

Muon Anomaly and Dark Parity Violation

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The muon anomalous magnetic moment exhibits a 3.6σ discrepancy between experiment and theory. One explanation requires the existence of a light vector boson, Z_d (the dark Z), with mass 10–500 MeV that couples weakly to the electromagnetic current through kinetic mixing. Support for such a solution also comes from astrophysics conjectures regarding the utility of a $U(1)_d$ gauge symmetry in the dark matter sector. In that scenario, we show that mass mixing between the Z_d and ordinary Z boson introduces a new source of “dark” parity violation, which is potentially observable in atomic and polarized electron scattering experiments. Restrictive bounds on the mixing $(m_{Z_d}/m_Z)\delta$ are found from existing atomic parity violation results, $\delta^2 < 2 \times 10^{-5}$. Combined with future planned and proposed polarized electron scattering experiments, a sensitivity of $\delta^2 \sim 10^{-6}$ is expected to be reached, thereby complementing direct searches for the Z_d boson.

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For a number of years, there has been a persistent disagreement between the experimental value of the muon anomalous magnetic moment, $a_\mu \equiv (g_\mu - 2)/2$

$$a_\mu^{\text{exp}} = 116\,592\,089(63) \times 10^{-11} \quad (1)$$

and the theoretical $SU(3)_C \times SU(2)_L \times U(1)_Y$ standard model (SM) prediction

$$a_\mu^{\text{SM}} = 116\,591\,802(49) \times 10^{-11}. \quad (2)$$

The above 3.6σ discrepancy [1]

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 287(80) \times 10^{-11} \quad (3)$$

could be indicative of problems with the theoretical calculations and/or experimental measurements. Alternatively, it could be a harbinger of “new physics” effects beyond SM expectations [2]. One possibility, receiving support from dark matter conjectures [3,4], envisions the existence of a relatively light $U(1)_d$ gauge boson, Z_d , coming from the “dark” sector that indirectly couples to our world via $U(1)_Y \times U(1)_d$ kinetic mixing [5], parametrized by ε such that [6]

$$\mathcal{L}_{\text{int}} = -e\varepsilon Z_d^\mu J_\mu^{\text{em}}, \quad J_\mu^{\text{em}} = \sum_f Q_f \bar{f} \gamma_\mu f, \quad (4)$$

where Q_f is the electric charge of fermion f . The coupling of Z_d to the weak neutral current from kinetic mixing is suppressed at low energies because of a cancellation between the ε dependent field redefinition and leading Z - Z_d mass matrix diagonalization effects induced by ε [6]. [We do not consider here the possibility that some ordinary fermions may have explicit $U(1)_d$ charges.]

The $Z_d \mu \bar{\mu}$ vector current coupling in Eq. (4) gives rise to an additional one-loop contribution [7,8] to a_μ

$$a_\mu^{Z_d(\text{vector})} = \frac{\alpha}{2\pi} \varepsilon^2 F_V(m_{Z_d}/m_\mu), \quad (5)$$

$$F_V(x) \equiv \int_0^1 dz \frac{2z(1-z)^2}{(1-z)^2 + x^2 z}, \quad F_V(0) = 1. \quad (6)$$

The effect in Eq. (5) has the right algebraic sign, such that for $10 \text{ MeV} \lesssim m_{Z_d} \lesssim 500 \text{ MeV}$ and ε^2 roughly in the range 10^{-6} – 10^{-4} , the discrepancy Δa_μ in Eq. (3) can be eliminated. We plot [9] in Fig. 1 the band in (m_{Z_d}, ε^2) space that reduces the discrepancy to within 90% C.L., i.e.,

$$a_\mu^{Z_d} = 287 \pm 131 \times 10^{-11}. \quad (7)$$

There, we also give a (roughly) 90% C.L. bound from the electron anomalous magnetic moment [10,11] constraint $|a_e^{Z_d}| < 10^{-11}$ using m_e in place of m_μ in Eq. (5) as well as

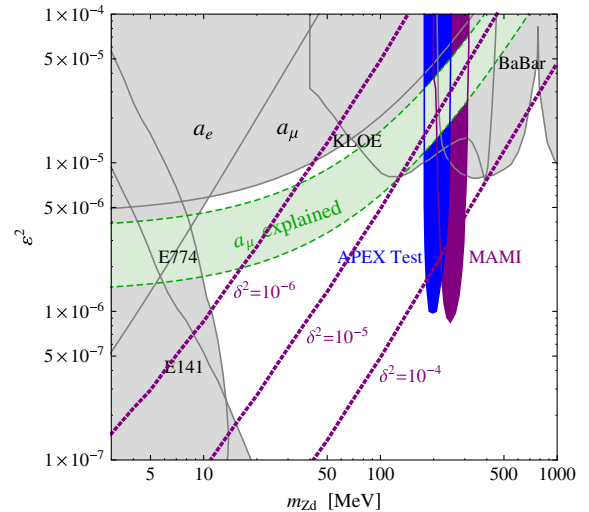


FIG. 1 (color online). Dark Z boson exclusion regions (partly adapted from Ref. [9]) in the (m_{Z_d}, ε^2) plane along with the band that explains the Δa_μ discrepancy (90% C.L.) and exclusion regions from atomic parity violation (above the lines) for Z - Z_d mixing δ values.

a $3\sigma a_{\mu}^{Z_d}$ bound. Constraints from other direct experimental searches for Z_d are also given [12,13]. However, those bounds are somewhat model dependent since they assume the Z_d decays primarily into e^+e^- or $\mu^+\mu^-$ pairs. They will be diluted if, for example, Z_d decays primarily into light “dark particles” that escape the detector as $Z_d \rightarrow$ missing energy [6].

Recently [6], we generalized the $U(1)_d$ kinetic mixing scenario to include possible Z - Z_d mass mixing by introducing the 2×2 mass matrix

$$M_0^2 = \begin{pmatrix} 1 & -\varepsilon_Z \\ -\varepsilon_Z & m_{Z_d}^2/m_Z^2 \end{pmatrix} m_Z^2, \quad (8)$$

where m_{Z_d} and m_Z (with $m_{Z_d}^2 \ll m_Z^2$) represent the “dark” Z and SM Z masses (before diagonalization). The off-diagonal mixing is parametrized by

$$\varepsilon_Z = \frac{m_{Z_d}}{m_Z} \delta, \quad 0 \leq |\delta| < 1, \quad (9)$$

where the m_{Z_d}/m_Z factor allows a smooth $m_{Z_d} \rightarrow 0$ limit for nonconserved current amplitudes, and δ is expected to be a small quantity that depends on the Higgs scalar sector of the theory [6]. Z - Z_d mixing induced by ε_Z leads to an additional coupling of Z_d to fermions via the weak neutral current

$$\begin{aligned} \mathcal{L}_{\text{int}} &= -\frac{g}{2 \cos \theta_W} \varepsilon_Z Z_d^\mu J_\mu^{NC} \\ J_\mu^{NC} &= \sum_f (T_{3f} - 2Q_f \sin^2 \theta_W) \bar{f} \gamma_\mu f - T_{3f} \bar{f} \gamma_\mu \gamma_5 f, \end{aligned} \quad (10)$$

with $T_{3f} = \pm 1/2$ and $\sin^2 \theta_W \simeq 0.23$ the SM weak mixing angle. Because of its axial-vector coupling, this new interaction violates parity and current conservation. As a result, it can lead to potentially observable effects in atomic parity violation (APV) and polarized electron scattering experiments, as well as rare flavor changing K and B or Higgs boson decays ($H \rightarrow ZZ_d$) to longitudinally polarized Z_d bosons (phase space permitting). We pointed out in Ref. [6] that the nonobservation of such effects already leads to bounds $|\delta| \lesssim 10^{-2}$ – 10^{-3} depending on m_{Z_d} and in some cases ε . Here, we further explore such constraints, but focus on that part of parameter space $10 \text{ MeV} \lesssim m_{Z_d} \lesssim 500 \text{ MeV}$ and $|\varepsilon| \approx 10^{-3}$ – 10^{-2} favored by a Z_d explanation of the Δa_μ discrepancy in Eq. (3). Also, to keep our analysis independent of the Z_d decay properties, we concentrate on low-energy parity violation, i.e., atomic and polarized electron scattering experiments. A variety of direct searches for Z_d have been discussed in the literature [6,9,12,13].

We begin by considering changes to $a_{\mu}^{Z_d}$ due to $\delta \neq 0$. The additional $Z_d \mu \bar{\mu}$ vector coupling in Eq. (10) modifies the contribution in Eq. (6) via the replacement

$$\varepsilon^2 \rightarrow \left(\varepsilon + \varepsilon_Z \frac{1 - 4 \sin^2 \theta_W}{4 \sin \theta_W \cos \theta_W} \right)^2 \simeq (\varepsilon + 0.02 \varepsilon_Z)^2, \quad (11)$$

where $\sin^2 \theta_W \simeq 0.24$ appropriate for low $Q^2 \simeq m_\mu^2$ scales [14] has been employed. For the Δa_μ favored range of m_{Z_d} and ε^2 in Fig. 1, the shift in Eq. (11) is small ($\lesssim 2\%$) for all δ and can be ignored.

The axial-vector part of the $Z_d \mu \bar{\mu}$ coupling in Eq. (10) gives rise to a negative contribution [8]

$$\begin{aligned} a_{\mu}^{Z_d}(\text{axial}) &= -\frac{G_F m_\mu^2}{8\sqrt{2}\pi^2} \delta^2 F_A(m_{Z_d}/m_\mu) \\ &\simeq -117 \times 10^{-11} \delta^2 F_A(m_{Z_d}/m_\mu) \end{aligned} \quad (12)$$

$$F_A(x) \equiv \int_0^1 dz \frac{2(1-z)^3 + x^2 z(1-z)(z+3)}{(1-z)^2 + x^2 z}, \quad (13)$$

where $G_F \simeq 1.166 \times 10^{-5} \text{ GeV}^{-2}$, $F_A(0) = 1$, and $F_A(\infty) = 5/3$. For $\delta^2 \lesssim 0.1$ (a mild requirement [6]), that contribution is also negligible throughout the Δa_μ favored region in Fig. 1. So, we conclude that the effect of Z - Z_d mass mixing plays little direct role in any discussion of the Δa_μ discrepancy and its interpretation as due to ε^2 .

Next, we examine constraints on the m_{Z_d} , ε , δ parameter space coming from low-energy, parity-violating experiments and their implications for a Z_d interpretation of the Δa_μ discrepancy.

It is well known that the classic cesium atomic parity-violation experiment [15] provides a stringent constraint on heavy Z' bosons [16] that violate parity, often implying $m_{Z'} \gtrsim \mathcal{O}(1 \text{ TeV})$. However, its application to relatively light gauge bosons such as Z_d has been less explored. Such a connection was first made by Bouchiat and Fayet [17] for a light “ U boson” with very general parity violating couplings to fermions. They found strong constraints and argued against axial-vector couplings. We recently [6] revisited the application of low-energy parity violation experimental constraints within the general Z - Z_d mass mixing formalism of Eq. (8). We updated the cesium constraint to include more recent atomic theory [18], expanded the analysis to polarized electron scattering [19], and applied our study specifically to the “dark” Z boson. Here, we focus on the connection of that analysis with the Δa_μ discrepancy and its interpretation via $10 \text{ MeV} \lesssim m_{Z_d} \lesssim 500 \text{ MeV}$ with $\varepsilon^2 \sim 10^{-6}$ – 10^{-4} .

The additional parity violation from Eq. (10) manifests itself as replacements in low-energy SM parity violating weak neutral current amplitudes [6]

$$G_F \rightarrow \rho_d G_F, \quad \sin^2 \theta_W \rightarrow \kappa_d \sin^2 \theta_W, \quad (14)$$

where for (momentum transfer) $Q^2 = -q^2$

$$\rho_d = 1 + \delta^2 f(Q^2/m_{Z_d}^2), \quad (15)$$

$$\kappa_d = 1 - \varepsilon \delta \frac{m_Z}{m_{Z_d}} \frac{\cos \theta_W}{\sin \theta_W} f(Q^2/m_{Z_d}^2) \quad (16)$$

giving rise to

$$\Delta \sin^2 \theta_W \simeq -0.42 \varepsilon \delta \frac{m_Z}{m_{Z_d}} f(Q^2/m_{Z_d}^2). \quad (17)$$

As pointed out in Ref. [17], for parity violation in heavy atoms, such as cesium, there is a correction factor $f = K(\text{Cs})$ relevant for very small m_{Z_d} . For example, $K(\text{Cs}) \simeq 0.5$ at $m_{Z_d} \simeq 2.4$ MeV, which sets the typical momentum transfer $\langle Q \rangle$ in this case, whereas $K(\text{Cs}) \simeq 0.74, 0.98$ at $m_{Z_d} \simeq 10, 100$ MeV. In the case of polarized electron scattering asymmetries, the Z_d propagator effect gives

$$f(Q^2/m_{Z_d}^2) = \frac{1}{1 + Q^2/m_{Z_d}^2} \quad (18)$$

with $\langle Q \rangle$ ranging from 50–170 MeV for the experiments we consider.

Currently, the SM prediction for the weak nuclear charge $Q_W(Z, N) \simeq -N + Z(1 - 4\sin^2 \theta_W)$ in the case of ^{133}Cs (including electroweak radiative corrections) [20]

$$Q_W^{\text{SM}}(^{133}\text{Cs}) = -73.16(5) \quad (19)$$

is in excellent agreement with experiment (including the most up-to-date atomic theory) [15,18]

$$Q_W^{\text{exp}}(^{133}\text{Cs}) = -73.16(35). \quad (20)$$

The 90% C.L. bound on the difference

$$|\Delta Q_W(\text{Cs})| = |Q_W^{\text{exp}}(^{133}\text{Cs}) - Q_W^{\text{SM}}(^{133}\text{Cs})| < 0.6 \quad (21)$$

can be compared with the potential Z_d contribution [6]

$$\Delta Q_W(^{133}\text{Cs}) = \left(-73.16 \delta^2 + 220 \varepsilon \delta \frac{m_Z}{m_{Z_d}} \sin \theta_W \cos \theta_W \right) \times K(\text{Cs}). \quad (22)$$

In principle, there could be a cancellation between the two terms in Eq. (22) for $\varepsilon(m_Z/m_{Z_d}) \sim 0.8\delta$. However, for

the Δa_μ preferred band in Fig. 1, $|\varepsilon(m_Z/m_{Z_d})| \gtrsim 2$; the second term in Eq. (22) always dominates. In fact, a conservative self-consistent assessment of the bound (at 90% C.L.) from Eqs. (21) and (22) yields

$$|\delta^2 - 2\delta| < 0.008 \rightarrow \delta^2 < 2 \times 10^{-5} \quad (23)$$

for the entire Δa_μ motivated band in Fig. 1. That means the first term in Eq. (22) can be neglected and the $Q_W(^{133}\text{Cs})$ bound becomes for arbitrary ε^2 and m_{Z_d} essentially a bound

$$\varepsilon^2 < \frac{4 \times 10^{-5}}{\delta^2 K^2} \left(\frac{m_{Z_d}}{m_Z} \right)^2 \quad (24)$$

on the allowed $\sin^2 \theta_W$ shift. The atomic parity violation bound on ε^2 is illustrated in Fig. 1 for various values of δ^2 . Note that for $\delta^2 \gtrsim 2 \times 10^{-5}$, the entire Δa_μ discrepancy motivated band is already ruled out. Alternatively, if a light Z_d is responsible for the Δa_μ discrepancy, the Z - Z_d mixing $|\varepsilon_Z| = |(m_{Z_d}/m_Z)\delta|$ must be very tiny ($\delta^2 < 2 \times 10^{-5}$). Of course, the Δa_μ discrepancy may have nothing to do with Z_d . In that case, larger δ^2 values can be accommodated by going to smaller ε^2 or larger m_{Z_d} values, although other constraints [6] then come into play.

Atomic parity violation already provides a powerful constraint on δ^2 over an interesting m_{Z_d} range. Future experiments employing ratios of isotopes could, in principle, eliminate the atomic theory uncertainty and further probe Z_d mass and mixing as well as other “new physics” scenarios [21].

Another type of low-energy, parity-violating experiment involves polarized electron scattering on electrons, protons, or other targets. They measure the parity-violating asymmetry [19] $A_{LR} \equiv \sigma_L - \sigma_R / \sigma_L + \sigma_R$ due to γ - Z interference at low Q^2 . In some cases, such as ee and ep , those experiments are particularly sensitive to $\sin^2 \theta_W$ at

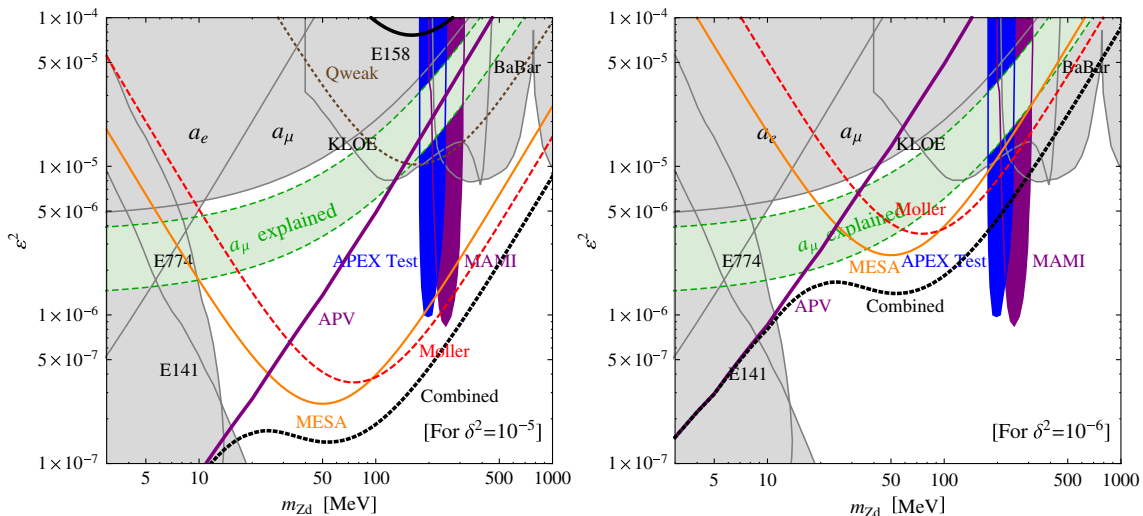


FIG. 2 (color online). Dark Z boson exclusion regions from various parity violating experiments (existing and proposed) and their combined sensitivity for $\delta^2 = 10^{-5}$ (left) and 10^{-6} (right) at 90% C.L.

TABLE I. Existing and possible future constraints on dark Z from various parity violating experiments.

Experiment	$\langle Q \rangle$	$\sin^2 \theta_W(m_Z)$	Bound on dark Z (90% C.L.)
Cesium APV	2.4 MeV	0.2313(16)	$\varepsilon^2 < \frac{39 \times 10^{-6}}{\delta^2} \left(\frac{m_{Z_d}}{m_Z} \right)^2 \frac{1}{K(m_{Z_d})^2}$
E158 (SLAC)	160 MeV	0.2329(13)	$\varepsilon^2 < \frac{62 \times 10^{-6}}{\delta^2} \left(\frac{(160 \text{ MeV})^2 + m_{Z_d}^2}{m_Z m_{Z_d}} \right)^2$
Qweak (JLAB)	170 MeV	± 0.0007	$\varepsilon^2 < \frac{7.4 \times 10^{-6}}{\delta^2} \left(\frac{(170 \text{ MeV})^2 + m_{Z_d}^2}{m_Z m_{Z_d}} \right)^2$
Moller (JLAB)	75 MeV	± 0.00029	$\varepsilon^2 < \frac{1.3 \times 10^{-6}}{\delta^2} \left(\frac{(75 \text{ MeV})^2 + m_{Z_d}^2}{m_Z m_{Z_d}} \right)^2$
MESA (Mainz)	50 MeV	± 0.00037	$\varepsilon^2 < \frac{2.1 \times 10^{-6}}{\delta^2} \left(\frac{(50 \text{ MeV})^2 + m_{Z_d}^2}{m_Z m_{Z_d}} \right)^2$
Combined		± 0.00021	$\varepsilon_{\text{comb}}^2 < \frac{1}{\sum_i (1/\varepsilon_i^2)}$

low Q^2 , where the effective $\sin^2 \theta_W$ is expected [14] to be about 0.24, thereby leading to very small asymmetries (proportional to $1-4\sin^2 \theta_W$). Already, experiment E158 at SLAC has measured [22] (evolving to $Q^2 = m_Z^2$)

$$\sin^2 \theta_W(m_Z)_{\overline{\text{MS}}} = 0.2329(13) \quad (\text{E158 at SLAC}), \quad (25)$$

which is to be compared with the Z pole average [1]

$$\sin^2 \theta_W(m_Z)_{\overline{\text{MS}}} = 0.23125(16). \quad (26)$$

The relatively good agreement between Eqs. (25) and (26) already constrains many types of “new physics” at a sensitivity similar to APV. In the case of Z_d at low masses, cesium APV has the advantage of a low [17] $\langle Q \rangle \simeq 2.4$ MeV while for E158, $\langle Q \rangle^{\text{E158}} \simeq 160$ MeV such that Z_d propagator effects suppress the sensitivity by $m_{Z_d}^2/(Q^2 + m_{Z_d}^2)$ at the amplitude level.

A comparison of E158 constraints, using [see Eq. (17)]

$$\varepsilon^2 < \frac{6 \times 10^{-5}}{\delta^2} \left(\frac{0.026 \text{ GeV}^2 + m_{Z_d}^2}{m_Z m_{Z_d}} \right)^2 \quad (27)$$

with APV, is illustrated in Fig. 2. The one-sided 90% C.L. coefficient in that bound has been increased due to the $\sim 1\sigma$ difference between Eqs. (25) and (26). For a given δ^2 , the bounds at large m_{Z_d} are similar, but APV is superior for $m_{Z_d} \lesssim 160$ MeV.

An ongoing polarized ep experiment [9,23], Qweak at JLAB, aims to measure $\sin^2 \theta_W$ to ± 0.0007 at $\langle Q \rangle \simeq 170$ MeV. That represents an improvement by about a factor of 2 over E158, but the similar $\langle Q \rangle$ means that it also lacks low m_{Z_d} sensitivity. In the longer term, a new polarized ee (Moller) [24] experiment at JLAB would measure $\sin^2 \theta_W$ to ± 0.00029 at $\langle Q \rangle \simeq 75$ MeV, and a very low-energy polarized ep experiment at a new proposed MESA facility [25] in Mainz, Germany, would measure $\sin^2 \theta_W$ to ± 0.00037 for $\langle Q \rangle$ perhaps as low as 50 MeV. The sensitivities of these (proposed) experiments are also illustrated in Fig. 2, using the constraints in Table I derived from Eq. (17).

In Fig. 2, we give a combined sensitivity bound for $\delta^2 = 10^{-5}$ and $\delta^2 = 10^{-6}$ from all existing and proposed low-energy, parity-violating experiments. That plot illustrates the complementarity of atomic and polarized electron scattering experiments. In addition to providing overlapping probes of new physics, collectively they span a large range of (m_{Z_d}, ε^2) space and probe down to δ^2 of $\mathcal{O}(10^{-6})$. Of course, it is possible that a light Z_d exists that is consistent with the Δa_μ discrepancy and will be discovered. For example, if $m_{Z_d} \simeq 75$ MeV, $|\varepsilon| \simeq 3 \times 10^{-3}$ and $|\delta| \simeq 2 \times 10^{-3}$, the proposed Moller and MESA experiments should find shifts $|\Delta \sin^2 \theta_W| \simeq 0.0015$ and 0.0021, respectively, corresponding to about 5σ discovery sensitivities.

In conclusion, we have found that existing atomic parity-violating results already require $\delta^2 \lesssim 2 \times 10^{-5}$ for the entire range of (m_{Z_d}, ε^2) , i.e. $10 \text{ MeV} \lesssim m_{Z_d} \lesssim 500 \text{ MeV}$, $\varepsilon^2 \simeq 10^{-6} - 10^{-4}$, favored by the Z_d interpretation of the Δa_μ discrepancy. That requirement calls into question the Z_d interpretation of the Δa_μ unless Z - Z_d mixing is naturally small, for example, if the mass m_{Z_d} is primarily generated by an $SU(2)_L \times U(1)_Y$ Higgs singlet [6]. Future polarized electron scattering experiments will provide additional Z_d sensitivity, particularly for $m_{Z_d} \gtrsim 75$ MeV (where 5σ effects are possible) and will nicely complement atomic parity-violation experiments as well as direct Z_d searches.

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