Enhanced Subthreshold e^+e^- Production in Short Laser Pulses

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The emission of e^+e^- pairs off a probe photon propagating through a polarized short-pulsed electromagnetic (e.g., laser) wave field is analyzed. A significant increase of the total cross section of pair production in the subthreshold region is found for decreasing laser pulse duration even in the case of moderate laser pulse intensities.

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The history of the study of e^+e^- production in $\gamma'\gamma$ interaction starts with the pioneering work by Breit and Wheeler [1] published in 1934. About 30 years later, Reiss [2] and Narozhnyi, Nikishov, and Ritus [3,4] analyzed the e^+e^- emission off a photon γ' propagating in the field of an intensive polarized monochromatic electromagnetic (em) plane. The e^+e^- production probabilities were found using the nonperturbative Volkov solutions for the electron and positron wave functions [5].

If one identifies the external em field with a laser pulse, then most of the early work considers long lasting pulses where the temporal shape can be neglected. We denote this approach as the infinite pulse approximation (IPA). In IPA, electrons e^- and positrons e^+ become quasiparticles with effective quasimomenta and effective (dressed) masses. Differential and total probabilities of the e^+e^- pair emission depend on the reduced strength of the em field A^μ, $\xi^2 = -\frac{e^2\langle A^2\rangle}{M_e^2} \equiv \frac{e^2a^2}{M_e^2}$, where M_e is the electron mass (we use $c = \hbar = 1$, $e^2/4\pi = \alpha = 1/137$). Furthermore, the dimensionless variable $\zeta = \frac{s_{\rm thr}}{s}$ is introduced, where s is the square of the total energy in the center-of-mass system (c.m.s.) of the Breit-Wheeler process $\gamma' + \gamma \to e^+ + e^-$ and $s_{\rm thr} = 4M_e^2$ is its threshold value. The Ritus variable is then defined by $\kappa = 2\xi/\zeta$ [4]. The case of $\zeta > 1$ corresponds essentially to multiphoton processes. Within IPA, the minimum number of photons γ in the reaction $\gamma' + n\gamma \to e^+ + e^-$ is defined as $n_{\rm min} = I(\zeta) + 1$, where $I(\zeta)$ is the integer part of ζ. First evidence of the multiphoton Breit-Wheeler process with $\zeta = 3.83$ and $0.1 < \xi < 0.35$ was detected at SLAC in the E-144 experiment [6], where the application of IPA is justified since the used laser pulses contain around 10^3 cycles in a shot.

The rapidly evolving laser technology [7] can provide the laser power up to 10^{24}–10^{25} W/cm^2 in the near future, which is sufficient for the formation of positrons from cascade processes in the photon-electron-positron plasma [8–10] generated by photon-laser [11–13], electron-laser, [14,15] or laser-laser interactions [16,17] (see [18] for surveys). The next generation of optical laser beams is expected to be essentially short (femtosecond duration) with only a few oscillations of the em field in the pulse to be expected at ELI [19] and CLF [20] facilities. This requires the generalization of the IPA multiphoton process $\gamma' + n\gamma \to e^+ + e^-$ to a finite pulse duration. Formally, this generalization may be done in a straightforward manner by substituting the expansion in Fourier series into Fourier integrals taking into account the Volkov solution for the finite wave field. In practice, an evaluation of the total cross section requires the calculation of five-dimensional integrals with rapidly oscillating integrands which is rather demanding. Therefore, previous considerations are often restricted to the analysis of the three-dimensional differential cross sections; see, for example, [12] for finite beam size effects in e^+e^- pair production (cf., also [21] and references therein).

The aim of the present Letter is to elaborate a method for the calculation of the total cross section in the subthreshold (multiphoton) region accounting for the effect of finite laser-pulse duration in e^+e^- pair production off a probe photon. We denote such a process with a finite pulse and plane wave fronts as finite pulse approximation (FPA). In this case, the in or out fermion states refer to the vacuum. Moreover, due to the modulation of the pulse envelope function, the power spectrum contains frequencies $>\omega$ (see below) which enhance the pair production in the subthreshold region even for moderately strong laser intensities.

We consider the em four-potential $A \sim (0, \mathbf{A})$ in FPA, depending solely on the invariant phase $\phi = k \cdot x$,

$$\mathbf{A}(\phi) = f(\phi)(\mathbf{a}_1 \cos\phi + \mathbf{a}_2 \sin\phi), \quad (1)$$

where $|\mathbf{a}_1| = |\mathbf{a}_2| = a$, $\mathbf{a}_1\mathbf{a}_2 = 0$ for circular polarization. We employ here the envelope function $f(\phi) = 1/\cosh(\phi/\Delta)$, where $\Delta = \pi\frac{\tau}{\tau_0} = \pi N$, and N characterizes the number of cycles in a pulse; $\tau_0 = 2\pi/\omega$ is the time of one cycle for the laser frequency ω. Thus, τ is the time scale of the pulse duration. The case of pulses obeying $\omega\tau \gg 1$ has been analyzed in [22].

Utilizing the em potential (1) in the Volkov solutions leads to two significant modifications of the transition amplitude. Besides physical asymptotic momenta and masses, the finite time τ requires Fourier integrals in the integrand of invariant amplitudes, and the discrete harmonics become continuous. Thus, the S matrix element of the process $\gamma' \to e^+(\gamma) + e^-(\gamma)$, where $e^\pm(\gamma)$ refers to Volkov states in the field (1), is expressed as

$$S = \int_\zeta^\infty dl M(l) \frac{(2\pi)^4 \delta^4(k' + lk - p - p')}{\sqrt{2p_0 2p_0' 2\omega'}}, \quad (2)$$

where the transition matrix $M(l)$, similarly to the case of the nonlinear Compton effect [23–26] as a crossed channel of the pair production, consists of four terms

$$M(l) = \sum_{m=0}^{3} M^{(m)} C^{(m)}(l), \quad (3)$$

where

$$C^{(m)}(l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\phi \chi^{(m)}(\phi) e^{il\phi - i\mathcal{P}(\phi)}. \quad (4)$$

Here, $\chi^{(m)} = (1, f^2(\phi), f(\phi)\cos\phi, f(\phi)\sin\phi)$ with $m = 0, 1, 2, 3$ and

$$\mathcal{P}(\phi) = z\mathcal{P}_0(\phi, \phi_0) - \xi^2 \zeta u \int_{-\infty}^{\phi} d\phi' f^2(\phi'), \quad (5)$$

$$\mathcal{P}_0(\phi, \phi_0) = \int_{-\infty}^{\phi} d\phi' \cos(\phi' - \phi_0) f(\phi'), \quad (6)$$

where $u = (k\cdot k')^2/[4(k\cdot p)(k\cdot p')]$, $z = 2l\xi\sqrt{u(u_0 - u)}/u_0$, $u_0 = l/\zeta$. The angle ϕ_0 is related to the azimuthal angle of the positron in the e^-e^+ rest frame by $\phi_0 = \phi_p + \pi$ and can be determined through invariants $\alpha_{1,2} = e[(a_{1,2}\cdot p)/(k\cdot p) - (a_{1,2}\cdot p')/(k\cdot p')]$ as $\cos\phi_0 = \alpha_1/z$, $\sin\phi_0 = \alpha_2/z$. Here, $p_{e^-} \equiv p' \sim (p_0', \mathbf{p}')$ and $p_{e^+} \equiv p \sim (p_0, \mathbf{p})$. The transition operators $M^{(2,3)}$ are the same as in IPA [4], while the operators $M^{(0,1)} = \bar{u}_{p'} \hat{M}^{(0,1)} v_p$, read now

$$\hat{M}^{(0)} = \not\varepsilon', \qquad \hat{M}^{(1)} = \frac{e^2 \not A \not k \not\varepsilon' \not k \not A}{4(k\cdot p)(k\cdot p')}, \quad (7)$$

where $u_{p'}$ and v_p are the free-field Dirac spinors of the outgoing electron and positron, respectively; ε' is the polarization four-vector of the probe photon γ' with four-momentum $k' \sim (\omega', \mathbf{k}')$, and $k \sim (\omega, \mathbf{k})$ is the four-momentum of the em (laser) field (1). Feynman's slash notation is employed, e.g., $\not A = A\cdot\gamma$, is the four-product with the Dirac γ matrices. The integrand of the function $C^{(0)}$ does not contain the envelope function and needs a regularization, e.g., using a prescription given in Ref. [23]

$$C^{(0)}(l) = \frac{1}{2\pi l} \int_{-\infty}^{\infty} d\phi e^{il\phi - i\mathcal{P}(\phi)} \times [z\cos(\phi - \phi_0) f(\phi) - \xi^2 \zeta u f^2(\phi)]. \quad (8)$$

The probability is normalized to some time unit. In IPA, one can use the time of one cycle, τ_0. In FPA, a proper time unit is provided by the pulse width, which is N times greater, $\tau = N\tau_0$, where N is the number of the cycles in a pulse. Therefore, for a convenient comparison of IFA and FPA results, the latter one is scaled by $1/N$. Thus, the probability of the e^+e^- pair emission reads

$$W^{\mathrm{FPA}} = \frac{\alpha M_e^2}{4\omega' N} \int \frac{d\phi_p}{2\pi} \int_\zeta^\infty dl \int_1^{u_0} du \frac{w(l, \xi, u, \phi_p)}{u^{3/2}\sqrt{u-1}}, \quad (9)$$

$$w(l, \xi, u, \phi_p) = (2u_0 + 1)|C^{(0)}(l)|^2 + \xi^2(2u - 1) \times (|C^{(2)}(l)|^2 + |C^{(3)}(l)|^2) + \mathrm{Re}C^{(0)}(l) \times \left(\xi^2 C^{(1)}(l) - \frac{2}{\zeta}[\alpha_1 C^{(2)}(l) + \alpha_2 C^{(3)}(l)]\right)^* \quad (10)$$

with $u_0 = l/\zeta$. This expression will be used below for direct numerical evaluations of the probability.

Inspection of the functions $\mathcal{P}(\phi)$ and $C^{(m)}(l)$ shows however that Eq. (10) may be simplified to get, in some cases, a more suitable analytical expression for $w(l)$. Integrating by parts, the function $\mathcal{P}_0(\phi, \phi_0)$ might be expressed in the following form

$$\mathcal{P}_0(\phi, \phi_0) = \sin(\phi - \phi_0) f(\phi) + \mathcal{O}(\Delta), \quad (11)$$

where $\mathcal{O}(\Delta) = -\frac{1}{\Delta}\int_{-\infty}^{\phi} \sin(\phi' - \phi_0) f'(\phi') d\phi'$ is a rather small contribution for a finite pulse duration $\Delta = \pi N$ with $N \geq 2$ because of (i) the factor $1/\Delta$ and (ii) the derivative $f'(\phi)$ in the integrand has a maximum value at the boundaries of the pulse with $\phi \sim 0.9\Delta$, where this function is suppressed. In fact, the numerical evaluation shows that the contribution of $\mathcal{O}(\Delta)$ can be omitted [we find $|\mathcal{O}(\Delta)| < 0.1$ (0.05) for $\Delta = 2\pi$ (5π)]. This approximation allows us to express the basic functions $C^{(m)}(l)$ via new functions Y_l and X_l

$$C^{(0)}(l) = \tilde{Y}_l(z) e^{il\phi_0}, \quad C^{(1)}(l) = X_l(z) e^{il\phi_0},$$
$$C^{(2)}(l) = \frac{1}{2}[Y_{l+1}(z) e^{i(l+1)\phi_0} + Y_{l-1}(z) e^{i(l-1)\phi_0}], \quad (12)$$

with

$$\tilde{Y}_l(z) = \frac{z}{2l}[Y_{l+1}(z) + Y_{l-1}(z)] - \xi^2 u \frac{\zeta}{l} X_l(z),$$
$$Y_l(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\psi \tilde{f}^{(1)}(\psi + \phi_0) e^{il\psi - izf(\psi+\phi_0)\sin\psi},$$
$$X_l(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\psi \tilde{f}^{(2)}(\psi + \phi_0) e^{il\psi - izf(\psi+\phi_0)\sin\psi},$$
$$\tilde{f}^{(m)}(\phi) = f^m(\phi) \exp\left[i\xi^2 \zeta u \Delta \tanh\frac{\phi}{\Delta}\right]. \quad (13)$$

The function $C^{(3)}$ emerges from $C^{(2)}$ by the substitutions $1/2 \to 1/2i$ and sign "+" between two terms in the bracket to "−." In the last line, "m" ($= 1, 2$) is a label on the left-hand side, while on the right-hand side it is the power of the envelope function, as follows from Eqs. (4) and (8); the exponential term results from an analytic evaluation of the last term in Eq. (5) for the chosen envelope function.

The partial probability $w(l)$ in Eq. (10) reads

$$w(l, \xi, u, \phi_p) = 2\tilde{Y}_l^2(z) + \xi^2(2u - 1) \times [Y_{l-1}^2(z) + Y_{l+1}^2(z) - 2\,\mathrm{Re}\tilde{Y}_l(z)X_l^*(z)], \quad (14)$$

which resembles the expression for the probabilities w_n in case of IPA (cf., Ref. [4]) arising upon the substitutions $\int dl w(l) \to \sum_n w_n$, $\tilde{Y}_l^2 \to J_n^2$, $Y_{l\pm1}^2 \to J_{n\pm1}^2$, $\mathrm{Re}\tilde{Y}_l(z)X_l^*(z) \to J_n^2$ with Bessel functions J_n.

In the case of small field intensity, $\xi \ll 1$, implying $z \ll 1$, and denoting $l = n + \epsilon$, where n is the integer part of l, one can use the following decomposition

$$Y_l \simeq \frac{1}{2\pi}\int_{-\infty}^{\infty} d\psi\, e^{il\psi - izf(\psi+\phi_0)\sin\psi} f(\psi + \phi_0) \to \frac{1}{2\pi}\int_{-\infty}^{\infty} d\psi \sum_{k=0}^{\infty} \frac{(iz)^k}{k!} \sin^k\psi\, e^{i(n+\epsilon)\psi} f^{k+1}(\psi + \phi_0) \quad (15)$$

and analog for the function $X_l(z)$ with the substitution $f^{k+1} \to f^{k+2}$. The dominant contribution to the integral with a rapidly oscillating integrand stems from the term with $k = n$, which results in

$$Y_{k+\epsilon} \simeq \frac{z^k}{2^k k!} e^{-i\epsilon\phi_0} f_F^{(k+1)}(\epsilon), \qquad X_{k+\epsilon} \simeq \frac{z^k}{2^k k!} e^{-i\epsilon\phi_0} f_F^{(k+2)}(\epsilon), \quad (16)$$

where the function $f_F^{(k)}(\epsilon)$ is the Fourier transform of the function $f^k(\psi)$. For the above envelope function, it can be calculated analytically using the theory of residues. Results of the leading orders $n = 0, 1$ are

$$Y_{0+\epsilon}(z) = \frac{\Delta e^{-\pi|\epsilon|\Delta/2}}{1 + e^{-\pi|\epsilon|\Delta}} e^{-i\epsilon\phi_0},$$
$$Y_{1+\epsilon}(z) = \frac{z}{2}\frac{\Delta^2|\epsilon| e^{-\pi|\epsilon|\Delta/2}}{1 - e^{-\pi|\epsilon|\Delta}} e^{-i\alpha\phi_0}, \quad (17)$$
$$X_{1+\epsilon}(z) = \frac{z}{4}\frac{\Delta(\Delta^2\epsilon^2 + 1) e^{-\pi|\epsilon|\Delta/2}}{1 + e^{-\pi|\epsilon|\Delta}} e^{-i\epsilon\phi_0}.$$

The representation of Eq. (16) evidences (i) a fast decrease of $Y_{n+\epsilon}^2$ with increasing $|\epsilon|$ and (ii) the ϕ_p dependence disappears in $Y_{n+\epsilon}^2$ and $X_{n+\epsilon}^2$. This allows us to express the integral over dl in (9) in a form useful for a qualitative analysis:

$$\left(\frac{4\omega' N}{\alpha M_e^2}\right) W^{\mathrm{FPA}} = \int_{\zeta - n_0}^{1} d\epsilon \int_1^{u_0} du \frac{w(n = n_0, \epsilon, \xi, u)}{u^{3/2}\sqrt{u-1}} + \sum_{n=n_0+1}^{\infty} \int_{\nu}^{1} d\epsilon \int_1^{u_0} du \frac{w(n, \epsilon, \xi, u)}{u^{3/2}\sqrt{u-1}} \quad (18)$$

with $u_0 = (n + \epsilon)/\zeta$; $n_0 = 1$ for $\zeta \le 1$, and $n_0 = I(\zeta)$ for $\zeta > 1$; the lower limit in integral over $d\epsilon$ in the second term reads $\nu = \zeta - n$ for $\zeta > 1$ and $n = n_0 + 1$, and $\nu = -1$ in other cases. This equation shows that, contrary to IPA where at given $\zeta > 1$ (i.e., below threshold, $s < s_{\mathrm{thr}} = 4M_e^2$), only harmonics with $n > I(\zeta + 1)$ contribute, in FPA the harmonic with $n = I(\zeta)$ also contributes.

Consider, as a check of the normalization, the pair production above threshold with $\zeta = 1 - \delta s_e/s < 1$, where δs_e is the energy excess $\delta s_e = s - s_{\mathrm{thr}}$. Utilizing the explicit expressions (17) for the leading contribution $\tilde{Y}_{1+\epsilon}$ in (14), one can get a relation between emission probabilities in IPA (cf. [4]) and FPA:

$$W^{\mathrm{FPA}} = W^{\mathrm{IPA}}(n = 1, \xi, \bar{u}_1)\mathcal{I}(\Delta, \zeta), \quad (19)$$

$$\mathcal{I}(\Delta, \zeta) = \frac{\Delta^2}{N}\int_{\zeta-1}^{1} d\epsilon \frac{e^{-\pi|\epsilon|\Delta}}{(1 + e^{-\pi|\epsilon|\Delta})^2}, \quad (20)$$

where $\bar{u}_n$ is an effective value of u in nth term of Eq. (18). The dependence of W on $\bar{u}_n$ is rather weak compared to the dependence on ξ and can be disregarded. Thus, in the limit $\pi\Delta\delta s_e/s \gg 1$, IPA and FPA practically coincide since $\mathcal{I}(\Delta, \zeta) \simeq \Delta[\pi N(1 + e^{-\pi\Delta\delta s_e/s})]^{-1} \simeq \frac{\Delta}{\pi N} = 1$.

Consider now the case of subthreshold pair production with $\zeta = 1 + \delta s_l/s > 1$, where $\delta s_l = s_{\mathrm{thr}} - s$ is the "lack of energy." The probability has the following form

$$W^{\mathrm{FPA}} = \mathcal{I}_1 W^{\mathrm{IPA}}(n = 1) + CW^{\mathrm{IPA}}(n = 2) + \ldots, \quad (21)$$

with $\mathcal{I}_1(\Delta, \delta s_l/s) \simeq e^{-\pi\Delta\delta s_l/s}/(1 + e^{-\pi\Delta\delta s_l/s})$ and $C = (1/\pi^2)\int_{\pi\Delta(\zeta-2)}^{\pi\Delta} x^2 \exp(-x)[1 - \exp(-x)]^{-2} dx \simeq 2/3$ for $\delta s_l/s \lesssim 1 - 0.65/N$. The terms in the right-hand side of (21) are meant to have the same functional dependence on ξ and $\bar{u}_{1,2}$ as in IPA. One can expect a significant enhancement of pair production for the short pulse because the probability of single-photon events ($n = 1$) is much greater than the probability of the two-photon events ($n = 2$): $W^{\mathrm{IPA}}(n = 1)/W^{\mathrm{IPA}}(n = 2) \sim \xi^{-2} \gg 1$. When the length of the pulse increases, the contribution of the first term in Eq. (21) decreases exponentially due to $\mathcal{I}_1$, and the prediction of FPA approaches to the IPA one.

The probability and the cross section are related to each other [27] as $dW = 2[\omega M_e^2\xi^2/(4\pi\alpha)]d\sigma$. The total cross section of e^+e^- production is calculated using Eqs. (9), (14), and (16). The cross sections are exhibited in Fig. 1 as a function of $\sqrt{s}$ in the threshold region for finite pulses with $\Delta = \pi N$. The left and right panels correspond to $\xi = 0.01$ and 0.1, respectively. The dashed and thick solid curves are for $N = 2$ and 5, respectively. The thin solid curve is the IPA result. The thin dashed curve, labelled by

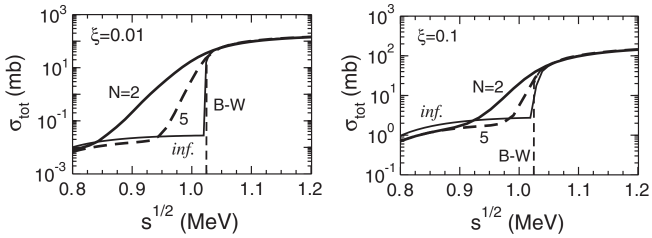

FIG. 1. The total cross section of e^+e^- pair production as a function of the total energy in the c.m.s., $\sqrt{s}$, for the finite pulse. Notation is given in the text.

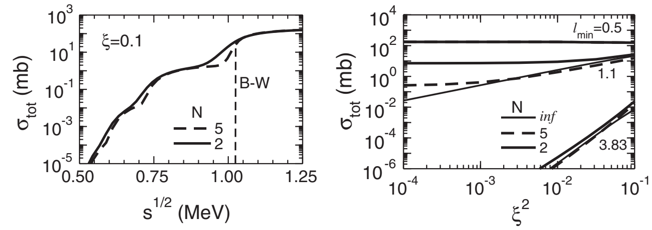

FIG. 2. Left panel: As Fig. 1 (right panel) but for a wider region of $\sqrt{s}$. Right panel: The total cross section of the e^+e^--pair production as a function of ξ^2 at three values of $l_{\rm min} = \zeta = 0.5$, 1.1 and 3.8. Notation as in Fig. 1.

"B-W," corresponds to the Breit-Wheeler process [1] practically coinciding with the lowest harmonic ($n = 1$). One can see that in the subthreshold region, $\sqrt{s} = 0.85$–1.02 MeV, the cross section for short pulses is significantly greater than in IPA and the difference may reach one or two orders of magnitude for $\xi = 0.1$ and $\xi = 0.01$, respectively. When ξ and/or ζ increase, the contribution of higher terms with $n \geq 1$ becomes finite that brings an additional (increasing) dependence on Δ [cf. Eq. (16)].

The total cross section in a wider region of $\sqrt{s}$ is exhibited in Fig. 2, left panel. At $\sqrt{s} \simeq 0.55$ MeV, the multiphoton events with $l \geq 4$ become important. In general, the total cross section in FPA has also the steplike structures similar to IPA. However, a decrease of the pulse duration leads to a smoothing. One can also see some enhancement of the cross section for a short pulse with $N = 2$ compared to the case of a longer pulse with $N = 5$. The total cross sections of the e^+e^- pair production as a function of ξ^2 at three values of $l_{\rm min} = \zeta = s_{\rm thr}/s$ are presented in Fig. 2, right panel. The case of $\zeta = 0.5$ corresponds to the production above the threshold. Here, the predictions for IPA and FPA coincide. Examples of $\zeta = 1.1$ and 3.8 correspond to the subthreshold production. In the first case, we are slightly below the threshold and one can see a large difference between predictions for pulses with $N = 2$ and 5, which has been explained above. The last example ($\zeta = 3.83$) corresponds to the kinematics of the SLAC E-144 experiment. In this case, the predictions of IPA and FPA are qualitatively similar with some enhancement for a shorter pulse duration. Finally note that we do not take into account radiation reaction effect discussed in [4] and recently in Ref. [28] because it influences the fermions in the final state and is not expected to change significantly the total e^+e^- yield.

In summary, we have considered the total cross section of e^+e^- production off a probe photon interacting with a semi-intensive short laser pulse in the subthreshold region defined by multiphoton interactions. We find a nontrivial dependence of the cross section (production probability) on the pulse duration. Just below the threshold of the weak-field Breit-Wheeler process, the short laser pulses increase the cross section up to two orders of magnitude relative to a monochromatic plane wave. This effect must be taken into account in the evaluation of e^+e^- pair production in cascade processes produced by high-power laser fields.

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