Delay Coupled Oscillators for Frequency Tuning of Solid-State Terahertz Sources

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(Received 23 December 2011; published 4 June 2012)

We present a solid-state tunable terahertz source that exploits the theory of coupled oscillators to simultaneously achieve high output power and frequency tuning. Our proposed structure effectively generates and combines high power harmonics from multiple synchronized solid-state oscillators in a loop configuration. We study the dynamics of the system, find the stable modes, and show how the structure can dynamically select a desired coupling mode. Using this method, we fabricated 0.29 and 0.32 THz tunable sources with peak output powers of 0.76 and 0.5 mW both in a standard 65 nm bulk complementary metal-oxide semiconductor technology. This power level is around 10 000 times larger than the state of the art which demonstrates that the proposed concept achieves significantly higher output power for a given solid-state process.

DOI: 10.1103/PhysRevLett.108.234101 PACS numbers: 05.45.Xt, 07.57.Hm, 84.30.Ng

The terahertz range (0.3–3 THz) is one of the last frontiers in the electromagnetic spectrum. Many molecules have absorption bands inside this frequency range which gives terahertz systems niche applications in medical imaging and molecular spectroscopy. In addition, terahertz signals are non-ionizing which is crucial for noninvasive imaging.

Different approaches to generate power at this band include free-electron lasers, solid-state quantum cascade lasers, and Josephson arrays at cryogenic temperatures [1–3]. Despite recent advances, these methods are still complex, bulky, and cost-inefficient. Therefore, it is desirable to generate terahertz signals using low-cost solid-state devices. This approach results in reliable, portable, and cost-efficient terahertz systems that can be integrated with other functions on a single die.

The challenge with solid-state implementation is that the maximum oscillation frequency $f_{\max}$ in most devices—in particular the silicon metal-oxide semiconductor (MOS) transistor—is below the terahertz frequency band. Previous attempts have either used a fundamental oscillator using compound semiconductor (e.g., InP HBT) transistors [4] or employed the device nonlinearity to generate harmonic power from an oscillator below $f_{\max}$ [5]. However, because of the narrow spectral bandwidth of electrical oscillators—typically less than 1 MHz—frequency tuning is crucial for such a source to be applicable to spectroscopy and imaging.

Frequency tuning above the $f_{\max}$ of transistors is a major challenge. The well-established tuning scheme introduces varactors inside the resonator to tune the center frequency of oscillation. This method is not applicable to sub-mm wave frequencies mainly because of the poor quality factor of varactors at this frequency range. As an example, attempts to generate tunable power using MOS devices have only achieved sub-μW output powers, which is not sufficient for most applications [6]. In this Letter and for the first time we address this concern by proposing a fundamentally different tuning methodology. The proposed approach of frequency tuning is based on coupling between multiple core oscillators.

The dynamics of coupled oscillators has been the subject of extensive work. The collective dynamics of randomly distributed oscillators has been a key to understanding biological and chemical systems [7,8]. Electrical oscillators have also exploited frequency synchronization for power combining and beam steering [9,10].

Our proposed method uses N coupled oscillators in a ring structure as shown in Fig. 1. In this scheme all oscillators have the same free running frequency of ω_0 and each one is unidirectionally coupled to its neighbor through an adjustable coupling block. We show that the coupling characteristics control the dynamics of the loop, including the locking frequency. This system has two distinct

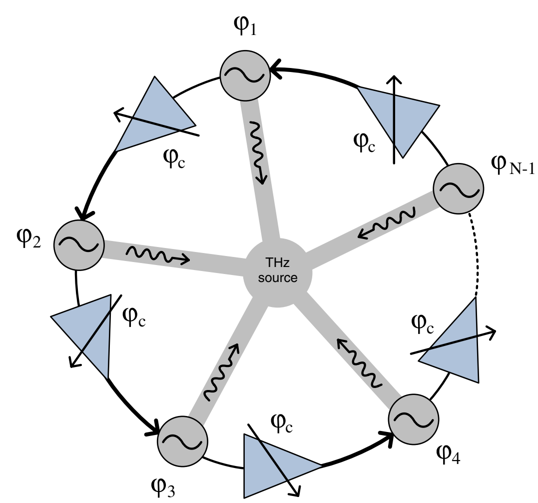

FIG. 1 (color online). A ring of N actively coupled electrical oscillators. The gray circles represent the core oscillators and the blue blocks are the coupling phase shifters.

advantages. First, since the core oscillators do not use low-quality factor tuning elements, they can generate high harmonic power. Second, by controlling the loop dynamics, we impose certain phase shifts between the cores. This results in constructive power combining only at the desired harmonic from all core oscillators at the center of the loop.

In order to analyze the behavior of this coupled system, we define ϕ_i as the phase of the ith oscillator and assume $K\exp(i\phi_c)$ to be the transfer function of the coupling block. As a result, the coupling block is represented by an electrical phase shifter with a phase shift equal to ϕ_c and a coupling gain of K. This is a good approximation for the implemented coupling block using complementary metal-oxide semiconductor transistors. The coupling factor K in an LC oscillator is equal to $i_{\text{inj}}\omega_0/(2Qi_{\text{core}})$ [11]. Here, i_{inj} is the injected current to each oscillator from the coupling block, i_{core} is the current produced inside the oscillator, and Q is the quality factor of the resonator of each oscillator. Note that both K and ϕ_c are controllable parameters of the coupling block.

Next, we consider the coupling between the $(i-1)$th and the ith cores. Based on Adler's equation, the phase dynamics of the ith core in this structure is

$$\dot{\phi}_i = \omega_0 + K\sin(\phi_{i-1} + \phi_c - \phi_i). \quad (1)$$

For a loop of N cores, by defining $\psi_i = \phi_i - \phi_{i-1}$, we can reformulate this relation into

$$\dot{\psi}_i = K\sin(\phi_c - \psi_i) - K\sin(\phi_c - \psi_{i-1}), \quad (2)$$

for $1 \le i \le N$. This set of N equations along with the consistency condition

$$\sum_{i=1}^{N} \psi_i = 2n\pi, \qquad n = 0, 1, \ldots, \quad (3)$$

fully describe the dynamics of the system. For this analysis we assume the following: All coupling blocks are similar with a controllable phase shift ϕ_c and a constant magnitude K. We are interested in the stable solutions of the system when all core oscillators are synchronized. Under locking conditions, the phase difference between oscillators should be constant, resulting in $\dot{\psi}_i = 0$ in Eq. (2). This can be written in the matrix form as

$$\mathbf{K}\,\mathbf{x} = \mathbf{0}, \quad (4)$$

where $\mathbf{x}_i = \sin(\phi_c - \psi_i^0)$ and ψ_i^0 represents the steady state solution of ψ_i. The coupling matrix, $\mathbf{K}$ is equal to

$$\mathbf{K} = \begin{bmatrix} K & 0 & \ldots & \ldots & -K \\ -K & K & 0 & \ldots & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \ldots & \ldots & -K & K \end{bmatrix}. \quad (5)$$

This coupling matrix is asymmetric which stems from the unidirectional coupling of this structure. The trivial solution of Eq. (4) is $\mathbf{x}_i = 0$. This along with Eq. (3) results in $\phi_c = 2n\pi/N$. Such a solution imposes discrete values for ϕ_c, which is not physically stable.

The nontrivial solution of Eq. (4) is in the form of $\mathbf{x}_i = \mathbf{x}_1$ which leads to two choices for ψ_i^0:

$$\psi_i^0 = \psi_1^0 \pm 2n\pi, \quad (6a)$$

$$\psi_i^0 = 2\phi_c - \pi - \psi_1^0 \pm 2n\pi. \quad (6b)$$

This suggests 2^{N-1} distinct coupling modes for a given ψ_1^0. However, in the next theorem we show that only solutions with all ψ_i^0's chosen from Eq. (6a) are stable.

Theorem.—If m of ψ_i^0's are chosen from Eq. (6b) and $N-m$ from Eq. (6a), only solutions corresponding to $m = 0$ are stable.

Proof.—In order to show this, we consider the nontrivial case of $N \ge 3$. We perturb Eq. (4) around each solution. By letting $\psi_i = \psi_i^0 + \eta_i(t)$ and after linearizing the perturbed equations, we obtain

$$\dot{\boldsymbol{\eta}} = \mathbf{J}_{\{\psi_1^0\ldots\psi_n^0\}}\boldsymbol{\eta}, \quad (7)$$

where the elements of the Jacobian matrix are equal to $J_{ij} = -K_{ij}\cos(\phi_c - \psi_i^0)$. For a solution to be stable, all eigenvalues of the Jacobian matrix should be nonpositive [12]. It is straightforward to find the characteristic polynomial of $\mathbf{J}$ to be

$$P(\lambda) = \prod_{i=1}^{N}(J_{ii} - \lambda) - \prod_{i=1}^{N} J_{ii}$$
$$= (-1)^N(\lambda^N + P_{N-1}\lambda^{N-1} + \cdots + P_1\lambda). \quad (8)$$

Thus, $\lambda_1 = 0$ is an eigenvalue of the Jacobian. We show that for stable solutions, all other eigenvalues are nonzero. By elaborating on the characteristic polynomial we find that the sum (Σ) and product (Π) of the other $N-1$ eigenvalues are

$$-P_{N-1} = \sum_{i=2}^{n} \lambda_i = J_{11}(N - 2m), \quad (9a)$$

$$(-1)^{N-1}P_1 = \prod_{i=2}^{N} \lambda_i = J_{11}^{N-1}(N-2m)(-1)^m. \quad (9b)$$

Without loss of generality we can assume $J_{11} \neq 0$, and thus a second zero only happens when $N = 2m$, requiring both Π and Σ to be zero. Since in a stable solution no eigenvalue can be positive, $\Sigma = 0$ forces all eigenvalues to be zero which is not possible for the nonzero Jacobian matrix. As a result, for a stable solution all the other $N-1$ eigenvalues of the coupled system are nonzero.

Next, based on an extended Gershgorin theorem [13], the eigenvalues of $\mathbf{J}$ are inside circles centered at J_{ii} with equal radii of $|J_{11}|$. These are essentially two circles, one on the left half-plane and the other on the right half-plane which overlap at the origin.

If $m = 1$ and $J_{11} > 0$, $N - m (\geq 2)$ Gershgorin circles lie on the right side; hence, at least one eigenvalue is positive. On the other hand if $J_{11} < 0$ we consider two cases: (i) For an even N, from Eq. (9b), the product of an odd number of eigenvalues becomes positive, which means that at least one eigenvalue is positive; (ii) for an odd N, the product of even number of eigenvalues becomes negative, which again means that at least one eigenvalue is positive. Similarly, one can show that the solutions corresponding to $m = N - 1$ are also unstable.

For $2 \leq m \leq N - 2$ there are more than one Gershgorin circles on either half-planes, and thus at least one positive eigenvalue exists which again means that the corresponding steady-state solution is unstable.

Finally, the only stable solutions correspond to $m = 0$ and $J_{11} < 0$ which confines all Gershgorin circles and eigenvalues to the left half plane. ■

As a result of the above theorem, by applying Eq. (3) we find N distinct stable modes of oscillation,

$$\psi_k^0 = \frac{2k\pi}{N}, \qquad 0 \leq k \leq N - 1, \tag{10}$$

where the kth mode is stable as long as $J_{11} < 0$, which means

$$2\pi n - \frac{\pi}{2} + \psi_k^0 < \phi_c < 2\pi n + \frac{\pi}{2} + \psi_k^0. \tag{11}$$

Figure 2 shows the stable regions of the coupled system with respect to ϕ_c. The stable modes have overlap for $N > 2$, meaning that for a given ϕ_c the system can settle to one of multiple states depending on the initial dynamics of the system.

Now, let us consider the coupling dynamics for $N = 4$. There are no continuous regions of ϕ_c where an initial condition corresponds to only one stable mode. Consider the case that the system has settled to ψ_0^0. By increasing ϕ_c, the system will reach the point that ϕ_c passes $\pi/2$. At this instant, the only stable mode is ψ_1^0 and the system jumps to this mode. This is because the noise present in the physical system acts as a perturbation on ϕ_c making both ψ_0^0 and ψ_2^0 unstable at values of ϕ_c close to $\pi/2$.

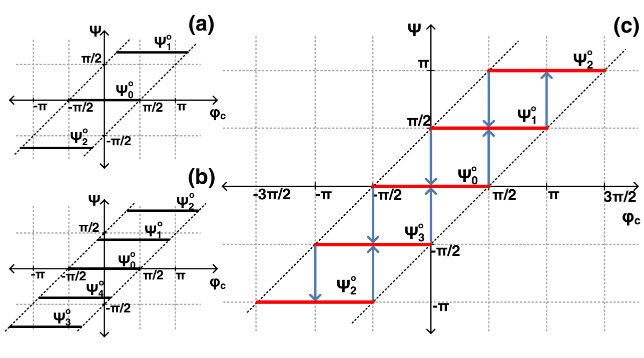

FIG. 2 (color online). Stable modes and their corresponding regions (a) for $N = 3$ and (b) for $N = 5$. (c) The stable modes for $N = 4$ and mode transitions at the edge of stability.

Since the operating frequency of the system is orders of magnitude higher than the rate of change of ϕ_c, a continuous sweep in ϕ_c will keep the system long enough in the narrow region which corresponds to only one stable solution, ψ_1^0. Figure 3 shows simulations for the system with $N = 4$ and the hysteresis bifurcation resulting from sweeping ϕ_c. We conclude that even in the case of $N = 4$, regardless of the initial mode, we can ensure a desired coupling mode through proper manipulation of ϕ_c.

After the system settles to a particular mode ψ_k^0, the frequency of all cores are locked together. This locking frequency form Eq. (1) is equal to

$$\omega = \omega_0 + K \sin(\phi_c - \psi_k^0), \tag{12}$$

which gives the frequency tuning range. We apply this tuning method to implement a tunable frequency source. The proposed system consists of four core oscillators coupled together through four tunable phase shifters.

In this scheme, the core oscillators can efficiently generate and deliver the harmonic power because they do not include any low quality factor elements (e.g., varactors) inside the resonator. The fundamental frequency of all four cores is carefully chosen for generating the highest possible power around 0.3 THz. The transistors used for this design have a simulated $f_{\max}$ of 200 GHz. By lowering the oscillation frequency, more power can be generated at the fundamental and the harmonics but the achievable frequency is reduced. On the other hand, at oscillation frequencies close to $f_{\max}$ the power generated at the fundamental and the harmonics are significantly reduced. Based on this argument, using an oscillation frequency of 75 GHz and delivering the fourth harmonic power is optimal for the employed devices.

The harmonic power of the cores are combined at the center of the loop. The coupled system is designed to

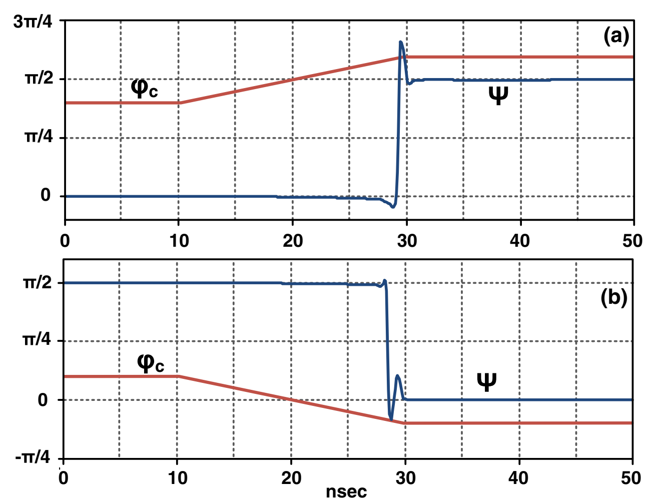

FIG. 3 (color online). Simulated mode transition (ψ) for $N = 4$ as a function of ϕ_c. (a) Transition form ψ_0^0 to ψ_1^0. (b) Transition form ψ_1^0 to ψ_0^0.

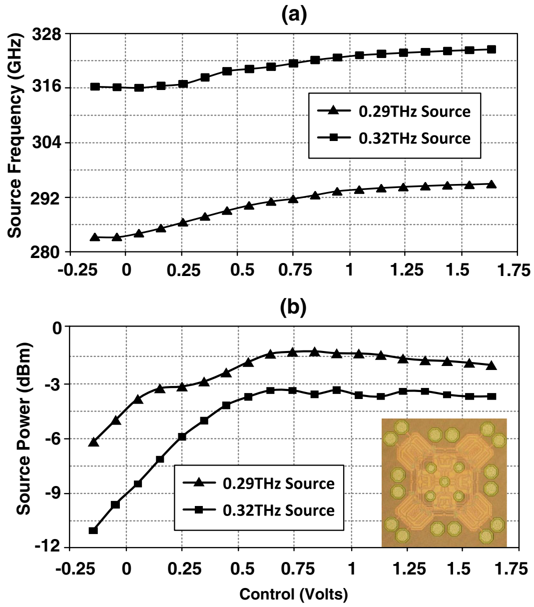

FIG. 4 (color online). (a) Measured output frequency and (b) measured output power as a function of the control voltage of the coupler for the two fabricated versions of the source. The top view of the chip is included.

operate at the $\psi_1^0 = \pi/2$ coupling mode. In this mode, the generated power at the kth harmonic shows a phase shift equal to $k\pi/2$ between consecutive cores. As a result, only the generated harmonics at multiples of four of the fundamental add constructively at the output, which means the signal source acts like a fundamental oscillator at 4ω.

The coupling block is a tunable phase shifter composed of two back-to-back resonance amplifiers with a tunable center frequency. The resonators of phase shifters are isolated from the core oscillators in order to avoid any degradation of the quality factor of the core resonators. The phase shift of the coupling block resonator depends on its center frequency ω_c as well as the operating frequency ω. While we have control over ω_c, ω is determined by the dynamics of the loop including the phase shifters. This results in an iterative mechanism for tuning ω. From Eq. (12) the tuning range depends on K and the range of ϕ_c, both of which depend on the resonator of the coupling block. The highest variation of ϕ_c happens around ω_c and as a result, for the highest tuning range this frequency has to match ω_0.

We have fabricated two prototypes at center frequencies of 0.29 and 0.32 THz in a standard 65 nm bulk complementary metal-oxide semiconductor process. The output power is measured using a calibrated wideband calorimeter through matched rectangular waveguides. Figure 4 shows the measured power for the two oscillators. The peak output power is around 0.76 mW, which is around 10 000 higher than conventional tunable signal sources around this frequency range using a similar process. Moreover, a higher output power can be achieved by incorporating more cores (i.e., a larger N) in the coupled system. The silicon process that we use has a fairly low $E_b v_{\rm sat}$ product where E_b is the breakdown field and $v_{\rm sat}$ is the saturation velocity of carriers. If this structure is used in processes with higher E_b such as GaN, higher power levels are achievable. Furthermore, if we implement this system in a process with higher $f_{\max}$ such as InP, it is possible to generate this power level well above 1 THz.

The output frequency and the tuning range shown in Fig. 4 are measured using a diode harmonic mixer. The gain of the phase shifter varies with respect to both ω and ω_c, which result in variation of K with frequency. This explains the variation of the output power and the slope of the output frequency in Fig. 4. The phase shift ϕ_c is designed to center around $\pi/2$, which according to Fig. 2 is the optimal point for tuning the frequency at the ψ_1^o coupling mode.

It is intriguing to consider the asymmetric tuning case where ϕ_c is controlled independently for the N coupling blocks. Here, the same theoretical treatment results in stable modes only for $m = 0$. However, in this case Eq. (6a) should be replaced with

$$\psi_i^0 - \phi_c^i = \psi_{i-1}^0 - \phi_c^{i-1}, \qquad (13)$$

where ϕ_c^i is independently set for the ith coupling block. In other words, the phase shift ψ between the cores can be manipulated. In this design, it is possible to allow each oscillator to radiate separately and spatially combine their outputs. It is therefore possible to perform beam steering and frequency tuning at the same time.

We thank Steven Strogatz and Mohammad Soltani for helpful discussions.

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