and Chan [1,2] and the interpretation of the possible existence of a supersolid [3], there has been renewed interest in solid ^{4}He. Some have questioned the supersolid interpretation and imply that some experiments carried out to date may show no clear or only weak direct evidence for supersolid behavior [4,5]. Experiments designed to create flow in solid ^{4}He in confined geometries by directly squeezing the solid lattice have not been successful [6–9]. We took a different approach and, by creation of chemical potential differences across bulk solid samples in contact with superfluid helium, have demonstrated mass transport by measuring the mass flux F through a cell filled with solid ^{4}He [10,11] at temperatures that extend to values above those where torsional oscillator or other experiments have focused attention. Indeed, these experiments revealed interesting temperature dependence [12,13] in the vicinity of 80 mK, where the major changes in torsional oscillator period or shear modulus [14] were seen.

Here, we seek to understand the behavior of F for $T > 100$ mK in more detail. We apply a temperature difference, ΔT, to create an initial chemical potential difference, $\Delta\mu_0$, between two superfluid-filled reservoirs in a series with a cell filled with solid ^{4}He. We then measure in some detail the behavior of the ^{4}He flux through the solid-filled cell for $T > 100$ mK that results from the imposed ΔT as the pressure difference between the two reservoirs changes (the fountain effect) and the chemical potential difference between the two reservoirs, $\Delta\mu$, changes from $\Delta\mu_0$ to zero. For $T > 100$ mK, modest period shifts have been seen in a number of torsional oscillator experiments, in some cases even above 400 mK [15].

Since the apparatus [12,13] used for this work has been described in detail previously, our description here will be concise. A temperature gradient is present across the superfluid-filled Vycor [16–18] rods $V1$ and $V2$ (Fig. 1), which ensures that the reservoirs $R1$ and $R2$ remain filled with superfluid, while the solid-filled cell (1.84 cm^3) remains at a low temperature. For the present experiments, a chemical potential difference can be imposed by the creation of a temperature difference, $\Delta T = T1 - T2$, between the two reservoirs. The resulting change in the fountain pressure [19] between the two reservoirs results in a mass flux through the solid-filled cell to restore equilibrium. The experimental protocol is designed to minimize what has been described as the "syringe effect"[20,21] by which sequential net injections of atoms to the cell increase the density of the solid. By a reduction in the base temperatures of $R1$ and $R2$, we also eliminate the flow restriction that would be present for too high a Vycor temperature [13].

To fill the cell initially, the helium gas (ultrahigh purity, assumed to contain ~300 ppb ^{3}He) is condensed through a direct-access heat-sunk capillary (not shown in Fig. 1). To grow a solid at constant temperature from the superfluid, which is our standard technique, we begin with the pressure in the cell just below the bulk melting pressure

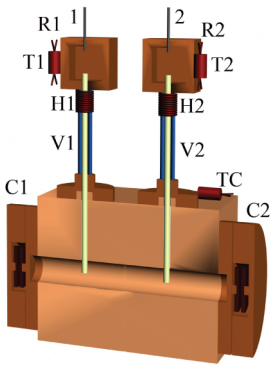

FIG. 1 (color online). Diagram of the apparatus. The pressure of the solid is measured by capacitance strain gages [33] C1 and C2, the pressures in the superfluid-filled reservoirs, $P1$, $P2$, are measured at room temperature, and the sample cell temperature is measured by thermometer TC. (Not to scale; $V1$ and $V2$ are longer than shown here.)

for ^{4}He at the growth temperature and then add atoms simultaneously through lines 1 and 2. Once we have created a solid at the desired pressure, we close the fill lines and change the cell temperature.

With stable solid ^{4}He in the cell, we use heaters $H1$ ($H2$) to vary $T1$ ($T2$) to create chemical potential differences between the reservoirs and then measure the resulting changes [19] in the pressures $P1$ and $P2$. An example of the behavior seen from an application of this approach is shown in Fig. 2, where $P1$, $P2$, $\Delta P = P1 - P2$ and $T1$ and $T2$ are shown as a function of time. A baseline reservoir temperature is first selected, T_0, with $T1 = T2 = T_0$. Then $T1$ is decreased by δT, while $T2$ is increased by the same interval: $\Delta T = T1 - T2 = -2\delta T$. After chemical potential equilibrium is reached (A, Fig. 2), the values of $T1$ and $T2$ are interchanged (B, Fig. 2): $\Delta T = T1 - T2 = +2\delta T$. Later δT is changed by a small amount (C, Fig. 2) and the process is continued as time elapses. With each switch in the value of $T1 - T2$ there is a response of $P1 - P2$. This approach is expected to create a smaller perturbation on the solid and allows us to obtain larger $\Delta\mu$ values without exceeding the upper Vycor temperature at which a significant flow limitation is encountered [13]. We take $F = d(P1 - P2)/dt$ to be proportional to the flux of atoms that passes through the solid. We study F as a function of T and $\Delta\mu$, the chemical potential difference between $R1$ and $R2$, where $\Delta\mu = m_4[\int(dP/\rho) - \int(sdT)]$, where m_4 is the ^{4}He mass, ρ is the density, and s is the entropy per unit mass. We report $\Delta\mu$ in units of J/g instead of J/atom. We will report our flux values in mbar/s, where a typical value of 0.1 mbar/s corresponds to a mass flux through the cell of $\approx 4.8 \times 10^{-8}$ g/sec.

We typically consider the data in two ways: (1) F as a function of ΔT at a sequence of fixed solid ^{4}He temperatures and (2) F for fixed ΔT as a function of T. For data of the first sort, we measure the dependence of the flux F on the imposed temperature difference between the reservoirs $R1$ and $R2$, ΔT. Following the application of the imposed ΔT, the system responds with a flux to create an increasing ΔP (the fountain effect). Thus the net chemical potential difference $\Delta\mu$ between the two reservoirs decreases to zero as ΔP increases. Since the flux should depend on $\Delta\mu$ we document that behavior. An example of the relationship between F and $\Delta\mu$ is shown in Fig. 3 for several solid ^{4}He temperatures. These data have error bars that are related to our ability to determine the flux from the measured $d(P1 - P2)/dt$ and this becomes more difficult at small values of $\Delta\mu$. We find that a reasonable characterization of the data is given by $F = A(\Delta\mu)^b$. The results of fitting the data to this functional form for several sets of data at various cell temperatures are shown along with the data in Fig. 3. We find that A has temperature dependence, but that the exponent b is constant within our errors, as is illustrated in Fig. 4 for several data sets. We conclude from the behavior seen in Fig. 3, which we have seen in other samples, that our measurements here are primarily in the dissipative regime. This dissipation may come from phase slippages, a many-body tunneling phenomena expected in a superfluid-like system. Historically, some have explored the approach to the dissipative regime in a superfluid system by study of the flow velocity associated with a pressure gradient [22] or a decreasing gravitational pressure head[23], under quasi-isothermal conditions. An interchange of the axes of Fig. 3 is reminiscent of such studies.

Next, we study F vs. T for several fixed values of $\Delta\mu$. We find that the data of this sort can be reasonably well

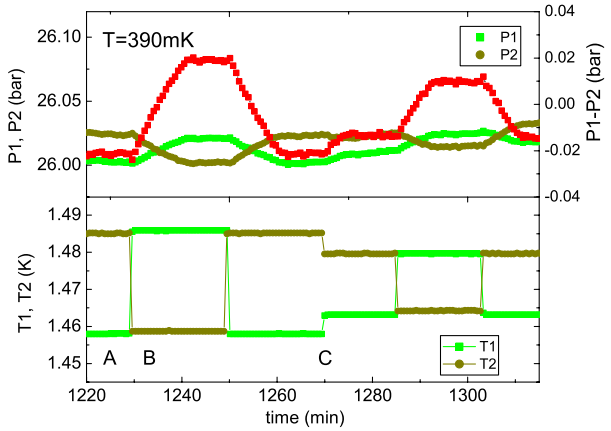

FIG. 2 (color online). Response of pressures $P1$ and $P2$ to the application of two different δT; $T = 390$ mK. Use of heaters $H1$ and $H2$ results in changes in $T1$ and $T2$. The resulting changes in $P1$ and $P2$ are best seen as $P1 - P2$, shown here (uppermost data). The small drift in $P1$, $P2$ of the sort seen here is typical and variable and appears to have no influence on $P1 - P2$.

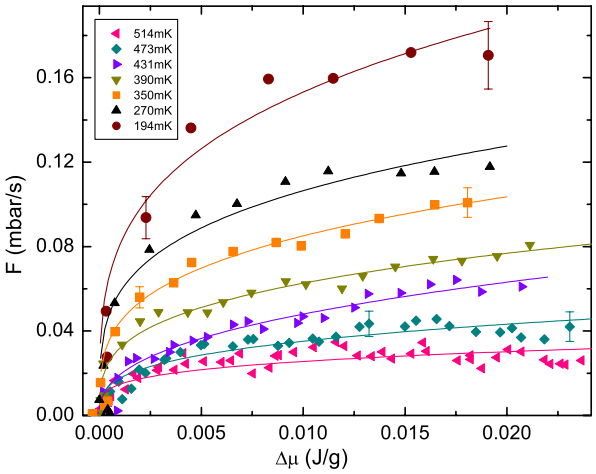

FIG. 3 (color online). Values of $F = d(P1 - P2)/dt$ for a solid sample at a cell pressure of $(C1 + C2)/2 = 25.6$–25.8 bar shown as a function of $\Delta\mu$ determined for the case $\Delta T = 27$ mK; $|\Delta T| = 2\delta T$. Small shifts in $\Delta\mu$ have been applied to align the data at $\Delta\mu = 0$. The flux depends on $\Delta\mu$ and can be represented by $F = A(\Delta\mu)^b$.

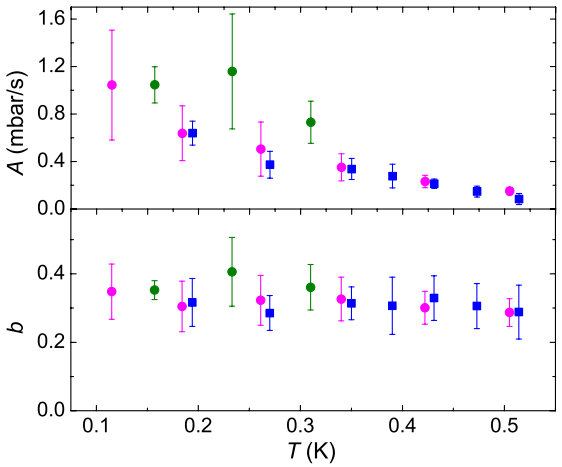

FIG. 4 (color online). Values of the fit parameters A and b as a function of temperature as determined from data including that in Fig. 3 (squares), obtained from a stable sample studied sequentially over several days; different symbols are for data separated by helium transfers to the apparatus. Here data that results from $|\Delta T|$ values in the range 10–37.5 mK have been averaged at each T. For these data sets b is nearly independent of temperature with an average value ≈ 0.32.

represented by a function of the form $F \sim -\ln(T/\tau)$, where τ is a fitting parameter. As a specific example, in Fig. 5 we show data deduced from that shown in Fig. 3 for which we find that $\tau \approx 630 \pm 20$ mK, which is consistent with earlier observations in this pressure range [13], which showed no evidence for flux above ≈ 650 mK. Typically in the temperature range studied here the behavior of the flux is reproducible and not hysteretic; for a given value of $\Delta\mu$ one can reproduce the measured $F(T)$ value (e.g., shown in Figs. 3 and 5) for increases or decreases in temperature. But, at times the flux becomes unstable and

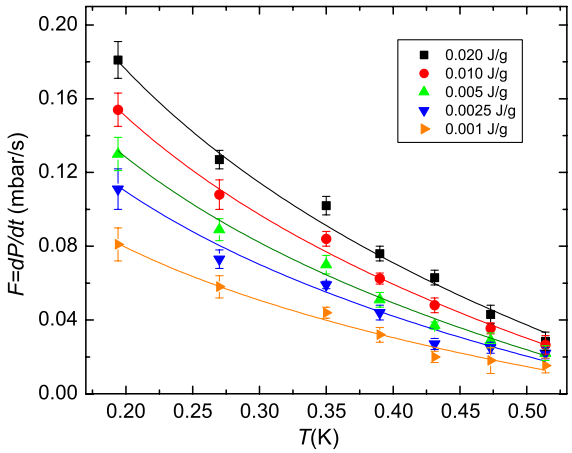

FIG. 5 (color online). F determined at $(C1 + C2)/2 = 25.6$–25.8 bar as a function of T for different values of $\Delta\mu$, interpolated from the data shown in Fig. 3, for the case of an applied $|\Delta T| = 27$ mK. These and data for other imposed ΔT values can be represented by $F \sim -\ln(T/\tau)$.

can fall to values which are indistinguishable from zero. Once this happens, or the temperature is raised above $\sim$650 mK and then lowered, it almost never is the case that a finite value of the flux will reappear when the temperature is lowered unless net atoms are added to or removed from the cell, which presumably changes the disorder in the solid.

We find that a single function $F = F_0(\Delta\mu)^b \ln(T/\tau)$ fits the data in the range 25.6–25.8 bar reasonably well. From simultaneous fits to the data for the dependence on $\Delta\mu$ and T for the data of Figs. 3 and 5, we find average values for the parameters to be $F_0 = -0.50 \pm 0.03$ mbar/s, $b = 0.29 \pm 0.01$ and $\tau = 0.63 \pm 0.01$ K.

The behavior of F vs. $\Delta\mu$ appears to be qualitatively different from the behavior seen by Sasaki *et al.* [24] for flow along grain boundaries on the ^{4}He melting curve; our flux decreases with decreasing $\Delta\mu$, theirs did not. In a saturated vapor pressure study of the decay of a superfluid ^{4}He level, h, due to flow through narrow ($\sim$ 2–5 μm) slits formed between two flat plates Rorschach [25] found that $dh/dt = ah^{1/3}$, a dependence on h consistent with Gorter-Mellink friction. Presumably $\Delta\mu \sim h$ in each case. The temperature dependence found by Rorschach closely followed the temperature dependence of the bulk superfluid density, which is distinctly different from the temperature dependence we observe, $F \sim -\ln(T/\tau)$.

Given the fact that we have a solid-filled cell off the melting curve, an unusual dependence on T, dissipative flux and sensitivity to disorder, we suggest a possible explanation for our observations. In confined one-dimensional geometries some authors have predicted that liquid helium might behave as a Luttinger liquid [26,27]. For example, in the context of liquid helium-filled carbon nanotubes, Del Maestro *et al.* [27] have recently used quantum Monte Carlo simulations to show that Luttinger liquid-like behavior should be present, with a Luttinger parameter that depends on the pore diameter. And Boninsegni *et al.* [28] have predicted that Luttinger-like behavior will be present in the cores of screw dislocations in solid helium.

In one dimension, the macroscopic behavior of bosons and fermions is the same [29]. A Luttinger liquid is traditionally thought of as a one-dimensional fermionic conductor in which the relaxation of current is due to the back scattering of single fermions dressed with bosonic phonon-type modes. The bosonic counterpart of this picture is the relaxation of a supercurrent due to quantum phase slippages [30,31]. In the low temperature limit in the presence of a finite chemical potential difference, a Luttinger liquid is predicted to carry a non-Ohmic current I, of the form $I \sim (\delta\mu)^p$, where $\delta\mu$ is the driving chemical potential difference, e.g., the applied voltage, and where p is a constant related to the Luttinger liquid parameter, g. For a single conduction channel with Luttinger liquid behavior, one expects such behavior for $k_B T/\hbar \ll J$, where J is the

flux in atoms/s. For our work, e.g., at $T \sim 0.2$ K, with $\Delta\mu \approx 0.01$ J/g, we have a flux of $J \sim 7 \times 10^{15}$ atoms/sec. $T \sim 0.2$ K results in $k_B T/\hbar = 2.6 \times 10^{10}$. This indicates that for Luttinger liquid behavior to be relevant to our results, the effective number of conducting channels that carry flux, N, should be $\lesssim 2 \times 10^5$. Estimating the effective diameter of a channel [28], this in turn indicates that the flow velocity in a given channel should be $\gtrsim 200$ cm/sec.

For a Luttinger liquid in the quantum regime where g is less than unity, and the impedance is due to impurities, Kane and Fisher [30] predict that $p = 2/g - 1$, independent of temperature. The data presented in Fig. 4, with $b = p \approx 0.32$, imply that $g \approx 1.52$ according to the above criterion. On the other hand, more recent work [32] based on an effective Hamiltonian of phase slippages suggests (for the impedance due to impurities) $p = 1/(2g - 1)$, which implies $g \approx 2.06$. [The result $p = 1/(2g - 1)$ is implied [32] by the impurity correction for $g > 1$ considered by Kane and Fisher [30].] If in our temperature range, we assume that the flux in solid helium is carried by the superfluid cores [28] of edge dislocations [20], a one-dimensional model seems relevant. Typically, one thinks of Luttinger liquids as finite-length one-dimensional systems. Here, the picture would perhaps be of a series of connected one-dimensional segments, e.g., dislocation cores.

In summary, we find that at constant solid ^{4}He temperature, the flux of atoms that pass through a cell filled with solid ^{4}He can be reasonably represented by $F = A(\Delta\mu)^b$, with the exponent b independent of temperature. We also find that at fixed $\Delta\mu$, the temperature dependence of the flux can be rather well represented by $F = f_0 \ln(T/\tau)$, consistent with the extinction of the flux above a characteristic temperature, τ. We suggest that solid helium in the temperature and pressure range of this study may be an example of a Bosonic Luttinger liquid. The relationship between this work and the many recent torsional oscillator experiments in solid helium is not clear.

We thank M. W. Ray for his previous work on the apparatus and helpful comments, B. Svistunov for a number of stimulating discussions, and N. Mikhin for technical advice. This work was supported by NSF Grants No. DMR 08-55954 and No. DMR 07-57701 and by Research Trust Funds administered by the University of Massachusetts Amherst.

[1] E. Kim and M. Chan, Nature (London) **427**, 225 (2004).
[2] E. Kim and M. Chan, Science **305**, 1941 (2004).
[3] A. J. Leggett, Phys. Rev. Lett. **25**, 1543 (1970).
[4] O. Syshchenko, J. Day, and J. Beamish, Phys. Rev. Lett. **104**, 195301 (2010).
[5] J. D. Reppy, Phys. Rev. Lett. **104**, 255301 (2010).
[6] D. S. Greywall, Phys. Rev. B **16**, 1291 (1977).
[7] J. Day, T. Herman, and J. Beamish, Phys. Rev. Lett. **95**, 035301 (2005).
[8] J. Day and J. Beamish, Phys. Rev. Lett. **96**, 105304 (2006).
[9] Ann Sophie C. Rittner, W. Choi, E. J. Mueller, and J. D. Reppy, Phys. Rev. B **80**, 224516 (2009).
[10] M. W. Ray and R. B. Hallock, Phys. Rev. Lett. **100**, 235301 (2008).
[11] M. W. Ray and R. B. Hallock, Phys. Rev. B **79**, 224302 (2009).
[12] M. W. Ray and R. B. Hallock, Phys. Rev. Lett. **105**, 145301 (2010).
[13] M. W. Ray and R. B. Hallock, Phys. Rev. B **84**, 144512 (2011).
[14] J. Day and J. Beamish, Nature (London) **450**, 853 (2007).
[15] A. Penzev, Y. Yasuta, and M. Kubota, Phys. Rev. Lett. **101**, 065301 (2008).
[16] J. R. Beamish, A. Hikata, L. Tell, and C. Elbaum, Phys. Rev. Lett. **50**, 425 (1983).
[17] Cao Lie-zhao, D. F. Brewer, C. Girit, E. N. Smith, and J. D. Reppy, Phys. Rev. B **33**, 106 (1986).
[18] E. Adams, Y. Tang, K. Uhlig, and G. Haas, J. Low Temp. Phys. **66**, 85 (1987).
[19] M. W. Ray and R. B. Hallock, Phys. Rev. B **82**, 012502 (2010).
[20] Ş. G. Söyler, A. B. Kuklov, L. Pollet, N. V. Prokof'ev, and B. V. Svistunov, Phys. Rev. Lett. **103**, 175301 (2009).
[21] M. W. Ray and R. B. Hallock, Phys. Rev. B **81**, 214523 (2010).
[22] J. N. Kidder and W. M. Fairbank, Phys. Rev. **127**, 987 (1962).
[23] E. Flint and R. B. Hallock, Phys. Rev. A **10**, 1285 (1974).
[24] S. Sasaki, R. Ishiguro, F. Caupin, H. Maris, and S. Balibar, Science **313**, 1098 (2006).
[25] H. Rorschach, Phys. Rev. **105**, 785 (1957).
[26] A. Del Maestro and I. Affleck, Phys. Rev. B **82**, 060515(R) (2010).
[27] A. Del Maestro, M. Boninsegni, and I. Affleck, Phys. Rev. Lett. **106**, 105303 (2011).
[28] M. Boninsegni, A. B. Kuklov, L. Pollet, N. V. Prokof'ev, B. V. Svistunov, and M. Troyer, Phys. Rev. Lett. **99**, 035301 (2007).
[29] F. D. M. Haldane, Phys. Rev. Lett. **47**, 1840 (1981).
[30] C. L. Kane and M. Fisher, Phys. Rev. Lett. **68**, 1220 (1992).
[31] V. A. Kashurnikov, A. I. Podivaev, N. V. Prokofev, and B. V. Svistunov, Phys. Rev. B **53**, 13091 (1996).
[32] B. V. Svistunov and N. V. Prokof'ev (to be published).
[33] G. C. Straty and E. E. Adams, Rev. Sci. Instrum. **40**, 1393 (1969).