Bounds on Spectral Dispersion from Fermi-Detected Gamma Ray Bursts

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Data from four Fermi-detected gamma-ray bursts (GRBs) are used to set limits on spectral dispersion of electromagnetic radiation across the Universe. The analysis focuses on photons recorded above 1 GeV for Fermi-detected GRB 080916C, GRB 090510A, GRB 090902B, and GRB 090926A because these high-energy photons yield the tightest bounds on light dispersion. It is shown that significant photon bunches in GRB 090510A, possibly classic GRB pulses, are remarkably brief, an order of magnitude shorter in duration than any previously claimed temporal feature in this energy range. Although conceivably $a > 3\sigma$ fluctuation, when taken at face value, these pulses lead to an order of magnitude tightening of prior limits on photon dispersion. Bound of $\Delta c/c < 6.94 \times 10^{-21}$ is thus obtained. Given generic dispersion relations where the time delay is proportional to the photon energy to the first or second power, the most stringent limits on the dispersion strengths were $k_1 < 1.61 \times 10^{-5}$ sec Gpc^{-1} GeV^{-1} and $k_2 < 3.57 \times 10^{-7}$ sec Gpc^{-1} GeV^{-2}, respectively. Such limits constrain dispersive effects created, for example, by the spacetime foam of quantum gravity. In the context of quantum gravity, our bounds set M_1c^2 greater than 525 times the Planck mass, suggesting that spacetime is smooth at energies near and slightly above the Planck mass.

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Gamma-ray bursts (GRBs) are the furthest known explosions in the Universe. Their rapid variability and great distances make them useful as probes of light properties as well as the intervening space. Were light to have fundamentally different speeds at different wavelengths (spectral dispersion), distant GRBs might show persistent energy-dependent arrival patterns [1]. Spacetime foam inherent in some formulations of quantum gravity, for example, might cause spectral dispersion [2–4]. Other properties of light or the Universe might also cause different wavelengths to propagate at different speeds [5,6].

GRBs have already been used to limit the cosmological density of compact objects through the nondetection of their gravitational lensing [7]. Lag-minimizing algorithms have been previously designed to search for quantum gravity based dispersion effects [8]. Although bounds on quantum-gravity dispersion in Fermi GRBs have been explored previously for two different Fermi GRBs [9,10], the present work limits more general parameters, considers four Fermi GRBs, considers only super-GeV photons, and yields substantially tighter bounds.

Given that two photons of different energies ΔE are emitted at the same place and time, the gap Δt between their arrivals can be quantified as

$$\Delta t = k_n D_n E^{n-1} \Delta E, \qquad (1)$$

where k_n is the dispersion strength and D_n is a cosmological lookback distance that also depends on the nature of the photon dispersion [11]. Specifically,

$$D_n = \frac{c}{H_o} \int_0^z \frac{(1+z')^n dz'}{\sqrt{\Omega_M (1+z')^3 + \Omega_\Lambda}}, \qquad (2)$$

where H_o is the present value of Hubble's constant, and Ω_M and Ω_Λ are the present values of the matter density and cosmological constant density [11,12] in a geometrically flat universe [13].

For clarity and following theoretical precedents [11,14,15], only three cases will be considered here: $n = -1$, $n = 1$, and $n = 2$. The first case, $n = -1$, is for a universe with no chromatic dispersion. Then, $k_{-1} = 0$ and D_{-1} corresponds with the classic cosmological look-back distance [12]. In the second case, $n = 1$, the dispersion delay scales with the energy difference between photons, a primary case expected were spacetime to have the foaminess inherent in some models of quantum gravity [15]. The third case, $n = 2$, is considered in some models of quantum gravity [15]. It will be assumed here that dispersion occurs uniformly along the light paths.

For a group of photons emitted over a source of finite size, an upper limit on Δt might relate primarily to an upper limit on source size and not to dispersion properties of light. Given limited information, one might not be able to disentangle the various contributions to Δt. Surely, though, an observed bound on Δt would constrain the combined processes, thereby limiting the individual magnitudes. An exception to this would be if the source and universe dispersion effects were of similar magnitudes but of opposite sign, a coincidence that is testable with a larger data set but here considered unlikely.

Because the largest energy ranges occur most commonly in the GRBs with the highest energy photons, and since these GRBs with many high-energy photons are rare, GRBs with numerous high-energy photons were initially sought—to find the finest temporal feature of statistical significance. A useful previous search included one by Rubtsov *et al.* [16] of the Fermi LAT photon database, although other previous studies also were influential [9,17,18]. Another clue came from a visual inspection of Fig. 1 of Abdo *et al.* [10], where a striking clustering of photons above 1 GeV was spotted for GRB 090510A. Other reasons for our 1 GeV threshold include the lower photon background at higher energies, and the possibility of extremely brief GRB pulses at higher energies. Four candidate GRBs eventually emerged: GRB 080916C, GRB 090510A, GRB 090902B, and GRB 090926A. "Pass 7" data from these GRBs were downloaded from the Fermi web interface at NASA's Goddard Space Flight Center in February 2012. Only photons within a 95% energy-dependent error radius of the sky position of the optical counterpart were considered. This error radius was interpolated from Fermi performance data given by Ref. [19].

The bottom four panels of Fig. 1 show time series for the arrivals of photons of the four Fermi GRBs. The origin $t = 0$ indicates the time that the GRB triggered on Fermi's GLAST Burst Monitor (GBM). On the left, at negative times, is 100 s of Fermi LAT data that occurred before the trigger time while on the right, at positive times, is 100 s of data that occurred after the trigger time. Individual counts are shown as vertical line segments. The height of the line segment indicates the recorded energy of the photon detected. Inspection of Fig. 1 shows that the background for stray photons, prior to the trigger time, for example, is very low. The top panel of Fig. 1 shows a closeup of the 1 s of GRB 090510A when the bunched photons arrived.

For a (short) GRB 090510A, consider the first 11 photons arriving over a $\Delta T = 0.1745$ s. The post-trigger arrival times of these photons were 3.702 234, 3.702 783, 3.706 941, 3.719 431, 3.763 108, 3.764 177, 3.799 190, 3.799 319, 3.800 096, 3.816 729, 3.875 767 s, respectively. For comparison, the next five photons, photons 12 through 16, arrived at 3.925 311, 3.953 093, 4.037 660, 4.140 611, and 4.152 783 s. The eighth photon had the unusually high energy of 30.9 GeV. Of the 11 photons considered, six photons arrived before the temporal midpoint and five photons arrived later. Notable is the closeness in arrival times of three photon groups. These groups are defined by the first and second photons, the fifth and sixth photons, and photons seven through nine. The time between the first and last photons in these groups are 0.549 ms, 1.069 ms, and 0.906 ms, respectively.

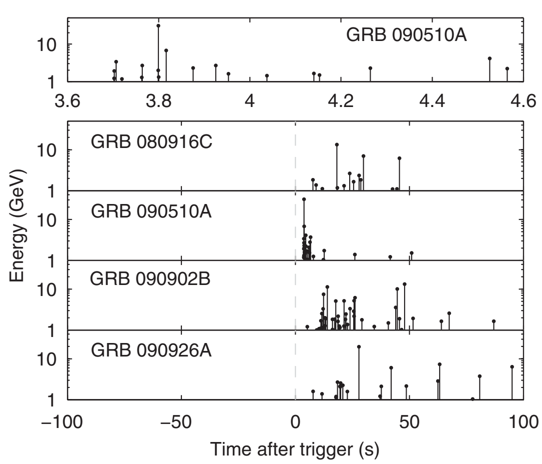

FIG. 1. Time series of photon arrivals for the four Fermi GRBs analyzed. Top panel is a closeup of the 1 s of GRB 090510A, containing the finest temporal features.

Is this arrival pattern of remarkably brief doublets separated by long pauses significant? Do these three brief pulses define the finest time scale yet? We argue that such "rhythm" is, most likely, not spurious. As shown below, this group of 11 photons is consistent with a constant overall arrival rate. Yet, the following simple, albeit crude, analytical argument shows that the odds of a uniformly emitting source producing the pattern described above, are below 3σ. This is then confirmed by a detailed Monte Carlo simulation.

For a perfectly random (Poisson) process, the waiting times (t) between consecutive photon arrivals are exponentially distributed and a sum of m such times is Γ-distributed with exponent m (convolution of m exponential variates). Given an estimated mean waiting time $\tau = 0.1745/10$ s, the probability of waiting $t \ll \tau$ is t/τ. For example, consider $t < 1.069$ ms a "success." The probability of success is then $\approx 0.1069/1.750 = 0.0613$ for the 11 photon group. Then the (binomial) probability of at least 4 "successes" in 10 trials (10 waiting times between the 11 photons) is $P(4, 11) = 1/455$. If one counts the triplet as three successes, the odds drop to $P(5, 11) = 1/5000$. These crude estimates bracket the result of the 10^9 uniformly random Monte Carlo runs, indicating that the chance that five photons would trail other photons by 1.069 ms or less occurs in only about 1 in 1190 trials (about 3.34σ). A sceptic might object that the mean rate need not be uniform, that both the first and the 11th arrivals ought to be regarded as fixed, etc. To that end, we now describe our data analysis as well as more elaborate Monte Carlo simulations in more detail.

To determine the briefest yet statistically meaningful time interval Δt in the data, we proceeded as follows. Groups of consecutive photon arrival times were considered, starting from the three photons arriving closest in time, then the four closest photons, and subsequently all numbers of GRB-associated photons for 500 s following the trigger. To ensure relatively uniform average arrival rates, we chose photon groups with roughly equal numbers of photon arrivals before and after the temporal midpoint of the group. Formally, a two-bin χ^2 statistic was computed. Given the single degree of freedom, "flat" groups with $\chi^2 < 1$ were considered as statistically consistent with a flat distribution, and then the search for Δt proper ensued, aided by a Monte Carlo simulation as follows.

For each photon in the time series except the last, the number of trailing photons arriving within a time window T was counted, for a wide range of Ts. This photon count was compared to that expected from a uniformly random arrival time distribution. The comparison distribution typically involved 10^6 trial time series. To avoid spurious bunching, only Δt delays such that the associated number of real photons was found in less than 1% of the equivalent Monte Carlo distributions, were considered for further analysis.

Returning to the GRB 090510A 11 photon group, do the three brief pulses define the finest time scale of significance? Could a variable mean rate, perhaps, produce such a pattern? To that end, we assumed that each of the three photon groups was randomly chosen from a single parent pulse form. This pulse form is the generic GRB "Norris pulse" shape first suggested by Norris [20] for the instance found most common by Nemiroff [21], specifically, $P = Ae^{-t/\alpha - \alpha/t}$, where P is the photon count rate, A is the pulse amplitude, t is time during the pulse, and α is the time scale of the pulse. To be conservative, we will focus on the broadest photon group, the central pair separated by 1.069 ms.

A simple simulation shows that randomly chosen pairs of photons from a Norris pulse form have a mean pair separation of about 1.20α. Additionally, in a Norris pulse, 68.2% of the photons arrive within a total time window of 1.74α surrounding the pulse peak, here called the pulse "width." Therefore, a parent pulse with width of $\Delta t = (1.74/1.20)1.069$ ms $= 1.55$ ms would yield a mean pair separation of 1.069 ms, the longest time between first and last photons of the three close photon groups of GRB 090510A. Therefore, in subsequent analysis, we will use $\Delta t = 1.55$ ms.

The conservative value of Δt estimated above for GRB 090510A is about a factor of 10 smaller than even the least conservative limit on Δt listed by Ref. [10] in row 5 of Table S1. A primary reason for this is that Ref. [10] measured the limiting Δt essentially as the time difference between the start of a sub-MeV spike and a possibly associated 0.75 GeV photon. Our analysis differs from this earlier analysis of GRB 090510A in that they looked at photons over a wide range of energies, whereas we looked at only the most energetic photons (> 1 GeV) because the pulse durations are known to decrease greatly as photon energy is increased, so the tightest limits on the dispersion delays will come from the highest energy photons. Therefore, the small Δt values presented here focus on extremely short doublets prominent at very high energies.

Of the four GRBs considered, only GRB 090510A and GRB 090902B have photons arriving close enough in time to eclipse the 0.01 s previously reported [10] as the smallest Δt record. We therefore conclude that analyzing the other GRBs at most increased the number of trials to two, which would decrease the statistical significance of the Δtreported here for GRB 090510A to about 3.14σ, still above 3σ.

For GRB 080916C, GRB 090902B, and GRB 090926A, none of the photon groups for which the two-bin χ^2 test was less than unity showed significant bunching on any time scale. On longer time scales, clearly distinct photon groups have their Δt values recorded in Table I.

Table I lists the measured parameters for the four GRBs selected. Column 1 lists the title of the GRB, coded with its date of detection. The Δt values as well as the number of photons N on which they are based as listed in columns 2 and 3, respectively.

Another measured parameter that limits spectral dispersion is ΔE, the energy between the highest and lowest energy photons arriving from the GRB in the Δt time window. Conservative 2σ values of the lowest and highest energy photons—E (low) and E (high)—are given, assuming a 10% single σ energy measurement uncertainty. They are listed in columns 4 and 5 of Table I. Values for the GRB redshifts were obtained by others from follow-up observations of the GRB optical afterglows and the 2σ lower limits are listed in column 6 of Table I, with references.

The ratio of Δt and ΔE has been used to set limits on Lorentz invariance previously, where Boggs *et al.* [25] derived an upper limit of $\Delta t/\Delta E$ of 0.7 s/GeV for GRB 021206. For GRB 090510A, Ref. [10] lists $\Delta t/\Delta E <$ 0.03 s/GeV at the 99% confidence level as their conservative limit (no least conservative limit is listed). The tightest bound from Table I, however, involving the upper limit on Δt for GRB 090510A, is $\Delta t/\Delta E < 6.71 \times 10^{-5}$ s/GeV, an improvement of greater than 2 orders of magnitude.

From the measured parameters listed in Table I, derived and limited parameters were computed and listed in Table II. Values of D_{-1}, D_1, and D_2 were computed from Eq. (2) under the assumption of a flat concordance cosmology with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, and a Hubble constant H_o of 72 km s^{-1} Mpc^{-1}, and are listed in columns 2, 4, and 7 of Table II, respectively.

TABLE I. Measured parameters of selected high-energy Fermi GRBs.

GRB name	Δt s	N	E (low) GeV (2σ)	E (high) GeV (2σ)	z [Ref.] (2σ)
080916C	37.9	14	1.32	10.6	4.05 [22]
090510A	0.00155	11	1.58	24.7	0.897 [10]
090902B	23.9	33	1.20	9.02	1.82 [23]
090926A	3.00	7	1.38	2.15	2.106 [24]

TABLE II. Derived and limited parameters of selected high-energy Fermi GRBs.

GRB name	D_{-1} Gpc	$\Delta c/c$	D_1 Gpc	k_1 s/(Gpc GeV)	M_1c^2 GeV	D_2 Gpc	k_2 s/(Gpc GeV^2)	M_2c^2 GeV
080916C	3.57	$1.03E-16$	16.8	$2.42E-01$	$4.25E+17$	48.8	$6.28E-03$	$4.96E+09$
090510A	2.17	$6.94E-21$	4.18	$1.61E-05$	$6.41E+21$	6.09	$3.57E-07$	$6.57E+11$
090902B	2.96	$7.84E-17$	8.38	$3.65E-01$	$2.82E+17$	16.0	$1.70E-02$	$3.01E+09$
090926A	3.10	$9.40E-18$	9.59	$4.05E-01$	$2.54E+17$	19.5	$7.41E-02$	$1.44E+09$

Given the above data, it is possible to place bounds for the difference between the speeds of light at different energies: $\Delta c/c$. Assuming Δc results from an inherent property of electromagnetic radiation itself, then the look-back distance each photon has traveled is D_{-1} as given by Eq. (2) [12]. Defining lookback time as $t = D_{-1}/c$, the time differential yields $\Delta c/c = c\Delta t/D_{-1}$. Limits on $\Delta c/c$, computed using our strictest upper limit on Δt, are listed in column 3 of Table II.

A previous limit on $\Delta c/c$ using GRBs was obtained in 1999 by Schaefer [14], where an analysis of GRB 930229 yielded $\Delta c/c < 6.3 \times 10^{-21}$ for photons of energies between 30 and 200 KeV. A comparable limit for $\Delta c/c < 6.94 \times 10^{-21}$ is derived here from the Δt listed in column 2 of Table I for GRB 090510A for photons of energy difference $\Delta E \gtrsim 23.5$ GeV.

Alternatively, it can be assumed that it is the intervening space that causes differential speed for photons of different energies. Following Eq. (2) and approximating $E \sim \Delta E$, it is clear that $k_n < \Delta t/(D_n \Delta E^n)$. In other words, were k_n greater than this, the Universe would have separated photons of an energy difference greater than ΔE by more than Δt. For $n = 1$ and $n = 2$, using the Δt limits listed in column 2 of Table I, limiting k_1 and k_2 values are listed in Table II's columns 5 and 8, respectively.

The k_1 parameter effectively limits dispersion expected in some versions of quantum gravity [15]. In particular, given that $\Delta t \sim (\Delta E/M_1c^2)(D_1/c)$ as delineated in Ref. [10], then $M_1c^2 = (k_1c)^{-1}$. In this parametrization, M_1c^2 is a minimum energy scale of the inherent foaminess of spacetime responsible for the dispersion. Note that the above data places an upper limit on k_1 which translates into a lower limit on M_1c^2. Similarly, it is found that $M_2c^2 = (3k_2c/2)^{-1/2}$. The limiting values of M_1c^2 and M_2c^2 are listed in Table II's columns 6 and 9, respectively.

Prior to NASA's Fermi, GRB published lower limits for M_1/M_{Planck} and M_2/M_{Planck} were on the order of 0.04 and 4×10^{-12}, respectively [14,25], where $M_{\mathrm{Planck}}c^2 = 1.22 \times 10^{19}$ GeV. Using Fermi data for GRB 090510A, however, the authors of Ref. [10] found $M_1/M_{\mathrm{Planck}} > 102$, while this was relaxed to $M_1/M_{\mathrm{Planck}} > 1.19$ for more conservative assumptions. Note that using the most stringent upper limit on Δt found here for conservative assumptions results in a rather tight bound of $M_1/M_{\mathrm{Planck}} > 525$, suggesting that space is smooth even at energies near and slightly above the Planck mass.

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