Clustering Fossils from the Early Universe

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Many inflationary theories introduce new scalar, vector, or tensor degrees of freedom that may then affect the generation of primordial density perturbations. Here we show how to search a galaxy (or 21-cm) survey for the imprint of primordial scalar, vector, and tensor fields. These new fields induce local departures to an otherwise statistically isotropic two-point correlation function, or equivalently, nontrivial four-point correlation functions (or trispectra, in Fourier space), that can be decomposed into scalar, vector, and tensor components. We write down the optimal estimators for these various components and show how the sensitivity to these modes depends on the galaxy-survey parameters. New probes of parity-violating early-Universe physics are also presented.

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Galaxy clustering has proven to be invaluable in assembling our current picture of a universe with a nearly scale-invariant spectrum of the primordial curvature perturbations [1]. The principal tool in clustering studies has been the two-point correlation function—or in Fourier space, the power spectrum—determined under the assumption of statistical homogeneity (SH). With the advent of new generations of galaxy surveys, as well as longer-term prospects for measuring the primordial mass distribution with 21-cm surveys of the epoch of reionization [2] and/or dark ages [3], it is worthwhile to think about what can be further done with these measurements.

Many inflationary models introduce new fields that may couple to the inflaton responsible for generating curvature perturbations. The effects of these fields may then appear as local departures from SH, or as non-Gaussianity, in the curvature perturbation. For example, models with an additional scalar field introduce a nontrivial four-point correlation function (or trispectrum, in Fourier space) [4], which we will describe below as local departures from statistical homogeneity; apart from this correlation, the scalar field may leave no visible trace. There may also be vector (spin-1) fields V^μ [5]—or vector spacetime-metric perturbations brought to life in alternative-gravity theories [6]—that, if coupled to the inflaton φ (e.g., through a term $(\partial_\mu\varphi)\times(\partial_\nu\varphi)\partial^\mu V^\nu$) may leave an imprint on the primordial mass distribution without leaving any other observable trace. Similar correlations with a tensor (i.e., spin-2) field $T^{\mu\nu}$ (e.g., $(\partial_\mu\varphi)(\partial_\nu\varphi)T^{\mu\nu}$) can be envisioned. Even in the absence of new fields, there are tensor metric perturbations (gravitational waves) that may have higher-order correlations with the primordial curvature perturbation [7,8]. Tensor distortions to the two-point correlation function (“metric shear”) may also be introduced at late times [9,10], and late-time nonlinear effects may induce scalar-like distortions to the two-point function [11].

Here we describe how the fossils of primordial tensor, vector, and scalar fields are imprinted on the mass distribution in the Universe today. We express these relics in terms of two-point correlations that depart locally from SH or off-diagonal correlations of the density-field Fourier components. This formalism allows the correlations to be decomposed geometrically into scalar, vector, and tensor components. We write down the optimal estimators for these scalar, vector, and tensor correlations, and quantify the amplitudes that can be detected if these perturbations have (as may be expected in inflationary models) nearly scale-invariant spectra.

We begin with the null hypothesis that primordial density perturbations are statistically isotropic and Gaussian. This implies that the Fourier modes $\delta(\mathbf{k})$ of the density perturbation $\delta(\mathbf{x})$ (at some fixed time) have covariances, $\langle\delta(\mathbf{k})\delta(\mathbf{k}')\rangle = V\delta^D_{\mathbf{k},-\mathbf{k}'}P(k)$, where the Kronecker (Dirac) delta on the right-hand side is zero unless $\mathbf{k} = -\mathbf{k}'$, $P(k)$ is the matter power spectrum, and V is the volume of the survey. In other words, the different Fourier modes of the density field are uncorrelated under the null hypothesis.

Coupling of the inflaton to some other field produces non-Gaussianity in the mass distribution that appears as off-diagonal (i.e., $\mathbf{k}_1 \neq -\mathbf{k}_2$) correlations of the density-field Fourier components in the presence of a given realization of the new field. Global SH requires that a given Fourier mode $h_p(\mathbf{K})$ of wave vector $\mathbf{K}$ and polarization p (about which we will say more below) of the new field induces a correlation,

$$\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle|_{h_p(\mathbf{K})} = f_p(\mathbf{k}_1,\mathbf{k}_2)h_p^*(\mathbf{K})\epsilon^p_{ij}(\mathbf{K})k_1^i k_2^j \delta^D_{\mathbf{k}_{123}}, \tag{1}$$

where $\delta^D_{\mathbf{k}_{123}}$ is shorthand for a Kronecker delta that sets $\mathbf{k}_1+\mathbf{k}_2+\mathbf{K}=0$. Note that $h_p(\mathbf{K})$ here are the new-field Fourier components during inflation when their effect on primordial perturbations is imprinted. The function

$f_p(\mathbf{k}_1, \mathbf{k}_2)$ is related to the density-density–new-field bispectrum $B_p(k_1, k_2, K)$ and new-field power spectrum $P_p(K)$ through $B_p(k_1, k_2, K) \equiv P_p(K) f_p(\mathbf{k}_1, \mathbf{k}_2) \epsilon^p_{ij} k_1^i k_2^j$. Statistical isotropy requires that $f_p(\mathbf{k}_1, \mathbf{k}_2)$ be a function only of k_1^2, k_2^2, and $\mathbf{k}_1 \cdot \mathbf{k}_2$.

The parameter p labels the polarization state of the new field and $\epsilon^p_{ij}(\mathbf{K})$ its polarization tensor, a symmetric 3×3 tensor. The most general such tensor can be decomposed into six orthogonal polarization states [12], which we label $s = \{+, \times, 0, z, x, y\}$, that satisfy $\epsilon^p_{ij} \epsilon^{p', ij} = 2\delta_{pp'}$. These states can be taken to be two scalar modes $\epsilon^0_{ij} \propto \delta_{ij}$ and $\epsilon^z_{ij} \propto K_i K_j - K^2/3$, two transverse-vector ("vector") modes $\epsilon^{x,y}_{ij} \propto K^{(i} w^{j)}$ with $K^i w^i = 0$, and two transverse traceless modes (the "tensor" modes) ϵ_+ and $\epsilon_\times$.

If $\mathbf{K}$ is taken to be in the $\hat{\mathbf{z}}$ direction, then the $+$ polarization of the tensor mode has $\epsilon^+_{xx} = -\epsilon^+_{yy} = 1$ with all other components zero, and the $\times$ polarization has $\epsilon^\times_{xy} = \epsilon^\times_{yx} = 1$ with all other components zero. These two tensor modes are thus characterized by a $\cos 2\phi$ or $\sin 2\phi$ dependence, for ϵ^+ and $\epsilon^\times$, respectively, on the azimuthal angle about the $\mathbf{K}$ direction of the tensor mode. The first two columns in Fig. 1 show the distortions induced to an otherwise isotropic two-point correlation function by correlation of the density field with a $+$ and $\times$ polarized tensor mode. Shown there is a quadrupolar distortion in the x-y plane that then oscillates in phase as we move along the direction $\hat{\mathbf{z}}$ of the Fourier mode.

The first scalar mode has $\epsilon^0_{ij} = \sqrt{2/3}\delta_{ij,}$ and as shown in Fig. 1 represents an isotropic modulation of the correlation function as we move along the direction $\hat{\mathbf{z}}$ of the Fourier wave vector. The other scalar (or longitudinal-vector) mode has $\epsilon^z_{ij} \propto \mathrm{diag}(-1, -1, 2)/\sqrt{3}$ that represents a stretching and compression along $\hat{\mathbf{z}}$. Both scalar modes represent local distortions of the two-point function that have azimuthal symmetry about $\mathbf{K}$.

Finally, the two transverse-vector modes have $\epsilon^x_{xz} = \epsilon^x_{zx} = 1$ with all other components zero, and $\epsilon^y_{yz} = \epsilon^y_{zy} = 1$ with all other components zero. These two modes represent stretching in the $\pm xz$ and $\pm yz$ directions, respectively, as shown in the last h_x and h_y columns in Fig. 1. These two transverse-vector modes have $\cos\phi$ and $\sin\phi$ dependences on the azimuthal angle ϕ about the direction of the Fourier mode.

The specific functional form of $f_p(\mathbf{k}_1, \mathbf{k}_2)$ depends on the coupling of the new field (scalar, vector, or tensor) to the inflaton. Statistical isotropy requires, though, that $f_p(\mathbf{k}_1, \mathbf{k}_2)$ will be the same for the two tensor polarizations and the same for the two vector polarizations: i.e., $f_\times(\mathbf{k}_1, \mathbf{k}_2) = f_+(\mathbf{k}_1, \mathbf{k}_2)$, and $f_x(\mathbf{k}_1, \mathbf{k}_2) = f_y(\mathbf{k}_1, \mathbf{k}_2)$. The same is not necessarily true for the scalar perturbations. In fact, the polar-angle dependence that distinguishes the 0 and z polarizations can be absorbed into $f_0(\mathbf{k}_1, \mathbf{k}_2)$ and $f_z(\mathbf{k}_1, \mathbf{k}_2)$. Thus, in practice, one can describe the most general scalar distortions to clustering in terms of either the 0 or the z polarization by an appropriate definition of $f_0(\mathbf{k}_1, \mathbf{k}_2)$ or $f_z(\mathbf{k}_1, \mathbf{k}_2)$. (This is the mixing between a scalar mode and a longitudinal-vector mode.) Thus, we merge these two polarizations into a single polarization below, which we label with a subscript s.

Suppose now that a correlation such as that in Eq. (1), for either a scalar, vector, or tensor distortion, is hypothesized. How would we go about measuring it? According to Eq. (1), each pair $\delta(\mathbf{k}_1)$ and $\delta(\mathbf{k}_2)$ of density modes with $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2$ (note that we have redefined the sign of $\mathbf{K}$ here) provides an estimator,

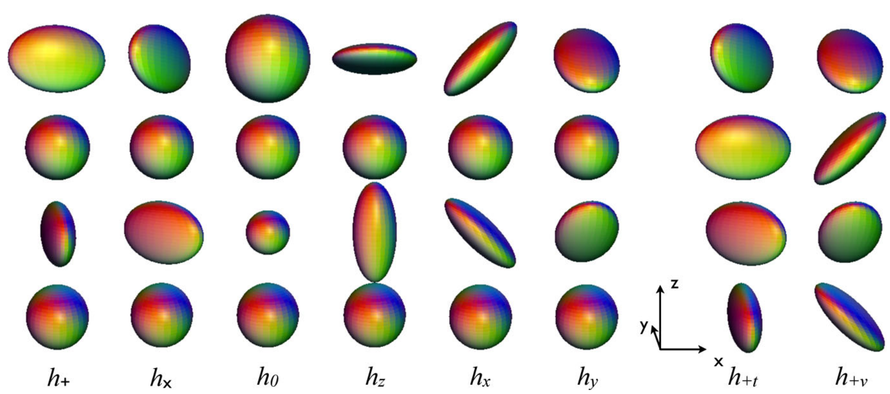

FIG. 1 (color online). Left six: The six possible types of distortions to an otherwise statistically isotropic two-point correlation function for a single Fourier mode, aimed in the $\hat{\mathbf{z}}$ direction, of the distortion pattern. The distortions to the sphere show the distortions of the two-point correlation function as one moves along the direction $\hat{\mathbf{z}}$ of the Fourier mode. The first two modes are the usual transverse-traceless tensor polarizations (gravitational waves), in which there are quadrupolar distortions in the plane transverse to the direction $\hat{\mathbf{z}}$ of the wave. The next two are scalar and longitudinal-vector distortions, respectively. The scalar mode represents an isotropic modulation while the longitudinal-vector mode stretches and compresses the correlations along $\hat{\mathbf{z}}$. The two transverse-vector modes induce quadrupolar distortions in the xz and yz directions, respectively. Right two: The circular polarizations of the tensor mode $(h + t)$ and vector mode $(h + v)$.

$$\widehat{h_p(\mathbf{K})} = \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)[f_p(\mathbf{k}_1, \mathbf{k}_2)\epsilon^p_{ij}k^i_1k^j_2]^{-1}, \quad (2)$$

for the Fourier-polarization amplitude $h_p(\mathbf{K})$. Since $\langle|\delta(\mathbf{k})|^2\rangle = VP^{\rm tot}(k)$, where $P^{\rm tot}(k) = P(k) + P^n(k)$ is the measured matter power spectrum, including the signal $P(k)$ and noise $P^n(k)$, the variance of this estimator is

$$2VP^{\rm tot}(k_1)P^{\rm tot}(k_2)|f_p(\mathbf{k}_1, \mathbf{k}_2)\epsilon^p_{ij}k^i_1k^j_2|^{-2}. \quad (3)$$

The minimum-variance estimator for $h_p(\mathbf{K})$ is then obtained by summing over all these individual $(\mathbf{k}_1, \mathbf{k}_2)$ pairs with inverse-variance weighting

$$\widehat{h_p(\mathbf{K})} = P^n_p(\mathbf{K})\sum_{\mathbf{k}}\frac{f^*_p(\mathbf{k}, \mathbf{K}-\mathbf{k})\epsilon^p_{ij}k^i(K-k)^j}{2VP^{\rm tot}(k)P^{\rm tot}(|\mathbf{K}-\mathbf{k}|)} \times \delta(\mathbf{k})\delta(\mathbf{K}-\mathbf{k}), \quad (4)$$

where the noise power spectrum,

$$P^n_p(K) = \left[\sum_{\mathbf{k}}\frac{|f_p(\mathbf{k}, \mathbf{K}-\mathbf{k})\epsilon^p_{ij}k^i(K-k)^j|^2}{2VP^{\rm tot}(k)P^{\rm tot}(|\mathbf{K}-\mathbf{k}|)}\right]^{-1}, \quad (5)$$

is the variance with which $\widehat{h_p(\mathbf{K})}$ is measured. This $P^n_p(K)$ is a function only of the magnitude K (not its orientation) as a consequence of statistical isotropy, and for the same reason, $P_\times(K) = P_+(K) \equiv P_t(K)$, for both the signal and noise power spectra, and similarly $P_x(K) = P_y(K) \equiv P_v(K)$.

In general, the amplitudes $h_p(\mathbf{K})$ arise as realizations of random fields with power spectra $P_h(K) = A_hP^f_h(K)$, for $h = \{s, v, t\}$, which we write in terms of amplitudes A_h and fiducial power spectra $P^f_h(K)$. We now proceed to write the optimal estimator for the amplitudes A_h.

Each Fourier-mode estimator $\widehat{h_p(\mathbf{K})}$ for the appropriate polarizations (s for scalar, x and y for vector, and $+$ and $\times$ for tensor) provides an estimator,

$$\widehat{A^{\mathbf{K},p}_h} = [P^f_h(K)]^{-1}[V^{-1}|\widehat{h_p(\mathbf{K})}|^2 - P^n_p(K)], \quad (6)$$

for the appropriate power-spectrum amplitude. Here we have subtracted the noise contribution to unbias the estimator. If $\widehat{h_p(\mathbf{K})}$ is estimated from a large number of $\delta(\mathbf{k}_1)$-$\delta(\mathbf{k}_2)$ pairs, then it is close to being a Gaussian variable. If so, then the variance of the estimator in Eq. (6) is, under the null hypothesis,

$$2[P^f_h(K)]^{-2}[P^n_p(K)]^2. \quad (7)$$

Adding the estimators from each Fourier mode with inverse-variance weighting leads us to the optimal estimator,

$$\hat{A}_h = \sigma^2_h\sum_{\mathbf{K},p}\frac{[P^f_h(K)]^2}{2[P^n_p(K)]^2}[V^{-1}|\widehat{h_p(\mathbf{K})}|^2 - P^n_p(K)], \quad (8)$$

where

$$\sigma^{-2}_h = \sum_{\mathbf{K},p}[P^f_h(K)]^2/2[P^n_p(K)]^2. \quad (9)$$

For the vector-power-spectrum amplitude, $\hat{A}_v$, we sum over $p = \{x, y\}$, and for the tensor-power-spectrum amplitude, $\hat{A}_t$, over $p = \{+, \times\}$. Following the discussion above, the sum on p is only for $p = s$ for $\hat{A}_s$.

The estimator in Eq. (8), along with the quadratic minimum-variance estimator in Eq. (4), demonstrates that the correlation of density perturbations with an unseen scalar, vector, or tensor perturbation appears in the density field as a nontrivial four-point correlation function, or trispectrum. The dependence of the trispectrum on the azimuthal angle about the diagonal of the Fourier-space quadrilateral distinguishes the shape dependences of the trispectra for scalar, vector, and tensor modes. To specify this trispectrum more precisely, though, requires inclusion of the additional contribution induced by modes $\mathbf{K}$ that involve the other two diagonals of the quadrilateral. Likewise, if a signal is detected—i.e., if the null-hypothesis estimators above are found to depart at $>3\sigma$ from the null hypothesis—then the optimal measurement and characterization of the trispectrum requires modification of the null-hypothesis estimators in a manner analogous to weak-lensing estimators [13].

We now evaluate the smallest amplitudes A_s, A_v, and A_t that can be detected with a given survey. To do so, we take for our fiducial models nearly scale-invariant spectra $P_h(K) = A_hK^{n_h-3}$, with $|n_h| \ll 1$. Moreover, we take the density-density–new-field bispectrum to be the squeezed limit of the density-density-tensor bispectrum form found in Ref. [7] for single-field slow-roll inflation, and assume for simplicity, the same $f_p(\mathbf{k}_1, \mathbf{k}_2)$ for scalar and vector modes. We then find that the integrand (using $\sum_{\mathbf{k}} \rightarrow V\int d^3k/(2\pi)^3$) in Eq. (5) is dominated by the squeezed limit ($K \ll k_1 \simeq k_2$) where $f_p(\mathbf{k}_1, \mathbf{k}_2) \simeq -(3/2)P(k_1)/k^2_1$. We then approximate $P(k)/P^{\rm tot}(k) \simeq 1$ for $k < k_{\rm max}$, where $k_{\rm max}$ is the largest wave number for which the power spectrum can be measured with high signal to noise, and $P(k)/P^{\rm tot}(k) \simeq 0$ for $k > k_{\rm max}$. This then yields a noise power spectrum $P^n_{\{v,t\}}(K) \simeq 20\pi^2/k^3_{\rm max}$ and $P^n_s(K) \simeq 8\pi^2/k^3_{\rm max}$. Evaluating the integral in Eq. (9), we find the scalar, vector, and tensor amplitudes detectable at $\gtrsim 3\sigma$ (for $n_h \simeq 0$) to be

$$3\sigma_h \simeq 30\pi\sqrt{3\pi}C_h\left(\frac{k_{\rm max}}{k_{\rm min}}\right)^{-3} \simeq 288C_h\left(\frac{k_{\rm max}}{k_{\rm min}}\right)^{-3}, \quad (10)$$

where $C_{\{t,v\}} = 1$ and $C_s = 2/5$. The smallest detectable power-spectra amplitudes are thus inversely proportional to the number of Fourier modes in the survey. We show the projected detection sensitivities for surveys with volumes of $200[{\rm Gpc}/h]^3$ and $10[{\rm Gpc}/h]^3$ in Fig. 2.

For example, if the new field is a scalar field that gives rise to a local-model trispectrum of amplitude $\tau_{\rm NL}$ [14], we may identify $A_s = 2.76 \times 10^{-7}\tau_{\rm NL}$ [15], suggesting a sensitivity, to $\tau_{\rm NL} \simeq 345$, from a galaxy survey of volume $V = 100[{\rm Gpc}/h]^3$ with $k_{\rm max} = 0.1[h/{\rm Mpc}]$. As another example, if there are tensor distortions to the two-point

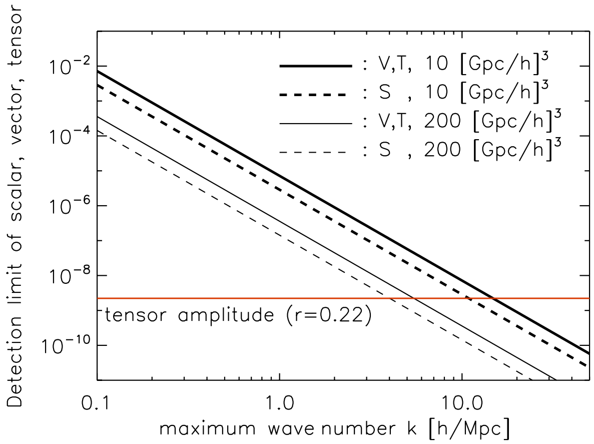

FIG. 2 (color online). The smallest scalar, vector, and tensor power-spectrum amplitudes A_s, A_v, and A_t, respectively, detectable at the 3σ level as a function of the maximum wave number $k_{\rm max}$ of the survey. Shown are results for survey volumes of 10 $[{\rm Gpc}/h]^3$ and 200 $[{\rm Gpc}/h]^3$, or minimum wave numbers $k_{\rm min} \simeq 0.001$ $[h/{\rm Mpc}]$ and $k_{\rm min} \simeq 0.003$ $[h/{\rm Mpc}]$, respectively.

correlation function induced by primordial gravitational waves, then a sensitivity to a tensor amplitude $A_t \simeq 2 \times 10^{-9}$ near the current upper limit requires $k_{\rm max}/k_{\rm min} \gtrsim 5200$. Such a dynamic range is probably beyond the reach of galaxy surveys, but it may be within reach of the 21-cm probes of neutral hydrogen during the dark ages envisioned in Refs. [10,16]. Of course, the signal could be larger if the inflaton is correlated with a scalar, vector, or tensor field that leaves no other trace.

Finally, several new tests for parity-violating early-Universe physics can be developed from simple modification of the estimators above. To do so, we substitute the x and y polarizations, and the $+$ and $\times$ polarizations, with circular-polarization tensors $\epsilon_{ij}^{\pm v} = \epsilon_{ij}^{x} \pm i\epsilon_{ij}^{y}$ and $\epsilon_{ij}^{\pm t} = \epsilon_{ij}^{+} \pm i\epsilon_{ij}^{\times}$. The two right-most patterns shown in Fig. 1 are the circular polarization patterns for tensor and vector modes. It may then be tested whether the power spectra for right- and left-circular polarizations are equal. For example, chiral-gravity models [17] may predict such parity-violating signatures in primordial gravitational waves, and similar models with parity-violating vector perturbations are easily imaginable.

Of course, "real-world" effects like redshift-space distortions, biasing, and nonlinear evolution, must be taken into account before the estimators written above can be implemented, but there are well-developed techniques to deal with these issues [18].

In summary, the most general two-point correlation function for the cosmological mass distribution can be decomposed into scalar, vector, and tensor distortions. We have presented straightforward recipes for measuring these distortions. Such effects may arise if the inflaton is coupled to some new field during inflation. We have avoided discussion of specific models, but the introduction of new fields during inflation is quite generic to inflationary models. We therefore advocate measurement of these correlations with galaxy surveys, and in the future with 21-cm surveys, as a simple and general probe of new inflationary physics.

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