Magnetic Dipole Moment of the Doubly-Closed-Shell Plus One Proton Nucleus ^{49}Sc

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(Received 21 April 2012; published 19 July 2012)

The nucleus ^{49}Sc, having a single $f_{7/2}$ proton outside doubly magic ^{48}Ca ($Z = 20$, $N = 28$), is one of the very few isotopes which makes possible testing of the fundamental theory of nuclear magnetism. The magnetic moment has been measured by online β NMR of nuclei oriented at milli-Kelvin temperatures to be (+)5.616(25) μ_N. The result is discussed in terms of a detailed theory of the structure of the magnetic moment operator, showing excellent agreement with calculated departure from the $f_{7/2}$ Schmidt limit extreme single-particle value. The measurement completes the sequence of moments of Sc isotopes with even numbers of $f_{7/2}$ neutrons: the first such isotopic chain between two major shells for which a full set of moment measurements exists. The result further completes the isotonic sequence of ground-state moments of nuclei with an odd number of $f_{7/2}$ protons coupled to a closed subshell of $f_{7/2}$ neutrons. Comparison with a recent shell-model calculation of the latter sequence is made.

DOI: 10.1103/PhysRevLett.109.032504 PACS numbers: 21.10.Ky, 21.60.Cs, 27.40.+z, 29.30.Lw

The extreme single-particle model of atomic nuclei gives estimates of nuclear magnetic dipole moments using the expression $\boldsymbol{\mu} = (g_l\mathbf{l} + g_s\mathbf{s})\ \mu_N$, where $\mathbf{l}$ and $\mathbf{s}$ are the orbital and spin-angular momenta of the single particle, and g_l and g_s are the appropriate g factors ($g_l = 1/0$ and $g_s = 5.586/-3.826$ for protons/neutrons). This gives rise to the Schmidt limits. Experimentation has shown that moments do not agree well with these estimates, and the observed deviations have been the subject of extensive theoretical study. These deviations are of two classes. The first is the mixing of single-particle configurations which, even if small, can have considerable impact on measured moments. The second is more fundamental in that it involves corrections to the magnetic moment operator in finite nuclei that arise from the fact that the nucleons are interacting with each other through the exchange of mesons.

Investigation of the second class of effects can only be made by measurements on states in which the first can be well controlled; that is, configuration mixing can be expected to be small and reliably estimated. In practice, this means making moment measurements on ground states of nuclei having a single odd nucleon outside a double-magic core.

This Letter describes such a measurement, of the magnetic dipole moment of the ^{49}Sc ground state by the technique of online low-temperature nuclear orientation combined with nuclear magnetic resonance. The structure of the ground state of ^{49}Sc has been established as a single $f_{7/2}$ proton coupled to the double-magic ^{48}Ca core. The experiment was performed at the NICOLE online nuclear orientation facility at ISOLDE (CERN). The result is discussed in terms of a detailed theory of the structure of the magnetic moment operator, showing excellent agreement with calculated departure from the $f_{7/2}$ Schmidt limit prediction.

A clean ^{49}Sc beam is not readily available at ISOLDE so the activity was produced by decay of an implanted ^{49}Ca beam. The 1.4 GeV proton beam on an ISOLDE UC$_X$ target with W surface ionizer produced both ^{49}Ca ($I^\pi = 3/2^-$, $T_{1/2} = 8.7$ m) and ^{49}K ($I^\pi = 1/2^+, 3/2^+$, $T_{1/2} = 1.26$ s). Unwanted ^{49}K $\beta^- n$ activity was eliminated by blocking the beam for 5 s after each proton pulse. ^{49}Ca decays primarily through an allowed β transition of end-point energy 2178 keV to the 3084 keV $3/2^-$ state in ^{49}Sc, followed by a pure E2 transition to the ($I^\pi = 7/2^-, T_{1/2} = 57.2$ m) ground state. 99.9% of the ^{49}Sc activity decays direct to the $7/2^-$ ground state of ^{49}Ti by allowed β decay of end-point energy 2006 keV. Thus, there is no measurable γ decay from ^{49}Sc, and the β spectrum cannot readily be separated from that of ^{49}Ca.

The ^{49}Ca beam was implanted into a pure iron foil soldered to the cold finger of the NICOLE online dilution refrigerator, which can maintain temperatures to below 10 mK. The foil was magnetized by a horizontal field in the plane of the foil that defined the axis of polarization of the implanted activity. A weak source of ^{60}Co in a Co single crystal was mounted on the back of the cold finger. Anisotropy of the 1333 keV and 1173 keV γ transitions detected in Ge detectors at 0° and 90° to the polarization axis acted as nuclear orientation thermometers. To detect polarization in the β activity, 4-mm thick, 19-mm diameter Ge detectors were used, set inside the cryostat at angles of approximately 15° and 165° to the polarization axis and operating at close to 4 K.

To estimate the observable β asymmetries and γ anisotropies from the ^{49}Ca and ^{49}Sc activities, we consider the expressions

$$\begin{aligned} W(\theta) &= 1 + A_1 B_1 P_1(\cos\theta) \ \ (\beta), \\ W(\theta) &= 1 + A_2 B_2 P_2(\cos\theta) + A_4 B_4 P_4(\cos\theta) \ \ (\gamma). \end{aligned} \tag{1}$$

Here, the B_λ are orientation parameters which reflect the degree of polarization of the nuclei in the sample and are functions of the nuclear magnetic dipole moment, the hyperfine field acting at the nuclei, the nuclear spin, and the lattice temperature. The A_λ are angular distribution coefficients, determined by the spins of the levels involved and the multipolarity of transitions, both observed and intermediate unobserved, P_λ are the ordinary Legendre polynomials, and θ is measured from the polarization axis. It is also necessary to consider whether the nuclei, implanted with random orientation, live long enough, compared to the nuclear spin-lattice relaxation time T_1, to become polarized in thermal equilibrium with the foil before they decay. This involves estimation of the nuclear spin-lattice relaxation time T_1 for each system.

To take ^{49}Ca first, the magnetic dipole moment ($-1.38\ \mu_N$ [1]) and hyperfine field in Fe (~ 10 T [2]) along with the value $A_1 = 0.298$ for the pure $3/2^- \rightarrow 3/2^-$ GT β transition predict a thermal equilibrium β asymmetry of close to 25% at 10 mK. Small A_2 and A_4 parameters lead to 3084 keV γ anisotropy less than 1% at 10 mK. Based on the measured T_1 for Sc in iron of 350 (70) s at 10 mK [3], combined with the fact that T_1 scales with the inverse square of the hyperfine splitting, we estimate T_1 for ^{49}Ca to be $\sim$1700 s much longer than the half-life of 520 s. Thus, the observable ^{49}Ca β asymmetry will be seriously reduced, with an expected value at 10 mK of $\sim$10%.

Turning to ^{49}Sc, the magnetic dipole moment should be close to that of ^{47}Sc ($+5.34\ \mu_N$ [4]) and the hyperfine field is precisely known $\{-13.17(5)$ T [3]$\}$. The A_1 coefficient for a pure $7/2^- \rightarrow 7/2^-$ GT β decay is small, 0.145. These parameters lead to a predicted asymmetry of $\sim$25% at 10 mK. The value of T_1, (350 s at 10 mK, taken equal to that of ^{47}Sc) is far shorter than the lifetime of the ground state so that complete thermal equilibrium will be achieved and the full asymmetry observed at all attainable temperatures.

There is a further complication in determining the observable β asymmetry. The sign of the magnetic moment of ^{49}Ca is opposite to that of ^{49}Sc, so their asymmetries will tend to cancel, given the similar end-point energy. Since only ^{49}Sc activity will show resonance in the explored frequency range (NMR of the ^{49}Ca would be at a frequency around 70 MHz), the presence of ^{49}Ca simply dilutes the resonance signal. To avoid this, and also because the cold finger of the NICOLE dilution refrigerator attains lower temperature when the beam is cut off, cyclic sample preparation for a period, followed by decay of the ^{49}Ca component and subsequent measurement of the remaining ^{49}Sc activity, was adopted. A series of cycles was made with β and γ data taken throughout, as illustrated in Fig. 1. The upper panel shows the counts recorded for the 3084 keV ^{49}Ca γ transition. The beam was cut off after about 50 minutes and the ^{49}Ca activity became small after about 90 minutes. The center panel shows the observed β asymmetry (the ratio of counts in the 15° and 165°

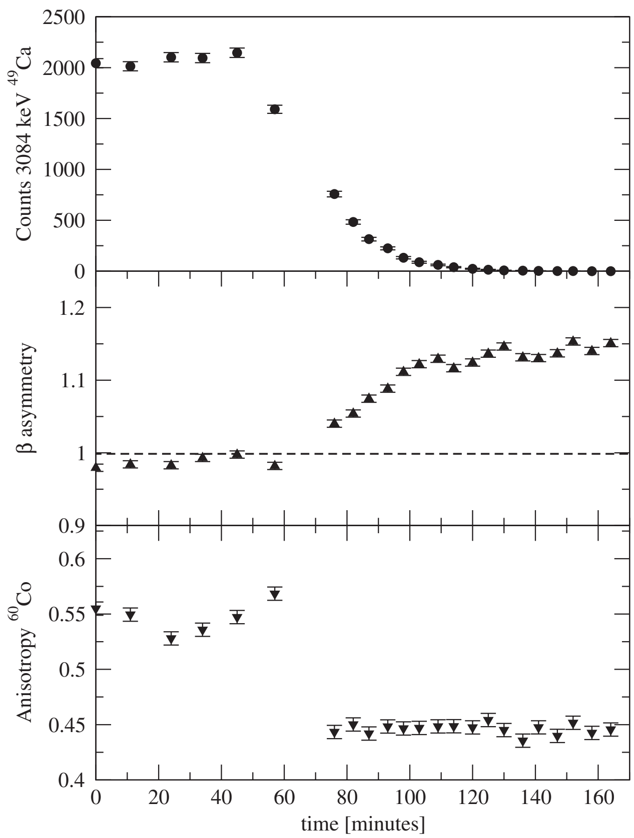

FIG. 1. Production of the sample of almost pure ^{49}Sc activity following the beam cutoff. Upper: Decrease of 3084 keV ^{49}Ca γ counts. Center: Combined ^{49}Ca + ^{49}Sc β asymmetry showing change from a negative few % to a positive $\sim$15% effect. Lower: Anisotropy in the ^{60}Co γ transition at 1173 keV showing a drop in implant foil temperature.

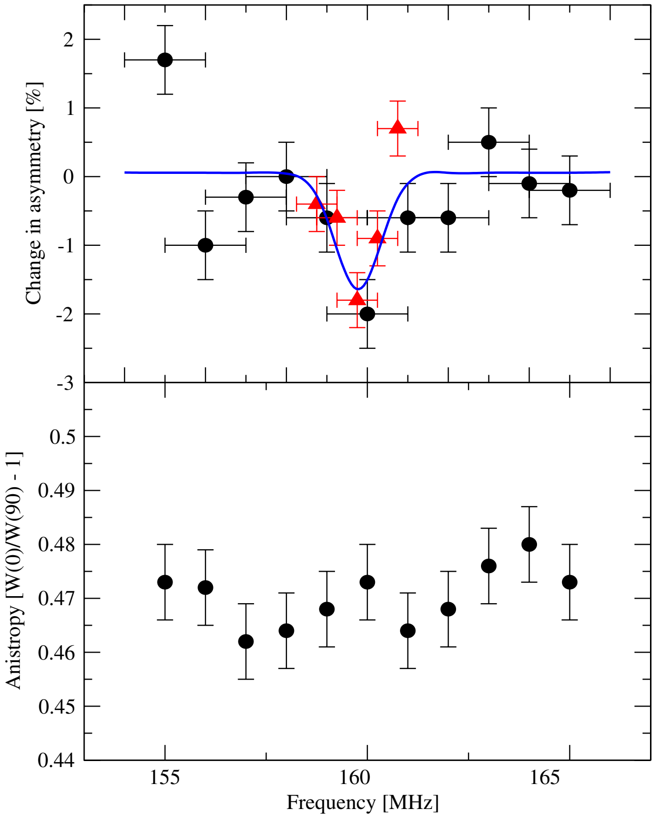

FIG. 2 (color online). Upper: Resonance of ^{49}Sc β activity in Fe. Black (circular) data are from the wide (1 MHz) step search and red (triangular) data from the narrow (0.5 MHz) step search. Horizontal bars—FM width. Vertical bars—statistical error. Lower: Anisotropy of the 1173 keV transition in ^{60}Co showing constancy of temperature during the resonance search.

β detectors) normalized to the ratio taken with the sample close to 1 K. The small negative asymmetry found during implantation of the ^{49}Ca activity (the first six data points taken when ^{49}Ca and ^{49}Sc activities were close to temporal equilibrium) was consistent with the expected close-to-cancellation of the asymmetries from the two isotopes. Closing off the beam revealed the asymmetry of ^{49}Sc, showing a ratio about 1.15, i.e., greater than unity, and steady after about 90 minutes. The lower panel of Fig. 1 shows the anisotropy observed in the pure E2 1173 keV transition of ^{60}Co. All parameters in the orientation and decay of this transition are known, so it serves as a thermometer. The data show a steady value (about 9 mK) during the implantation stage, falling to about 7 mK and remaining stable after implantation stops. During this latter period, the search for resonance of the Sc activity was carried out.

In a previous paper, we reported resonance on ^{47}Sc in iron at 150.94(1) MHz in an applied field of 0.2 T [3] with FWHM close to 1 MHz. The magnetic moment of ^{47}Sc is 5.34(2) μ_N [4]. The Schmidt limit prediction for the moment of a $7/2^-$ proton, which constitutes an effective upper limit for the search, is 5.794 μ_N. These indicated a search range of 145–165 MHz.

TABLE I. Magnetic dipole moments of odd-A Sc isotopes and $N = 28$ isotones.

A	Scandium Neutrons	Moment (μ_N)	Reference
^{41}Sc	0	5.431(2)	[5]
^{43}Sc	2	4.62(4)	[6]
^{45}Sc	4	4.756 487(2)	[7]
^{47}Sc	6	5.34(2)	[4]
^{49}Sc	8	5.616(25)	This work
B	$N = 28$ Protons	Moment (μ_N)	Reference
^{49}Sc	1	5.616(25)	This work
^{51}V	3	5.148 705 7(2)	[8]
^{53}Mn	5	5.024(7)	[9]
^{55}Co	7	4.822(3)	[10]

Initially resonance was sought, changing the applied rf frequency by 1 MHz steps, with sawtooth frequency modulation ± 1 MHz. Resonance was observed at the 160 MHz setting. In later cycles, smaller frequency steps were used with modulation ± 0.5 MHz. The combined results (see Fig. 2) show the resonance centered at 159.8(4) MHz with FWHM 0.8(2) MHz. During resonance searching, the applied field was set at 0.100(5) T.

Using the relation

$$h\nu = |\mu|(B_{\rm hf} + B_{\rm applied})/I, \qquad (2)$$

where I is the nuclear spin, the magnetic dipole moment can be deduced. Taking the measured hyperfine field of $-13.17(5)$ T, we obtain the magnetic dipole moment of ^{49}Sc

$$|\mu|(^{49}\text{Sc}, 7/2^-, \text{g.s.}) = 5.616(25)\ \mu_N. \qquad (3)$$

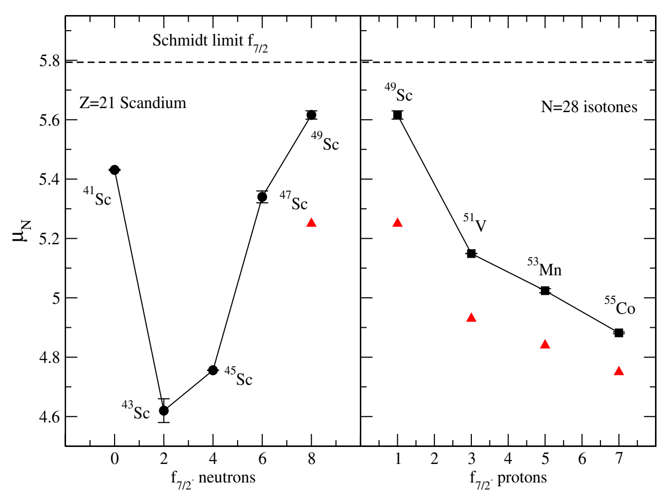

FIG. 3 (color online). Measured magnetic moments (left) of odd-A Sc $\pi(f_{7/2})^1$ isotopes with additional pairs of $7/2^-$ neutrons and (right) of isotones with $\pi(f_{7/2})^n$, n odd, and closed neutron shell $N = 28$. Also shown are the $\pi(f_{7/2})$ Schmidt limit and calculations by Honma *et al.* [11] (triangles).

TABLE II. Calculation of magnetic dipole moments of ^{41}Sc and ^{49}Sc. Each entry reflects a correction to the Schmidt limit values $g_l = 1.000$, $g_s = 5.847$, $g_p = 0.000$, and moment 5.794 μ_N.

^{41}Sc	δg_l	δg_s	δg_p	$\delta\mu[\mu_N]$
CP(RPA)	0	0	0	0
CP(2nd)	−0.106	−0.686	0.470	−0.629
MEC	0.124	0.207	−0.273	0.457
Isobar	0.000	−0.270	0.650	−0.093
MEC-CP	0.084	0.233	0.366	0.394
Relativistic	−0.026	−0.164	−0.044	−0.164
Total	0.076	−0.681	1.168	−0.035
Calculation result				5.759
Experiment				5.431(2)
^{49}Sc	δg_l	δg_s	δg_p	$\delta\mu[\mu_N]$
CP(RPA)	0.018	−0.330	−0.592	−0.150
CP(2nd)	−0.164	−0.925	0.557	−0.918
MEC	0.132	0.405	−0.464	0.568
Isobar	−0.006	−0.235	0.545	−0.100
MEC-CP	0.116	0.334	0.454	0.545
Relativistic	−0.025	−0.156	−0.042	−0.156
Total	0.071	−0.907	0.458	−0.211
Calculation result				5.583
Experiment				5.616(25)

Measurement of this magnetic moment completes two sequences of interest that pose challenges to the best nuclear models.

The sequence of odd-A ground states of the Sc isotopes from ^{41}Sc to ^{49}Sc ($Z = 21$, $N = 20$–28) is the first example of a single proton state combined with even neutron number progressing between two major shell closures for which we have a complete set of measured moments. [The sequence ^{57}Cu to ^{69}Cu ($Z = 29$, $N = 28$–40) ends at a minor shell closure.] The odd-A Sc moment values are given in Table I, part A and in Fig. 3. Also included in the figure is the Schmidt limit value for $7/2^-$ protons. The quasiparabolic form of the moment dependence clearly demonstrates the relative purity of the proton state which is at its maximum at the start and finish of the sequence.

The second sequence completed by this new measurement is of the $N = 28$ isotones from ^{49}Sc to ^{55}Co in which the $7/2^-$ proton subshell fills. This is shown in Fig. 3 and tabulated in Table I, part B. Again, it is clearly seen that as the double-shell closure at ^{48}Ca is approached, the $f_{7/2}$ single proton state becomes the ever more dominant component in the wave function. Calculations by Honma *et al.* [11], included in Fig. 3, are in a good agreement with the trend of observed behavior but show a systematic shift as compared to the experimental results.

As outlined in the introduction, only magnetic moments of nuclei having the simplest single-particle description can be used to examine in detail the deviations found with respect to the Schmidt limit predictions. At the limits of the Sc isotopic sequence, ^{41}Sc and ^{49}Sc, the double-magic cores, ^{40}Ca and ^{48}Ca, and the pure $f_{7/2}$ single-particle configurations, offer a rare opportunity to test fundamental calculations of magnetism in nuclei.

Such calculations are discussed in terms of an effective one-body magnetic moment operator [12,13]

$$\boldsymbol{\mu}_{\text{eff}} = (g_{l,\text{eff}}\boldsymbol{l} + g_{s,\text{eff}}\boldsymbol{s} + g_{p,\text{eff}}[\boldsymbol{Y}_2, \boldsymbol{s}])\ \mu_N, \tag{4}$$

where $g_{x,\text{eff}} = g_x + \delta g_{x,\text{eff}}$, with $x = l$, s, or p. g_x is the single-particle g factor and $\delta g_{x,\text{eff}}$ the correction to it. Note the effective operator includes an additional term involving a spherical harmonic of rank 2 coupled to a spin operator to give a tensor of multipolarity 1. The origin of this term, and detailed discussion of the various contributions to $\delta g_{x,\text{eff}}$, is given elsewhere [12,13]. The contributions are listed in Table II and are described briefly in order. The first, denoted CP(RPA), is the configuration mixing contribution due to core polarization (CP) in the random-phase approximation (RPA). It is identically zero for ^{41}Sc. This is followed by a second-order configuration mixing term CP(2nd). These two comprise the first class of corrections mentioned in the introduction. The second class consists of the meson exchange current (MEC), isobaric excitation (isobar), and combination MEC-CP terms with finally a small relativistic correction obtained by evaluating the operator to order $(p/M)^3$ where p is a typical nucleon momentum and M its mass. The final calculated moment values are

$$\mu(^{41}\text{Sc}, 7/2^-, \text{g.s.}) = 5.759\ \mu_N \quad \text{and}$$
$$\mu(^{49}\text{Sc}, 7/2^-, \text{g.s.}) = 5.583\ \mu_N. \tag{5}$$

Agreement with experiment for ^{49}Sc is extremely good, while that for ^{41}Sc is less impressive. The relative failure of

TABLE III. Theoretical and experimental magnetic moments of Sc and Cu isotopes with closed subshell neutron configurations. All entries for moments are in μ_N.

	Z	N	Configuration		Schmidt	Theory	Experiment	References
^{41}Sc	21	20	$\pi(f_{7/2})^1$	$\nu(f_{7/2})^0$	+5.794	+5.697	+5.431(2)	[4,13]
^{49}Sc	21	28	$\pi(f_{7/2})^1$	$\nu(f_{7/2})^8$	+5.794	+5.583	+5.616(25)	This work
^{57}Cu	29	28	$\pi(p_{3/2})^1$	$\nu(f_{7/2})^8$	+3.794	+2.404	+2.582(7)	[15]
^{69}Cu	29	40	$\pi(p_{3/2})^1$	$\nu(f_{5/2})^6$	+3.794	+2.874	+2.84(1)	[14,16]

the calculation where the doubly magic core is ^{40}Ca, compared to success with a ^{48}Ca core, is very similar to the situation found for the Cu isotopes. At ^{69}Cu, with a ^{68}Ni core, the theory gave very close agreement with experiment (also for ^{67}Ni), whereas at ^{57}Cu, with a ^{56}Ni core, agreement is not as good (see [14] and Table III).

This Letter has reported an accurate measurement of the magnetic moment of ^{49}Sc, one of the rare isotopes that allow testing of the fundamental theory of nuclear magnetism. The result shows excellent agreement with calculation. Consideration of this and other examples suggests that the theory is less successful in the case of doubly magic cores with equal number of protons and neutrons. However, it should be noted there is significant cancellation between the CP and MEC contributions (see Table II), and subtle changes in the calculation could easily alter the picture.

The research was supported by the U.S. DOE Office of Science, the Ministry of Education and Science of Serbia (Project No. 171002), the European Commission via the ENSAR project, the Fund of Scientific Research Flanders, and SNRS and IN2P3 (France).

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