Oscillons after Inflation

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(Received 22 June 2011; published 14 June 2012)

Oscillons are massive, long-lived, localized excitations of a scalar field. We show that in a class of well-motivated single-field models, inflation is followed by self resonance, leading to copious oscillon generation and a lengthy period of oscillon domination. These models are characterized by an inflaton potential which has a quadratic minimum and is shallower than quadratic away from the minimum. This set includes both string monodromy models and a class of supergravity inspired scenarios and is in good agreement with the current central values of the concordance cosmology parameters. We assume that the inflaton is weakly coupled to other fields so as not to quickly drain energy from the oscillons or prevent them from forming. An oscillon-dominated universe has a greatly enhanced primordial power spectrum on very small scales relative to that seen with a quadratic potential, possibly leading to novel gravitational effects in the early Universe.

DOI: 10.1103/PhysRevLett.108.241302 PACS numbers: 98.80.Cq

Simple, single-field models of inflationary cosmology are often associated with energy scales far beyond the reach of present day accelerators, and the properties of the postinflationary Universe are largely unknown. One constraint on this phase is that energy must be extracted from the oscillating inflaton condensate, ensuring that the Universe becomes radiation dominated, setting the scene for the hot big bang and the production of the cosmological neutrino background and nucleosynthesis. A widely-studied candidate for this process is parametric resonance [1–3]. In many cases, the potential can be self resonant, where resonance generates quanta of the inflaton field itself more efficiently than particles coupled to the inflaton. We show that in a class of well-motivated self-resonant models, the Universe may become dominated by oscillons: massive, localized, metastable configurations of a scalar field [4–13].

In this Letter, we study a single inflaton, ϕ, with a canonical kinetic term and potential, $V(\phi)$, minimally coupled to Einstein gravity. Oscillons can form if

$$V(\phi) = \frac{m^2\phi^2}{2} + U(\phi), \tag{1}$$

where $U(0) = 0$ and $U(\phi) < 0$ for some range of ϕ (see, e.g. [10]). Consider potentials with $V(\phi) \sim \phi^{2\alpha}$ during inflation and $\alpha < 1$. These are generated by a number of string and supergravity scenarios [14–20] and yield $U(\phi) < 0$ at large ϕ. We require that $V(\phi)$ has a stable minimum, which we chose to be at the origin, so it is natural to expect that $V(\phi) \sim \phi^2$ for small ϕ. Finally, by continuity there is necessarily some crossover scale, $\phi \approx M$, between these two regimes. We capture this with the following explicit potential:

$$V(\phi) = \frac{m^2M^2}{2\alpha}\left[\left(1 + \frac{\phi^2}{M^2}\right)^{\alpha} - 1\right]. \tag{2}$$

The precise forms of $V(\phi)$ in scenarios with $V \sim \phi^{2\alpha}$ can differ from Eq. (2); however, our results suggest that while oscillon formation is sensitive to M, it is insensitive to the detailed form of the potential. Moreover, for $\alpha = 1/2$ we reproduce the axion monodromy potential [15,16]. We stipulate that the couplings between the inflaton and other fields are small enough for them to be ignored.

The tensor-scalar ratio, r, and scale dependence in the scalar perturbations, $|n_s - 1|$, grows with α (see, e.g. [21]). Quartic inflation ($\alpha = 2$) is ruled out by current data [22–24] and even quadratic inflation ($\alpha = 1$) is somewhat disfavored relative to models with $\alpha < 1$ [25]. Consequently, the above potential is well motivated, both theoretically and phenomenologically. (Oscillon production in hybrid inflation models is studied in [9]. These models have $n_s \geq 1$ and are disfavored by observations.)

The postinflationary Universe is initially smooth, so even if a potential supports oscillon solutions, an actual oscillon-dominated phase requires a mechanism for generating inhomogeneity within the postinflationary horizon (see, e.g. [11]). Equation (2) supports parametric resonance when $\alpha < 1$, which leads to the explosive production of ϕ quanta and a highly inhomogeneous universe. However, when M is large the $V(\phi)$ is effectively quadratic during both the last portion of inflation and subsequent oscillatory phase, suppressing resonance and oscillon production. Conversely, if M is significantly sub-Planckian we see resonance and oscillons can form. Note that narrow resonance also occurs when $\alpha > 1$, but $V(\phi)$ cannot support oscillons.

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In what follows, we first summarize the inflationary dynamics and describe a Floquet analysis of the resonant phase. We show that strong resonance and a subsequent oscillon-dominated phase requires $0 \leq \alpha \lesssim 0.9$ and $M \lesssim 0.05\ M_{\rm pl}$ ($M_{\rm pl} \equiv 1/\sqrt{8\pi G_N}$), which may be realized in the physical scenarios that motivate these models. We then discuss the cosmological consequences of an oscillon-dominated phase.

Inflationary dynamics.—The observed amplitude of the primordial fluctuations effectively removes one free parameter from the potential in Eq. (2). Further, we will see that we are primarily interested in models where M is substantially smaller than the Planck mass so that $V(\phi) \approx m^2 M^2 (\phi/M)^{2\alpha}/2\alpha$ during inflation.

Astrophysically interesting perturbations are laid down when the remaining number of e-folds before the end of inflation, $N \sim 55$; though, in general, N is a function of the postinflationary expansion history [21]. Using standard slow-roll approximations, the amplitude of the power spectrum of curvature perturbations is

$$\Delta_R^2 = \frac{1}{96\pi^2\alpha^3}\left(\frac{m}{M_{\rm pl}}\right)^2\left(\frac{M}{M_{\rm pl}}\right)^{2-2\alpha}(4\alpha N)^{1+\alpha}. \qquad (3)$$

For a given α and $\beta \equiv M_{\rm pl}/M$, we use the above equation with $N = 55$ and $\Delta_R^2 = 2.4 \times 10^{-9}$ [23] to deduce m.

Resonance and oscillons.—Oscillon production at the end of inflation with $U(\phi) = -\lambda\phi^4/4 + g^2\phi^6/6m^2$ was studied in [11,13]. When $(\lambda/g)^2 \ll 1$, oscillons are copiously generated with properties matching analytic predictions [10,11]. However, an inflationary phase where $V(\phi)$ is dominated by a ϕ^6 term has an unphysical perturbation spectrum. If the above $U(\phi)$ is viewed as a truncation of Eq. (2), then $(\lambda/g)^2 \sim 1$, and we cannot appeal to the results of [10,11,13] for the properties of oscillons. Thus, to study oscillon formation in this physically reasonable scenario, we rely on numerical simulations.

We can gain a heuristic understanding of oscillon formation by looking at the instability diagram for the potential in Eq. (2), as resonance generates large inhomogeneities which then relax to form oscillons [11]. Ignoring expansion and working in the limit where ϕ is approximately homogeneous, Floquet theory allows us to write the individual momentum modes of ϕ as

$$\phi_k = P_+(t)e^{\mu_k t} + P_-(t)e^{-\mu_k t}, \qquad (4)$$

where $P_\pm(t)$ are periodic functions and $\pm\mu_k$ are called Floquet exponents. Our first task is to calculate these exponents: if the real part of μ_k, $\Re(\mu_k)$, is nonzero and its magnitude is larger than the Hubble parameter, $H \sim t^{-1}$, at the end of inflation the mode will grow. Roughly speaking, if $|\Re(\mu_k)|/H \gtrsim 10$, we have strong resonance. In an expanding universe, ϕ_k has a physical wave number $k/a(t)$ and, thus, moves through a number of Floquet bands as the scale factor, $a(t)$, grows, as shown in Fig. 1. For our potential with $\beta = M_{\rm pl}/M$, one can show that the maximum value of $|\Re(\mu_k)|/H$ as the modes traverse the Floquet bands is $[|\Re(\mu_k)|/H]_{\rm max} \approx A(\alpha)\beta$, where $A(\alpha) \approx \frac{1}{2}[(1-\alpha) - (1/10)(1-\alpha)^2]$.

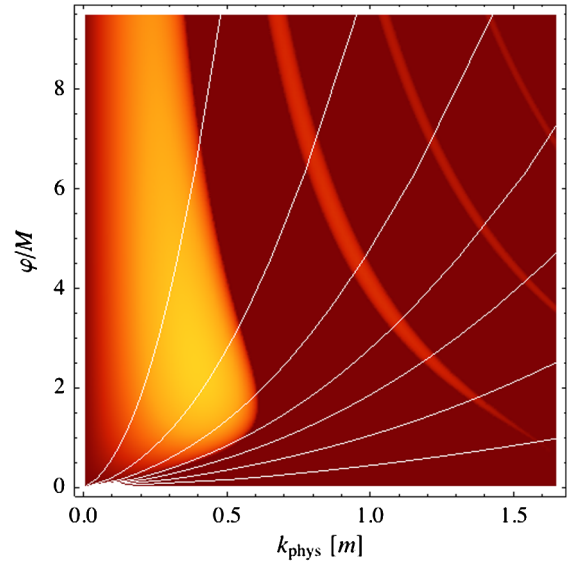

FIG. 1 (color online). Floquet diagram with $\alpha = 1/2$, $\beta = 100$. The stable regions are shaded dark and uniformly. Within the unstable bands, lighter shades correspond to larger real-valued Floquet exponents. White lines show $k/a(t)$ for representative modes in an expanding universe.

We studied the nonlinear dynamics of resonance following inflation driven by Eq. (2) using PSPECTRE [26]. PSPECTRE solves the fully nonlinear three-dimensional Klein-Gordon equation in an expanding background whose behavior is governed by the usual Friedmann equations, sourced by the average density and pressure. The backreaction of metric perturbations on the field is ignored. Our simulations begin at the first instant $\dot\phi = 0$, although our results are insensitive to the details of this choice. The scale factor $a = 1$ at the beginning of our simulations. We assume a standard spectrum of initial vacuum fluctuations, although we checked that our results are qualitatively insensitive to the detailed form of the initial conditions. We ignore backreaction of the metric perturbations on the field evolution—these can be shown to be small during resonance. The initial box size is $L = 25/m$ with 256^3 points in the (comoving) simulation volume.

A single time slice of a representative simulation is shown in Fig. 2. Given that oscillons are large overdensities, a necessary condition for oscillon domination is that

$$f = \frac{\int_{\rho>2\langle\rho\rangle}\rho dV}{\int \rho dV}, \qquad (5)$$

the fraction of the total energy density contributed by regions where $\rho/\langle\rho\rangle > 2$, is nontrivial. Oscillons are effectively fixed in space, persisting for a Hubble time or more, so the overall density in an oscillon-dominated universe at different times is strongly correlated. Heuristically,

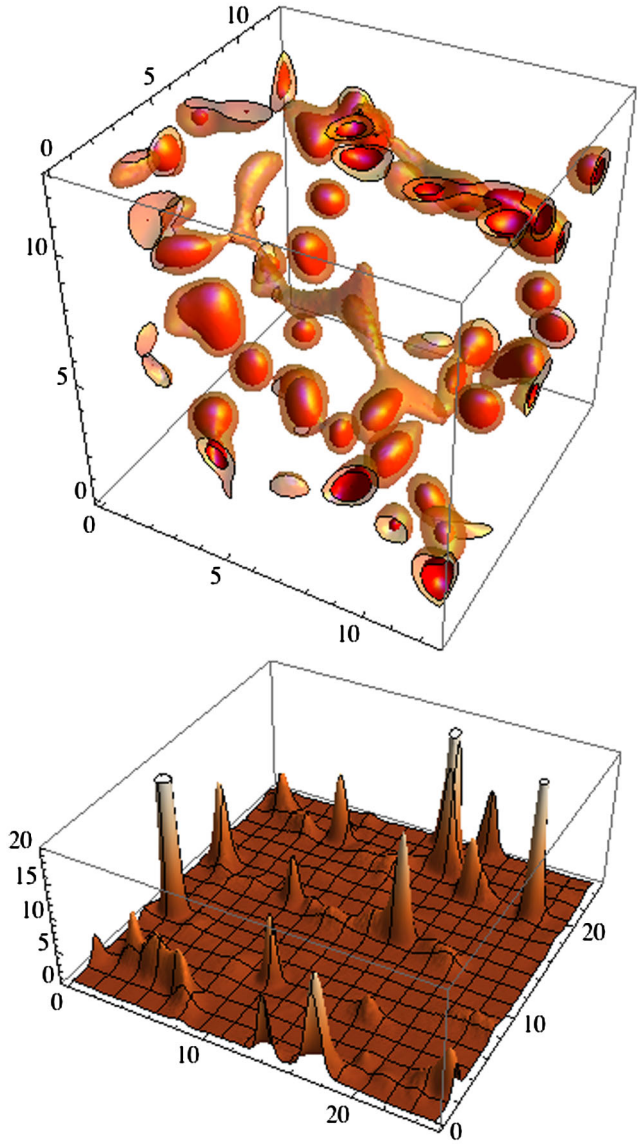

FIG. 2 (color online). Oscillon configuration with $\alpha = 1/2$ and $\beta = 50$. The top plot shows regions where $\rho/\langle\rho\rangle > 4$ (transparent) and 12 (solid), while the lower plot shows $\rho/\langle\rho\rangle$ on a two-dimensional slice through the simulation. Length units are $1/a(t)m$, and these plots were made when $a(t) = 5.46$.

$f \gtrsim 0.3$ is sufficient to guarantee that the field configuration (and thus the postinflationary Universe as a whole) was dominated by oscillons. Figure 3 shows f as a function of α and β, along with the maximal value of the resonance parameter $[|\Re(\mu_k)|/H]_{\max}$. We see that strong resonance, or $[|\Re(\mu_k)|/H]_{\max} \gtrsim 10$, is both necessary and sufficient for prompt, copious oscillon formation.

In models for which f is nonzero, it remains approximately constant for a several Hubble times after the onset of oscillon domination, demonstrating that this phase is long lived relative to prevailing cosmological time scales. Unlike the oscillons studied in [10,11,13] which have a stable, radial envelope, $\Phi(r)$, that evolves very slowly with

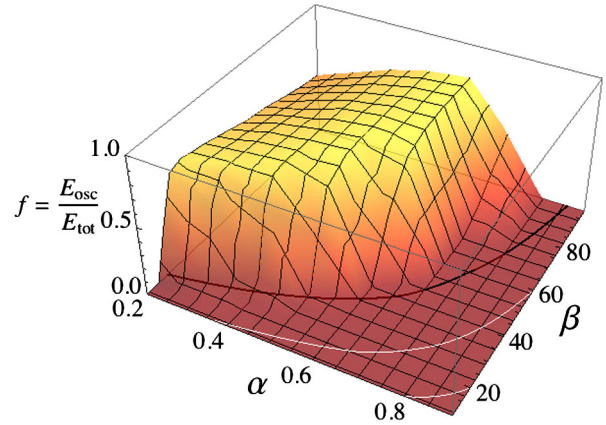

FIG. 3 (color online). The statistic f is shown at $a(t) = 7$ [$a(t) = 1$ at the beginning of the simulation] as a function of α and $\beta = M_{\rm pl}/M$. Contours show maximal value of the $[|\Re(\mu_k)|/H]_{\max}$. The thick black contour denotes $[|\Re(\mu_k)|/H]_{\max} = 7$, whereas the thin white ones correspond to $[|\Re(\mu_k)|/H]_{\max} = 1, 3$.

time, here the corresponding envelope is a periodic function of time, and the oscillon "breathes" in and out. The detailed dynamics of these oscillon solutions will be discussed in a future publication, but we have simulated a single oscillon (ignoring expansion) over a long interval for representative values of α and β after imposing strict radial symmetry, reducing the problem to a $1 + 1$ partial differential equation. Even though oscillons are not protected by a conserved charge and radiate energy [27–29], these simulations suggest that they live long enough for the Universe to grow by a factor of 100 or more, and we expect this to be true even if the assumption of radial symmetry is dropped. Also, the quantum radiation will be small in the regime where the self couplings, such as $\lambda \sim m^2/M^2$, are small [29].

Consequences and discussion.—We have demonstrated that for a large class of models, in excellent agreement with the current concordance cosmology, inflation is naturally followed by an oscillon-dominated phase, provided that the couplings to other fields are small. These oscillons are generated by parametric resonance, which occurs if the inflationary potential turns over from the slow-roll regime to a quadratic regime at a scale $M \ll M_{\rm pl}$.

The inflationary models here are self resonant, so oscillon production does not require specific couplings to other fields. It is likely that any significant couplings between the inflaton and other fields can inhibit the formation of oscillons by allowing resonant production of quanta of these additional fields. Further, couplings to other fields can reduce the stability of oscillons by providing an additional channel into which they can radiate energy. Lastly, the impact of interactions between oscillons is largely unexplored (however, see [30]).

Many resonant models include light fields, leaving the Universe in an intermediate state between matter and radiation [31,32], but massive self-resonant models lead

to an oscillon-dominated universe that is effectively matter dominated. Our simulations do not include local gravity, but perturbations with subhorizon wavelengths will grow gravitationally during the oscillon-dominated phase. The same behavior is seen in nonresonant models with an (almost) homogeneous inflaton condensate oscillating in a pure $m^2\phi^2$ potential [33]. However, in this case the primordial density fluctuations are $\mathcal{O}(10^{-5})$ at the scale of the horizon and take a long time to become nonlinear. By contrast, fluctuations grow rapidly in a self-resonant model, leading to a significant enhancement in the primordial power spectrum for high k. The possibility of gravitational collapse and even primordial black hole formation during this phase must be carefully analyzed [34,35]. Given that the oscillons exist on comoving scales vastly shorter than those which contribute to large-scale structure formation, oscillon formation is unlikely to directly modify the primordial power spectrum on present-day astrophysical scales. However, for any inflationary model the observed power spectrum is a function of the postinflationary expansion history [21,24]. Thus, it will be important to account for the existence and duration of any matter-dominated phase, oscillon-dominated or otherwise, when computing the detailed predictions of the model.

In summary, we have shown that a significant class of realistic inflationary models can naturally lead to copious oscillon production following inflation and that these oscillons can—for a time—dominate the overall matter density of the Universe. This provides a dramatic example of the potential importance of nonlinear dynamics in scalar fields to the properties of the very early Universe.

We thank Alan Guth, David Shirokoff, Ruben Rosales, and Evangelos Sfakianakis for useful conversations. R. E. and R. F. are partially supported by the Department of Energy (DE-FG02-92ER-40704) and NSF (CAREER-PHY-0747868). H. F. is supported, in part, by the United States Department of Energy under Award No. CSGF-DE-FG02-97ER25308. M. H. is supported by NSF Grant No. PHY-0756174. This work made use of the facilities and staff of the Yale University High Performance Computing Center.

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