Strong Running Coupling at τ and Z_0 Mass Scales from Lattice QCD

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This Letter reports on the first computation, from data obtained in lattice QCD with u, d, s, and c quarks in the sea, of the running strong coupling via the ghost-gluon coupling renormalized in the momentum-subtraction Taylor scheme. We provide readers with estimates of $\alpha_{\overline{\rm MS}}(m_\tau^2)$ and $\alpha_{\overline{\rm MS}}(m_Z^2)$ in very good agreement with experimental results. Including a dynamical c quark makes the needed running of $\alpha_{\overline{\rm MS}}$ safer.

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Introduction.—The confrontation of QCD, the theory for the strong interactions, with experiments requires a few inputs: one mass parameter for each quark species and an energy scale surviving in the limit of massless quarks, $\Lambda_{\rm QCD}$. This energy scale is typically used as the boundary condition to integrate the renormalization group equation for the strong coupling constant, α_S. The value of the renormalized strong coupling at any scale, or equivalently $\Lambda_{\rm QCD}$, has to be fitted to allow the QCD phenomenology to account successfully for experiments. A description of many precision measurements of α_S from different processes and at different energy scales can be found in Ref. [1]. The running QCD coupling can be alternatively obtained from lattice computations, where the lattice spacing replaces $\Lambda_{\rm QCD}$ as a dimensionful parameter to be adjusted from experimental inputs. This means that a lattice-regularized QCD can be a tool to convert the physical observation used for the lattice-spacing calibration, as for instance a mass or a decay constant, into $\Lambda_{\rm QCD}$. A review of most of the procedures recently implemented to determine the strong coupling from the lattice can be found in Ref. [2]. We also quoted in Ref. [3] many of the different methods proposed in the last few years.

The present "world average" for the strong coupling determinations [4], usually referred at the Z^0 mass scale, is dominated by the lattice determination included in the average [5], as discussed in Ref. [1]. Because of the importance of a precise and proper knowledge of the strong coupling for the LHC cross section studies and its exploration of new physics, independent alternative lattice determinations are strongly required. The latter is especially true when different lattice actions and procedures are applied, to gain thus the best possible control on any source of systematic uncertainty. Furthermore, the current lattice results have been obtained by means of simulations including only two degenerate up and down sea quarks ($N_f = 2$) or, as in Ref. [5], also including one more "tuned" to the strange quark ($N_f = 2 + 1$). Now, the European Twisted Mass (ETM) Collaboration has started a wide-ranging program of lattice QCD calculation with two light degenerate twisted-mass flavors [6,7] and a heavy doublet for the strange and charm dynamical quarks ($N_f = 2 + 1 + 1$) [8,9]. Within this ETM program, we have applied the method to study the running of the strong coupling, and so evaluate $\Lambda_{\rm QCD}$, grounded on the lattice determination of the ghost-gluon coupling in the so-called momentum-subtraction Taylor renormalization scheme [10,11]. We are publishing the results of this study in two papers: a methodological one [3], where the procedure is described in detail along with some results, and this short Letter aimed to update and emphasize the phenomenologically relevant results. In particular, as far as the lattice gauge fields with $2 + 1 + 1$ dynamical flavors that we are exploiting provide us with a very realistic simulation of QCD at the energy scales for the τ physics, we are presenting here the estimate for the coupling at the τ mass scale and directly comparing with the one obtained from τ decays. It should be noted that including the dynamical charm quark also makes the running up to the Z^0 mass scale safer.

The strong coupling in the Taylor scheme.—The starting point for the analysis of this Letter shall be the Landau-gauge running strong coupling renormalized in the momentum-subtraction-like Taylor scheme,

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda^2)}{4\pi} G(\mu^2,\Lambda^2)F^2(\mu^2,\Lambda^2), \quad (1)$$

obtained from lattice QCD simulations. F and G stand for the form factors of the two-point ghost and gluon Green functions (dressing functions). The procedure to compute the coupling defined by (1), and from it to perform an estimate of $\Lambda_{\overline{\rm MS}}$, is described in detail in Refs. [10,11]. We recently applied this in Ref. [3] to compute $\Lambda_{\overline{\rm MS}}$ from $N_f = 2 + 1 + 1$ gauge configurations for several bare couplings (β), light twisted masses ($a\mu_l$), and volumes. The prescriptions applied for the appropriate elimination of discretization artifacts, such as the so-called $H(4)$-extrapolation procedure [12], were also carefully explained in Ref. [3]. After this, we are left with the lattice estimates of the Taylor coupling, computed over a large range of momenta, that can be described above around 4 GeV by the following operator product expansion (OPE) formula [11]:

$$\alpha_T(\mu^2) = \alpha_T^{\rm pert}(\mu^2)\left[1 + \frac{9}{\mu^2}R(\alpha_T^{\rm pert}(\mu^2), \alpha_T^{\rm pert}(q_0^2)) \times \left(\frac{\alpha_T^{\rm pert}(\mu^2)}{\alpha_T^{\rm pert}(q_0^2)}\right)^{1-\gamma_0^{A^2}/\beta_0} \frac{g_T^2(q_0^2)\langle A^2\rangle_{R,q_0^2}}{4(N_C^2-1)}\right], \qquad (2)$$

where $1 - \gamma_0^{A^2}/\beta_0 = 27/100$ for $N_f = 4$ [13,14]. $R(\alpha, \alpha_0)$ for $q_0 = 10$ GeV {see Eq. (6) of [3]} is obtained as explained in the appendix of Ref. [11]. The purely perturbative running in Eq. (2) is given up to four loops by integration of the β function [4], where its coefficients are taken to be defined in the Taylor scheme [10,15]. Thus, $\alpha_T^{\rm pert}$ depends only on $\ln(\mu^2/\Lambda_T^2)$. This, however, allows us to fit both $g^2\langle A^2\rangle$ and Λ_T, the $\Lambda_{\rm QCD}$ parameter in the Taylor scheme, through the comparison of the prediction given by Eq. (2) and the lattice estimate of the Taylor coupling. The best fit of Eq. (2) to the lattice data published in Ref. [3] provided the estimates that can be read in Table I. In this Letter, we complete the previous analysis by including an *ad hoc* correction to account for higher power corrections (see Fig. 1) that allows us to extend the fitting window down to $p \simeq 1.7$ GeV and also apply the so-called plateau method to determine the best fit [10]. Furthermore, in addition to the lattice ensembles of gauge configurations described in Ref. [3], we study 60 more at $\beta = 2.1$ ($a\mu_l = 0.002$) and three new ensembles of 50 configurations at $\beta = 1.9$ and $a\mu_l = 0.003, 0.004, 0.005$ to perform a chiral extrapolation for the ratios of lattice spacings. We get $a(2.1, 0.002)/a(1.9, 0) = 0.685(21)$. The lattice scale at $\beta = 1.9$, 1.95, 2.1 is fixed by the ETM Collaboration through chiral fits to lattice pseudoscalar masses and decay constants, where $270 \lesssim m_{\rm PS} \lesssim 510$ MeV, that are required to take the experimental f_π and m_π at the physical point [8,9]: e.g., $a(1.9, 0) = 0.086\,12(42)$ fm.

TABLE I. The parameters for the best fit of Eq. (2) (see Ref. [3]) to lattice data (first row) and the same with Eq. (3) (second row). The conversion to the $\overline{\rm MS}$ scheme for $\Lambda_{\rm QCD}$ is done by applying Eq. (4). The renormalization point for the gluon condensate is fixed at $\mu = 10$ GeV. We quote statistical errors obtained by applying the jackknife method.

	$\Lambda_{\overline{\rm MS}}^{N_f=4}$ (MeV)	$g^2\langle A^2\rangle$ (GeV^2)	$(-d)^{1/6}$ (GeV)
Eq. (2) [3]	316(13)	4.5(4)	
Eq. (3)	324(17)	3.8(1.0)	1.72(3)

The Wilson OPE coefficient and the higher power corrections.—The OPE prediction for α_T given by Eq. (2) is dominated by the first correction introduced by the nonvanishing dimension-two Landau-gauge gluon condensate [16–21], where the Wilson coefficient is applied at the $\mathcal{O}(\alpha^4)$ order. In the previous methodological paper [3], we provided readers with a strong indication that the OPE analysis is indeed in order: it was clearly shown that the lattice data could be only explained by including nonperturbative contributions and that the Wilson coefficient for the Landau-gauge gluon condensate was needed to describe the behavior of data above $p \simeq 4$ GeV and up to $p \simeq 7$ GeV (see Fig. 2).

Now, in Fig. 1, the impact of higher power corrections is sketched: the plot shows the departure of the lattice data for the Taylor coupling from the prediction given by Eq. (2), plotted in terms of the momentum, with logarithmic scales for both axes. The data seem to indicate that the next-to-leading nonperturbative correction is highly dominated by a $1/p^6$ term. This might suggest that the $1/p^4$ OPE contributions are negligible when compared with the $1/p^6$ ones or that the product of the leading $1/p^4$ terms and the involved Wilson coefficients leaves with an effective $1/p^6$ behavior. Anyhow, this implies that we can effectively describe the Taylor coupling lattice data for all momenta above $p \simeq 1.7$ GeV with

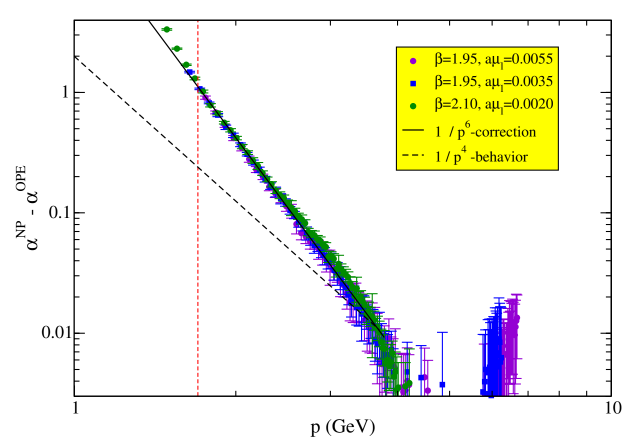

FIG. 1 (color online). The departure of lattice data from the leading nonperturbative (NP) OPE prediction for the running coupling plotted in logarithmic scales, in terms of the momentum, manifestly shows a next-to-leading $1/p^6$ behavior; the vertical dashed red line stands for the momentum scale, $p \simeq 1.7$ GeV, below which the lattice data do not follow the $1/p^6$ behavior any longer.

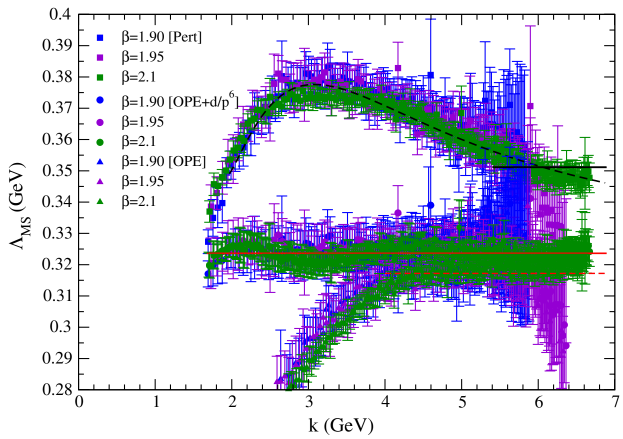

FIG. 2 (color online). $\Lambda_{\overline{\rm MS}}$ obtained by applying the plateau method to the lattice data labeled in the plot. The light gray (red) solid or dashed lines correspond to the plateaus for $\Lambda_{\overline{\rm MS}}$ obtained with Eq. (3) or (2). The black solid line is for Eq. (2) with $g^2\langle A^2\rangle = 0$, while the black dashed line corresponds to evaluate first Eq. (3) with the best-fit parameters in Table I and take then the resulting α_T to obtain $\Lambda_{\overline{\rm MS}}$ by inverting Eq. (2) with $g^2\langle A^2\rangle = 0$.

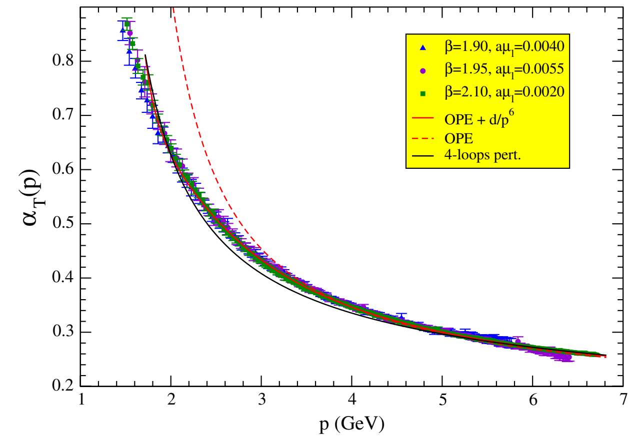

FIG. 3 (color online). Eq. (2) [light gray (red) dashed line] and Eq. (3) [light gray (red) solid line] for the parameters in Table I fitted to the lattice data for α_T defined by (1). The black line is for Eq. (2) with $g^2\langle A^2\rangle = 0$.

$$\alpha_T^d(p^2) = \alpha_T(p^2) + \frac{d}{p^6}, \tag{3}$$

where d is a free parameter to be fitted which we do not attribute to any particular physical meaning. Other possible *ad hoc* fitting formulas might be also applied and this can be thought to induce a systematic error on the determination of $\Lambda_{\overline{\rm MS}}$ in the next section. However, the comparison of perturbative and nonperturbative estimates will show this error not to be larger than around 20 MeV.

The strong coupling in the $\overline{\rm MS}$ *scheme.*—To obtain the $\overline{\rm MS}$ $\Lambda_{\rm QCD}$ from Λ_T is rather immediate, as the scale-independent $\Lambda_{\rm QCD}$ parameters in both the Taylor and $\overline{\rm MS}$ schemes are related through [11]

$$\frac{\Lambda_{\overline{\rm MS}}}{\Lambda_T} = \exp\left(-\frac{507 - 40N_f}{792 - 48N_f}\right) = 0.560\,832. \tag{4}$$

Then, one can numerically invert Eqs. (2) and (3) and apply Eq. (4) to determine $\Lambda_{\overline{\rm MS}}$ from all the lattice estimates of the Taylor coupling at any available momenta. $\Lambda_{\overline{\rm MS}}$ from different momenta must only differ by statistical fluctuations, provided that Eqs. (2) and (3) properly describe lattice data at those momenta. Thus, the parameters $g^2\langle A^2\rangle$ and d are to be fixed such that a constant fits with the minimum χ^2/d.o.f. to the $\Lambda_{\overline{\rm MS}}$ results obtained by the inversion of Eqs. (2) and (3). This is the plateau method applied in Fig. 2, which is equivalent to fitting Eqs. (2) and (3) directly to the Taylor coupling lattice data, as is done in Fig. 3. The best-fit parameters can be found in Table I. The best plateau with Eq. (3) is obtained for $\Lambda_{\overline{\rm MS}} = 0.324(17)$ GeV over a fitting window ranging from $p = 1.7$ GeV up to $p = 6.8$ GeV, where $\chi^2/\text{d.o.f.} = 146.9/516$, while $\chi^2/\text{d.o.f.} = 106.7/329$ over $4.1 < p < 6.8$ GeV for $\Lambda_{\overline{\rm MS}} = 0.316(13)$ with Eq. (2). For the sake of comparison, we also estimate $\Lambda_{\overline{\rm MS}}$ by inverting Eq. (2) with $g^2\langle A^2\rangle = 0$. A plateau is then possible for a narrow window only including the highest momenta, as for $5.5 < p < 6.8$ GeV, where we obtain $\Lambda_{\overline{\rm MS}} = 0.351(11)$ GeV with $\chi^2/\text{d.o.f.} = 107.2/154$. Indeed, these last estimates clearly show a systematic nonflat behavior that can be pretty well explained as described in the caption.

The $\overline{\rm MS}$ running coupling can be obtained again by the integration of the β function, with the coefficients now in the $\overline{\rm MS}$ scheme for $N_f = 4$. Thus, we can apply the two estimates of $\Lambda_{\overline{\rm MS}}$, that can be found in Table I, to run the coupling down to the scale of τ mass, below the bottom quark mass threshold, and compare the result with the estimate from τ decays [1], $\alpha_{\overline{\rm MS}}(m_\tau^2) = 0.334(14)$. This will produce, with the 1-σ error propagation, the two following results at the τ mass scale: $\alpha_{\overline{\rm MS}}(m_\tau^2) = 0.337(8)$ and $\alpha_{\overline{\rm MS}}(m_\tau^2) = 0.342(10)$. If we combine both estimates and conservatively add the errors in quadrature, we will be left with

$$\alpha_{\overline{\rm MS}}(m_\tau^2) = 0.339(13), \tag{5}$$

in very good agreement with the one from τ decays. This can be graphically seen in the plot of Fig. 4.

The determination of $\alpha_{\overline{\rm MS}}$ at the Z^0 mass scale implies first to run up to the MS running mass for the bottom quark, m_b, with β coefficients and $\Lambda_{\overline{\rm MS}}$ estimated for 4 quark flavors, and to apply next the matching formula [4]:

$$\alpha_{\overline{\rm MS}}^{N_f=5}(m_b) = \alpha_{\overline{\rm MS}}^{N_f=4}(m_b)\left[1 + \sum_n c_{n0}\left(\alpha_{\overline{\rm MS}}^{N_f=4}(m_b)\right)^n\right], \tag{6}$$

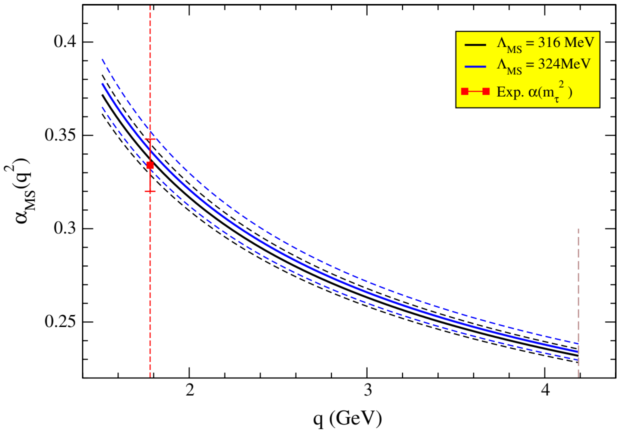

FIG. 4 (color online). The strong $\overline{\rm MS}$ coupling running for 4 quark flavors and for $\Lambda_{\overline{\rm MS}} = 316$ MeV (black lines) and $\Lambda_{\overline{\rm MS}} = 324$ MeV [dark gray (blue) lines] below the bottom mass threshold. The dashed lines represent the one-σ statistical deviations. The red point stands for the value of $\alpha_{\overline{\rm MS}}(m_\tau^2)$ obtained from τ decays [1].

where the coefficients c_{n0} can be found in Ref. [22], and then to run from the bottom mass up to the Z^0 mass scale. Thus, from our two estimates of $\Lambda_{\overline{\rm MS}}$, we obtain $\alpha_{\overline{\rm MS}}(m_Z^2) = 0.1198(9)$ and $\alpha_{\overline{\rm MS}}(m_Z^2) = 0.1203(11)$. Again, combining these two results and their errors added in quadrature, we will be left with

$$\alpha_{\overline{\rm MS}}(m_Z^2) = 0.1200(14), \tag{7}$$

lying in the same ballpark of lattice results from the PACS-CS Collaboration [23], $\alpha_{\overline{\rm MS}}(m_Z^2) = 0.1205(8)(5)$, estimated with 2 + 1 Wilson improved fermions but relatively large pion masses (~ 500 MeV), and from the HPQCD Collaboration [5], $\alpha_{\overline{\rm MS}}(m_Z^2) = 0.1183(8)$, with 2 + 1 staggered fermions. This last is consistently estimated from two different methods and 5 different lattice spacings and is included in the 2010 *world average* [4]: $\alpha_{\overline{\rm MS}}(m_Z^2) = 0.1184(7)$ {also in the very preliminary 2011 update [1]: $\alpha_{\overline{\rm MS}}(m_Z^2) = 0.1183(10)$}. Our estimate also agrees well with this world average, but still better with $\alpha_{\overline{\rm MS}}(m_Z^2) = 0.1197(12)$, the average obtained without the lattice HPQCD Collaboration result and without that from deep inelastic scattering nonsinglet structure functions [24], $\alpha_{\overline{\rm MS}}(m_Z^2) = 0.1142(23)$, which is more than 2 σ's away from most of the other involved estimates. However, if the HPQCD Collaboration lattice result, only including u, d, and s quarks, is replaced by the present one, also including the c quark, the world average would still be consistent: $\alpha_{\overline{\rm MS}}(m_Z^2) = 0.1191(8)$.

It should be noted that we applied two different fitting strategies, taking different fitting windows and studying the impact of higher order OPE corrections, and no systematic effects have been observed. Our error analysis is based on the jackknife method when we account for the fitted parameters, while the statistical uncertainties on the lattice sizes are properly propagated into the final estimates. Some other systematic effects (not included in our error budget), such as those related to the use of the twisted-mass action for the dynamical quarks or to the lattice size determination at the chiral limit, could also appear but can be only excluded by the comparison with other lattice and experimental estimates.

Conclusions.—We have presented the results for a first computation of the running strong coupling from lattice QCD simulations including u, d, s, and c dynamical flavors. We applied the procedure of determining the ghost-gluon coupling renormalized in the Taylor scheme over a large momenta window and then compared this with the perturbative running improved via nonperturbative OPE corrections. That procedure has been previously shown to work rather well when analyzing lattice simulations with $N_f = 0$ and 2 dynamical flavors and so happens here for $N_f = 2 + 1 + 1$. Our estimate for the running strong coupling at the τ mass scale nicely agrees with those from τ decays and, after being properly propagated up to the Z^0 mass scale, is pretty consistent with most of the estimates applied to obtain the current Particle Data Group (PDG) world average, although it is slightly larger than the $N_f = 2 + 1$ lattice result also used for this average.

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