Testing Parity with Atomic Radiative Capture of μ^-

David McKeen[1] and Maxim Pospelov[1,2]
[1]Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia V8P 5C2, Canada
[2]Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2J 2W9, Canada
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The next generation of ''intensity frontier'' facilities will bring a significant increase in the intensity of subrelativistic beams of μ^-. We show that the use of these beams in combination with thin targets of $Z \sim 30$ elements opens up the possibility of testing parity-violating interactions of muons with nuclei via direct radiative capture of muons into atomic $2S$ orbitals. Since atomic capture preserves longitudinal muon polarization, the measurements of the gamma ray angular asymmetry in the single photon $2S_{1/2}$–$1S_{1/2}$ transition will offer a direct test of parity. We calculate the probability of atomic radiative capture taking into account the finite size of the nucleus to show that this process can dominate over the usual muonic atom cascade and that the as-yet unobserved single photon $2S_{1/2}$–$1S_{1/2}$ transition in muonic atoms can be detected in this way using current muon facilities.

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Introduction.—The standard model (SM) of particles and fields has shown tremendous vitality under an onslaught of new tera-electron-volt-scale data from the Large Hadron Collider (LHC). Stringent limits are derived on new hypothetical vector particles Z' that mediate interactions between light quarks and charged leptons. For a sequential SM Z-like Z' particle, such limits extend to 2 TeV, rendering low-energy parity-violating tests not competitive with the LHC in the search for new heavy resonances with large couplings to SM particles. However, an alternative possibility—light and very weakly coupled particles—may easily escape the high-energy constraints while inducing some nontrivial effects at low energy [1]. In recent years the interest in this type of physics has intensified, largely due to the accumulation of various anomalous observations that such light particles may help to explain. (For a possible connection between light vectors and dark matter physics see, e.g., Ref. [2].) In parallel with this, attempts to detect such new states at ''intensity frontier'' facilities are becoming more frequent and more systematic [3].

Muon physics and its study with new high-intensity muon beams is a natural point of interest because of the lingering discrepancy between calculations and measurements of the muon anomalous magnetic moment [4] as well as the recent striking discrepancy of the proton charge radius extracted from the muonic hydrogen Lamb shift [5] as compared with other determinations of the same quantity [6]. While it is far from clear that these discrepancies are not caused by some poorly understood SM physics or experimental mistakes, it is still important to investigate models of new physics (NP) that could create such deviations. Models with light vector particles (see, e.g., Ref. [7]) are particularly interesting as they can remove the $g-2$ discrepancy quite naturally [8] or be responsible for extra muon-proton interactions that can be interpreted as a shift of the proton charge radius [9,10].

As was argued in Ref. [10], a lepton flavor-specific muon-proton interaction in combination with constraints in the neutrino sector may imply that right-handed muon number is gauged, leading to new parity-violating muon-proton neutral current interactions. We take this model as a representative example of new physics at the sub-GeV energy scale that can create stronger-than-weak effects in the interaction of muons with nuclei. In this Letter, we revisit the idea of searching for parity violation in the muon sector using muonic atoms, keeping in mind that no direct tests of the axial vector muon coupling have been performed at low energy and that the NP contribution could dominate over the SM [10]. To be specific, we consider a low-energy effective neutral current Lagrangian that includes the sum of the SM and NP contributions,

$$\mathcal{L}_\mu = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm NP},$$
$$\mathcal{L}_{\rm SM} = -\frac{G_F}{2\sqrt{2}}\bar{\mu}\gamma_\nu\gamma_5\mu(g_n\bar{n}\gamma_\nu n + g_p\bar{p}\gamma_\nu p), \qquad (1)$$

$$\mathcal{L}_{\rm NP} = \bar{\mu}\gamma_\nu\gamma_5\mu\frac{4\pi\alpha g_\mu^{\rm NP}}{m_V^2+\Box}(g_n^{\rm NP}\bar{n}\gamma_\nu n + g_p^{\rm NP}\bar{p}\gamma_\nu p), \quad (2)$$

where the SM vector couplings to nucleons are given by $g_n^V = -\frac{1}{2}$, $g_p^V = \frac{1}{2} - 2\sin^2\theta_W$. In the model with gauged right-handed muon number, the least constrained points in the parameter space correspond to the mass of the mediator gauge boson of $m_V \simeq 30$ MeV. In that case, the fit to the proton charge radius suggests [10]

$$\frac{4\pi\alpha g_\mu^{\rm NP} g_p^{\rm NP}}{m_V^2} \simeq \frac{2\times 10^{-5}}{(30\ {\rm MeV})^2} \gg G_F, \qquad (3)$$

which should be considered as perhaps the most optimistic value for the strength of the muon-proton interaction. In what follows we suggest a new way to search for the manifestation of Eqs. (1) and (2) in muonic atoms using the process of atomic radiative capture (ARC) to the $2S$ state: $\mu^- + Z \to (\mu^- Z)_{2S} + \gamma$. We show that probing $\mathcal{L}_{\rm NP}$ of maximal strength is possible with existing muon line facilities, while the SM values can eventually be tested at the next generation of high-intensity muon sources.

It is well-known that the suppressed $M1$ single photon $2S_{1/2}$–$1S_{1/2}$ transition in combination with the small energy difference between the $2S$ and $2P$ states enhances the parity-violating asymmetry in $M1$–$E1$ interference. This idea has received a significant amount of theoretical and experimental attention, summarized in a review [11]. The most promising scheme for the detection of parity violation to date was identified as a slow muon forming a highly excited atomic state with a nucleus followed by a cascade ending with

$$\ldots \to 2S_{1/2} \xrightarrow{M1-E1} 1S_{1/2} + \gamma; \quad (\mu^-)_{1S} \to e^- \nu_\mu \bar{\nu}_e, \tag{4}$$

with parity violation being encoded in the correlation between the directions of the outgoing γ and the muon decay electron. In Fig. 1 we show a level diagram for a typical muonic atom.

Despite considerable efforts, the single photon $2S$–$1S$ transition itself has never been detected in any muonic atoms. In light atoms, $Z \lesssim 10$, this transition cannot be distinguished from the far more dominant $2P$–$1S$, as the difference between gamma ray energies in this case is much smaller than the energy resolution of γ detectors. Combining this with the tiny branching ratio of the one-photon decay of the $2S_{1/2}$ state in light elements and the fact that it gets scarcely populated, $O(1\%)$, during the cascade makes the measurement of parity violation very challenging in light muonic atoms, even though the value of parity-violating asymmetries could be as large as a few percent [11]. Heavier muonic atoms, $Z \sim 30$, have been suggested as promising candidates to test parity [12] because the $2S$–$1S$ and $2P$–$1S$ transitions can be easily resolved, as the energy difference between the $2S$ and $2P$ states reaches

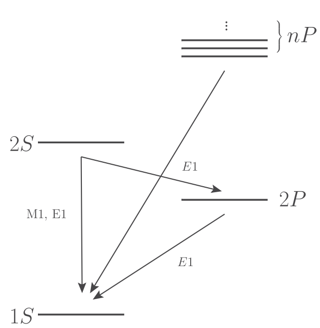

FIG. 1. Diagram of the atomic levels in typical muonic atoms. Also shown are some of the single photon transitions between states. The $2S \to 1S$ single photon transition is an admixture of a suppressed $M1$ transition and an $E1$ transition from $2S$–$2P$ mixing induced by parity violation.

$$\Delta E \equiv E_{2S} - E_{2P} = \frac{(Z\alpha)^4 m_\mu (m_\mu R_c)^2}{12}$$
$$\simeq 210 \text{ keV} \times (Z/36)^4 \times (R_c/4.2 \text{ fm})^2, \tag{5}$$

where we have normalized the nuclear charge Z and the nuclear charge radius R_c on the values for krypton and suppressed total J indices, effectively neglecting the splitting between $2P_{3/2}$ and $2P_{1/2}$ states. Unfortunately, as in the case of lighter elements, the $2S$–$1S$ transition was never detected in heavier atoms, because of the dominance of the background created by quanta from nP–$1S$ transitions, $n \geq 3$, whose energies have been degraded [11]. To elaborate on this, one can estimate the signal-to-background ratio of the single photon $2S$–$1S$ transition during the atomic cascade. The signal $S \sim N_{2S}\text{Br}_{1\gamma}$ is proportional to the fraction of cascade muons N_{2S} that end up in the $2S$ state, where N_{2S} is typically on the order of 10^{-2} [13], and the branching of $M1$ single photon transition from $2S$ states, which for $Z \sim 30$ [12] is given by

$$\text{Br}_{1\gamma} \simeq \frac{\Gamma_{2S-1S+1\gamma}}{\Gamma_{2S-2P} + \Gamma_{2S-1S+2\gamma} + \Gamma_{\text{Auger}}}$$
$$\simeq \frac{\Gamma_{2S-1S+1\gamma}}{\Gamma_{2S-2P}} \sim 2 \times 10^{-3}. \tag{6}$$

For smaller Z, $Z < 28$, the single photon branching is strongly suppressed by Auger processes [14] and by the two photon transitions. The cascade-related background consists of the number of energy-degraded nP–$1S$ ($n \geq 3$) photons (i.e., those that do not deposit their full energy in the detector) that fall into the energy resolution interval ΔE centered at the energy of the $2S$–$1S$ transition. From experimental studies [15], one can conclude that $O(20\%)$ of muons undergoing a cascade generate nP–$1S$ transitions. For realistic γ detectors, the number of energy-degraded photons is $\sim$50%, and the number of photons under the $2S$–$1S$ peak within the energy resolution window of $\Delta E \sim 2$ keV can be estimated as $B \sim 0.2 \times \Delta E/(2E_\gamma) \sim 10^{-4}$ for $E_\gamma \sim 2$ MeV. Therefore, one arrives at the following estimate of signal-to-background:

$$\left[\frac{S}{B}\right]_{\text{cascade}} \leq 0.2. \tag{7}$$

The actual ratio is smaller than this upper bound because of additional photon backgrounds caused by other sources, which explains why the $2S$–$1S$ transition has not been detected [11].

In addition to these challenges in detecting the $2S$–$1S$ transition in muon cascades, another difficulty in implementing the scheme in Eq. (4) lies in the fact that the final step, muon decay, for these elements is very subdominant to nuclear muon capture. Because of the combination of these two factors, parity experiments with $Z \sim 30$ elements were deemed impractical [11].

New proposal for a parity-violation measurement.—Our proposal is to abandon (4) and use thin targets of $Z \geq 30$ elements that only decrease the μ^- momentum but do not stop the particle completely. This removes most of the background related to the muonic cascade. A fraction of the muons undergo ARC directly into the $2S$ state. The signal consists of two γ quanta, one from the ARC process (γ_1) and the other from the single photon decay of the $2S$ state (γ_2),

$$\mu^-_{\rightarrow} + Z \rightarrow (\mu^-_{\rightarrow} Z)_{2S_{1/2}} + \gamma_1; \qquad 2S_{1/2} \xrightarrow{M1-E1} 1S_{1/2} + \gamma_2. \tag{8}$$

Here, $\mu^-_{\rightarrow}$ denotes the longitudinally polarized muon. While for the relevant range of Z the energy of γ_2 is on the order of 2 MeV, the energy of γ_1 is dependent on the muon momentum and for muon momentum of 50 MeV is in the 10 MeV range. The parity-violating signature is the forward-backward asymmetry of γ_2 relative to the direction of the muon spin.

To calculate the cross section for muonic ARC into the $2S$ state [the first step in Eq. (8)], we note that the analogous process involving an electron, electron-nucleus photorecombination, in the dipole approximation with a pointlike nucleus is a standard textbook calculation [16,17], as it can be obtained from the standard hydrogen-like photoelectric ionization cross section $\sigma^{(0)}_{PE}$. Here we adjust this for the muon case, which besides the substitution $m_e \rightarrow m_\mu$, involves accounting for the finite nuclear charge radius and the departure from the dipole approximation. This can be done by introducing a correction factor to the standard formula,

$$\sigma_{\rm ARC} = \frac{2\omega^2}{p^2}\sigma_{PE}; \quad \sigma_{PE} = \eta(p, R_c, Z, n, l) \times \sigma^{(0)}_{PE}(nl),$$
$$\sigma^{(0)}_{PE}(2S) = \frac{2^{14}\pi^2\alpha a^2 E_2^4}{3\omega^4}\left[1 + \frac{3E_2}{\omega}\right]\frac{\exp\{-\frac{4}{pa}\cot^{-1}\frac{1}{2pa}\}}{1-\exp(-2\pi/pa)}.$$

In these expressions, p is the momentum of the incoming muon, a is the Bohr radius, $a = (Z\alpha m_\mu)^{-1}$, $E_2 = Z^2\alpha^2 m_\mu/8$ is the (uncorrected) binding energy of the $2S$ muon, and $\omega = p^2/2m_\mu + E_2$ is the (uncorrected) energy of the photon emitted in the ARC process. The correction factor η is calculated by numerically solving the Schrödinger equation for a muon moving in the field of the nucleus with uniform charge distribution with charge radius R_c. The results for the cross sections are plotted in Fig. 2 for $Z = 36$ and $R_c = 4.2$ fm. As one can see, the corrections to the simple formula are significant, and mostly come from the finite charge of the nucleus, suppressing a naive cross section by more than a factor of $\sim$3 for $p_\mu >$ 60 MeV. Moreover, at $p \sim m_\mu$, this formula will need to be further corrected by relativistic effects that thus far have been ignored in our treatment.

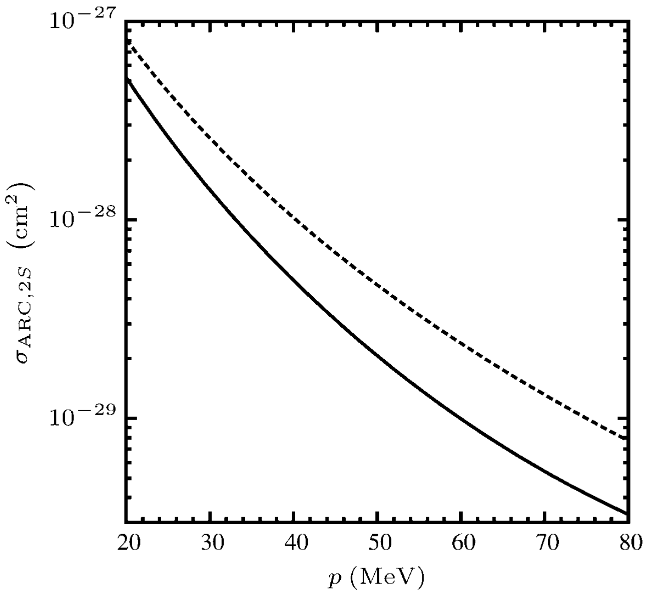

FIG. 2. $\sigma_{\rm ARC,2S}$ as a function of the incoming muon momentum, p (solid curve) for a muon scattering on krypton, $Z = 36$, with a uniform nuclear charge density and charge radius of $R_c = 4.2$ fm while taking the departure from the dipole approximation into account. Also shown is the cross section in the dipole approximation with a pointlike nucleus (dashed curve). ARC into the $2S$ state is a factor several times less probable than that into the $1S$ state.

Previously, the ARC process was considered theoretically in Ref. [18] for the case of muonic hydrogen, and searched for experimentally in Ref. [15] in muonic cascades in Mg and Al. The ARC process was not detected because in the case of stopped muons the cross section for forming muonic atoms via electron ejection is several orders of magnitude larger than $\sigma_{\rm ARC}$. Because of that, one should not expect that the muon cascade experiments can be sensitive to the ARC processes.

Below, we estimate the probability for the ARC process in a thin gaseous target of Kr that decreases the momentum of the muon beam from $p_{\rm max} = 30$ MeV to $p_{\rm min} = 25$ MeV,

$$P_{\rm ARC,2S} = \int_{p_{\rm min}}^{p_{\rm max}} dp \frac{n_{\rm Kr}\sigma_{\rm ARC,2S}}{|dp/dx|} \sim 2 \times 10^{-7}, \tag{9}$$

where the momentum loss, dp/dx, is given by standard Bethe-Bloch theory. For a target size of $\sim$5 cm, the number density of the krypton atoms would correspond to pressure of $p_{\rm Kr} \sim 8$ atm.

Combining the probability of the ARC process (9) with the branching ratio of the $M1$ photons (6), we arrive at the emission rate of $2S$–$1S$ photons as a function of the incoming muon flux,

$$\frac{dN_{2S-1S}}{dt} = P_{\rm ARC} \times {\rm Br}_{1\gamma} \times \Phi_{\mu^-} \sim \frac{1}{250\ {\rm s}} \times \frac{\Phi_{\mu^-}}{10^7{\rm s}^{-1}}. \tag{10}$$

The lifetime of the $2S$ state is extremely small: for $Z > 30$ it does not exceed 10 fs [12], which allows for a tight timing correlation between γ_1 and γ_2 in Eq. (8).

We can also estimate the intrinsic background created by the nP–$1S$ transitions in this case. For a transparent target, one source of background consists of the bremsstrahlung process $\mu + Z \to \mu + Z + \gamma$ that degrades the muon energy enough to trap it inside the target, with a subsequent muon cascade creating nP–$1S$ photons. To calculate the yield of nP–$1S$ photons, we estimate the probability for the process $\mu + Z \to \mu + Z + \gamma$ by taking the standard cross section [17] and modifying it by the correction coming from the finite nuclear charge. In this way we find, for the same parameters of the target,

$$P_{\text{cascade}} \sim P_{\mu+Z\to\mu+Z+\gamma} \sim 20 \times P_{\text{ARC},2S}, \tag{11}$$

requiring that the bremsstrahlung photon be at least as energetic as that coming from ARC into the $2S$ state for $p_{\min} = 25$ MeV. Only a small fraction of the cascade photons, $\sim O(10^{-4})$, will be degraded to mimic the $2S$–$1S$ transition, and we can conclude that the ratio of signal to irreducible background is

$$\left[\frac{S}{B}\right]_{\text{ARC}} = \frac{P_{\text{ARC},2S} \times \text{Br}_{1\gamma}}{P_{\text{cascade}} \times 10^{-4}} \sim O(1), \tag{12}$$

and the gain over (7) is rather significant. The contribution to the background due to direct capture on $n \geq 3$ orbits is even smaller. The background from bremsstrahlung and cascade photons in Eq. (11) is small enough that Ge detectors with μs response times can operate with muon fluxes of $O(10^{10}\ \text{s}^{-1})$ without photons from these processes arriving within the lifetime of the $2S$ state. We conclude that while the signal rate is small [Eq. (10)], the gain in the S/B can be substantial, making the search for the ARC processes and $2S$–$1S$ transitions worth pursuing experimentally. A further increase in S/B can be achieved by imposing a cut on the energy of γ_1 that can distinguish it from the lower-energy bremsstrahlung γ.

We are now ready to investigate the feasibility of the parity-violation experiment with the use of the ARC scheme in Eq. (8). The forward-backward asymmetry of the $2S$–$1S$ photon is related to the coefficient of $2S$–$2P$ mixing δ and the ratio of $E1$ and $M1$ amplitudes [12]

$$\mathcal{A}_{\text{FB}} = \frac{N_{\gamma_2}(\theta > \frac{\pi}{2}) - N_{\gamma_2}(\theta < \frac{\pi}{2})}{N_{\gamma_2}(\theta > \frac{\pi}{2}) + N_{\gamma_2}(\theta < \frac{\pi}{2})} = 2\delta \frac{(E1)_{2P-1S}}{(M1)_{2S-1S}} \simeq 680 \times \left(\frac{36}{Z}\right)^3 \times \delta, \quad i\delta = \frac{\langle 2S_{1/2}|H_{\text{PV}}|2P_{1/2}\rangle}{\Delta E}, \tag{13}$$

where the parity-violating Hamiltonian can be derived from Eqs. (1) and (2). The size of the parity-violating admixture in the SM [12] and in the presence of nonstandard interactions [10] is given by

$$\delta_{\text{SM}} \simeq \frac{3\sqrt{3}G_F}{8\sqrt{2}\pi Z\alpha R_c^2}\left(g_p + g_n\frac{A-Z}{Z}\right),$$
$$\delta_{\text{NP}} = \frac{3\sqrt{3}g_\mu^{\text{NP}}}{2Z\alpha R_c^2 m_\mu^2}\frac{m_V a}{(m_V a + 1)^3}\left(g_p^{\text{NP}} + g_n^{\text{NP}}\frac{A-Z}{Z}\right). \tag{14}$$

For the nonstandard interaction (2), we normalize its strength to the possible size of the effect suggested by the muonic hydrogen Lamb shift discrepancy, following Ref. [10]. This way, for $Z = 36$ we find

$$\mathcal{A}_{\text{FB}}[\text{SM}] \simeq 0.5 \times 10^{-4}, \quad \mathcal{A}_{\text{FB}}[\text{NP}] = (0.5\text{–}11)\%. \tag{15}$$

The lower value of the asymmetry $\mathcal{A}_{\text{FB}}[\text{NP}]$ is for small, ~ 10 MeV, masses of vector mediators, whereas larger values are for the scaling regime $m_V \gg 1/a$.

Using these asymmetries and a realistic efficiency factor of ~ 0.1 for the detection of a two-photon transition, we arrive at the following estimate of the time required to achieve the number of events $N \propto 1/\mathcal{A}_{\text{FB}}{}^2$:

$$T[\text{SM}] \sim 10^8\ \text{s} \times \frac{10^{11}\ \text{s}^{-1}}{\Phi_\mu},$$
$$T[\text{NP}] \sim 3 \times 10^5\ \text{s} \times \frac{10^7\ \text{s}^{-1}}{\Phi_\mu} \times \left(\frac{0.1}{\mathcal{A}}\right)^2. \tag{16}$$

One can see that, while the test of a muonic parity-violating $\mathcal{A}_{\text{FB}}$ down to the $O(10^{-4})$ value of the SM via the method suggested in this Letter is statistically possible only with future high-intensity muon beams, tests of some NP models [10] are feasible even at existing facilities.

In conclusion, let us summarize the main advantages of possible tests of parity using the atomic radiative capture scheme in Eq. (8): (i) The muon capture onto the $2S$ orbit proceeds via an $E1$ transition and does not depolarize the muons. Therefore, it is possible to capture a fully polarized muon onto the $2S$ orbit and study an angular asymmetry of the outgoing γ without the need to observe muon beta decay in the $1S$ state. (ii) The gain in S/B is significant, as the nP–$1S$ ($n > 3$) transitions of cascade muons that prevented the detection of the single photon $2S$–$1S$ decay in the past are greatly reduced. The detection of this transition can be realistically performed even with the existing sources of μ^-. (iii) The use of a transparent target allows one to study parity with muons in a "parasitic" setup, when the dominant part of the muon flux is used for other experiments. It also appears that the ARC-based method (8) can withstand the increase of the muon beam intensity more easily than the cascade-based methods (4).

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[1] P. Fayet, Phys. Lett. **95B**, 285 (1980); **96B**, 83 (1980).

[2] N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, and N. Weiner, Phys. Rev. D **79**, 015014 (2009); M. Pospelov and A. Ritz, Phys. Lett. B **671**, 391 (2009).

[3] H. Merkel *et al.* (A1 Collaboration), Phys. Rev. Lett. **106**, 251802 (2011); S. Abrahamyan *et al.* (APEX Collaboration), Phys. Rev. Lett. **107**, 191804 (2011).

[4] G. Bennett *et al.* (Muon G-2 Collaboration), Phys. Rev. D **73**, 072003 (2006).

[5] R. Pohl, A. Antognini, F. Nez, F. D. Amaro, and F. Biraben *et al.*, Nature (London) **466**, 213 (2010).

[6] P. J. Mohr, B. N. Taylor, and D. B. Newell, Rev. Mod. Phys. **80**, 633 (2008); R. J. Hill and G. Paz, Phys. Rev. D **82**, 113005 (2010).

[7] L. Okun, Zh. Eksp. Teor. Fiz. **83**, 892 (1982) [Sov. Phys. JETP **56**, 502 (1982)]; B. Holdom, Phys. Lett. **166B**, 196 (1986).

[8] S. Gninenko and N. Krasnikov, Phys. Lett. B **513**, 119 (2001); M. Pospelov, Phys. Rev. D **80**, 095002 (2009).

[9] J. Jaeckel and S. Roy, Phys. Rev. D **82**, 125020 (2010); V. Barger, C.-W. Chiang, W.-Y. Keung, and D. Marfatia, Phys. Rev. Lett. **106**, 153001 (2011); D. Tucker-Smith and I. Yavin, Phys. Rev. D **83**, 101702 (2011).

[10] B. Batell, D. McKeen, and M. Pospelov, Phys. Rev. Lett. **107**, 011803 (2011).

[11] J. H. Missimer and L. M. Simons, Phys. Rep. **118**, 179 (1985).

[12] G. Feinberg and M. Chen, Phys. Rev. D **10**, 190 (1974); **10**, 3145 (1974).

[13] K. Kirch *et al.*, Phys. Rev. Lett. **78**, 4363 (1997).

[14] D. Grechukhin and A. Soldatov, Zh. Eksp. Teor. Fiz. **77**, 2165 (1979) [Sov. Phys. JETP **50**, 1039 (1979)].

[15] F. J. Hartmann, R. Bergmann, H. Daniel, H.-J. Pfeiffer, T. von Egidy, and W. Wilhelm, Z. Phys. A **305**, 189 (1982); R. A. Naumann, H. Daniel, P. Ehrhart, F. J. Hartmann, T. von Egidy, Phys. Rev. A **31**, 727 (1985).

[16] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, New York, 1957).

[17] V. Berestetskii, E. Lifshitz, and L. Pitaevskii, *Quantum Electrodynamics* (Pergamon, New York, 1982), 2nd ed.

[18] L. Chatterjee, G. Das, and A. Chakraborty, Europhys. Lett. **18**, 145 (1992).