Constraints on Measurement-Based Quantum Computation in Effective Cluster States

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(Received 24 November 2011; published 8 June 2012)

The aim of this work is to study the physical properties of a one-way quantum computer in an effective low-energy cluster state. We calculate the optimal working conditions as a function of the temperature and of the system parameters. The central result of our work is that any effective cluster state implemented in a perturbative framework is fragile against special kinds of external perturbations. Qualitative aspects of our work are important for any implementation of effective low-energy models containing strong multisite interactions.

DOI: 10.1103/PhysRevLett.108.230508 PACS numbers: 03.67.Lx, 03.67.Ac, 05.50.+q, 75.10.Jm

A cluster state $|\psi_{\mathrm{CS}}\rangle$ is a quantum state defined on some lattice (we focus on a square lattice) of qubits, which fulfills the following eigenvalue equations:

$$K_a := X_a \bigotimes_{b\in\Gamma(a)} Z_b, \qquad K_a|\psi_{\mathrm{CS}}\rangle = |\psi_{\mathrm{CS}}\rangle, \tag{1}$$

with X_a and Z_b as Pauli operators acting on qubits a, b, and $\Gamma(a)$ is the set of nearest neighbors of site a. So-called one-way quantum computers (1-WQC) perform universal quantum computations just by one-qubit measurements on a cluster state [1]. While this saves the need to apply unitary transformations to the quantum register, it requires us to reliably prepare a cluster state. One possibility is to cool the Hamiltonian

$$H_{\mathrm{cl}} := -\sum_{a\in C} K_a \tag{2}$$

into its nondegenerate ground state, which is by definition the cluster state. A direct implementation of H_{cl} is not realistic, as it contains multiqubit interactions that are not realized in nature.

This suggests a search for a more realistic Hamiltonian with only two-qubit interactions having the same nondegenerate ground state. But such a Hamiltonian does not exist [2], so one is limited to an approximation of the cluster state. One approach is to use ancillary qubits to effectively mediate the many-qubit interactions. Unfortunately, this is not of practical use, since the necessary precision scales with the system size [3].

Alternatively, one can implement H_{cl} as an effective low-energy model of a realistic Hamiltonian containing only two-qubit interactions [4,5]. Two questions arise naturally: (1) What are the optimal working conditions to perform measurement-based quantum computation (MBQC) in an effective cluster state? (2) How robust are effective cluster states with respect to external perturbations?

In the following, we show that any such effective cluster state implemented in a perturbative framework is strongly affected by external perturbations. Most importantly, our results directly apply for any perturbative generation of multisite interactions in effective low-energy models, giving a fundamental barrier for this rather general concept used in theoretical physics.

Model.—We replace each qubit on a square lattice by four physical qubits and we encode the logical cluster qubit into the subspace defined by the projector $P := |0_{\log}\rangle\langle 0000| + |1_{\log}\rangle\langle 1111|$ [4]. The Hamiltonian $H := gH_0 + \lambda_{xz}V$ is then defined by

$$gH_0 := -g\sum_{\mu\in\mathcal{L}}\sum_{i\leftrightarrow j} Z_{(\mu,i)}\otimes Z_{(\mu,j)},$$

$$\lambda_{xz}V := -\lambda_{xz}\sum_{\mu\in\mathcal{L}}\sum_{i=1}^{4} X_{(\mu,i)}\otimes Z_{\xi(\mu,i)}.$$

The symbol $i \leftrightarrow j$ means that the lattice sites i and j are related in some connected graph structure. Other notations are illustrated in Fig. 1(a). The ground-state space of H_0 is the space of all logical qubits.

It is possible to solve this model exactly by transforming H into the base Σ^{loc} with the base transformation $CZ_{\mathcal{M}}$ (controlled-Z operators on every bond) [5]. Using $(CZ_{\mathcal{M}})X_{(\mu,i)}\otimes Z_{\xi(\mu,i)}(CZ_{\mathcal{M}}) = X_{(\mu,i)}\otimes I_{\xi(\mu,i)}$ and

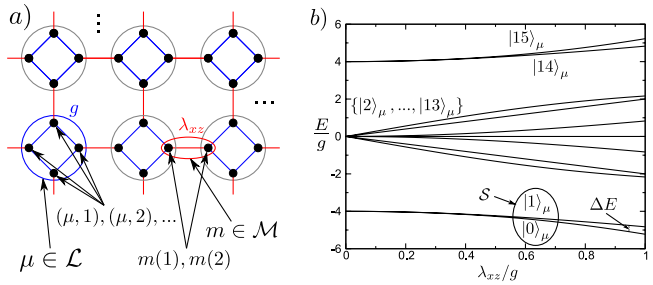

FIG. 1 (color online). (a) Physical qubits (black dots) on a CaVO lattice. The four physical qubits of a lattice site $\mu\in\mathcal{L}$ (gray circles) are named with the double indices $(\mu,1),\ldots,(\mu,4)$. The two physical qubits of a bond $m\in\mathcal{M}$ are called $m(1)$ and $m(2)$. To a physical qubit (μ,i), the neighboring qubit on the bond is $\xi(\mu,i)$. The ZZ interactions of H_0 are solid lines inside the gray circles. (b) Energy spectrum of H_μ^{loc} [Eq. (3)].

$[CZ_{\mathcal{M}}, H_0] = 0$, the transformed Hamiltonian reads $H^{\text{loc}} = \sum_{\mu\in\mathcal{L}} H^{\text{loc}}_\mu$ with

$$H^{\text{loc}}_\mu := -g\sum_{i\leftrightarrow j} Z_{(\mu,i)} \otimes Z_{(\mu,j)} - \lambda_{xz}\sum_{i=1}^{4} X_{(\mu,i)}. \quad (3)$$

The H^{loc}_μ are local transverse-field Ising models (TFIM) on each lattice site μ, which can be solved by exact diagonalization. If R_μ is the 16×16 matrix of the 16 eigenvectors $|0\rangle_\mu \ldots |15\rangle_\mu$ of H^{loc}_μ, then the diagonal form of H in the basis Σ^{diag} is $H^{\text{diag}} := [\otimes_{\mu\in\mathcal{L}} R_\mu] H^{\text{loc}} [\otimes_{\mu\in\mathcal{L}} R^\dagger_\mu]$ [Fig. 1(b)]. Most importantly, the gap ΔE between the (unique) ground state $|\psi_H\rangle$ and the first excited state arises perturbatively in order 4 in λ_{xz}/g [5,6]. Its analytic expression is given by

$$\frac{\Delta E}{-2g} = 1 + \sqrt{1+\frac{\lambda_{xz}^2}{g^2}} - \sqrt{2 + 2\frac{\lambda_{xz}^2}{g^2} + 2\sqrt{\frac{\lambda_{xz}^4}{g^4}+1}}.$$

It is useful to generalize the cluster stabilizers K_a into the physical space

$$K_\mu := \bigotimes_{i=1}^{4} X_{(\mu,i)} \otimes Z_{\xi(\mu,i)}.$$

The K_μ can be also transformed into the basis Σ^{loc}: $K^{\text{loc}}_\mu = (CZ_{\mathcal{M}}) K_\mu (CZ_{\mathcal{M}}) = \otimes^4_{i=1} X_{(\mu,i)}$. In this basis it is easy to show that the K^{loc}_μ commute with each other and with H^{loc}. Consequently, the eigenvalues ± 1 for each stabilizer K_μ are conserved quantities. For $\lambda_{xz}\to\infty$, the ground state of H^{loc} is the polarized state that is eigenvector to all K^{loc}_μ with eigenvalue $+1$. The ground state of H in the limit $\lambda_{xz}\to 0$ is therefore the cluster state of the logical qubits.

The two low-energy states $\{|0\rangle_\mu, |1\rangle_\mu\}$ of H^{loc}_μ represent an effective qubit on the according lattice site μ [Fig. 1(b)]. We can therefore derive an effective low-energy model in the space $\mathcal{S}$ of all effective qubits

$$H^{\text{diag}}|_{\mathcal{S}} = -\frac{\Delta E}{2}\sum_{\mu\in\mathcal{L}} \tilde{Z}_\mu = -\frac{\Delta E}{2}\sum_{\mu\in\mathcal{L}} K^{\text{diag}}_\mu|_{\mathcal{S}}, \quad (4)$$

where $\tilde{Z}_\mu$ is the Pauli Z operator acting on the effective qubit on the lattice site μ and $K^{\text{diag}}_\mu = R^\dagger_\mu K^{\text{loc}}_\mu R_\mu$. It follows that the effective low-energy approximation of H in the limit $\lambda_{xz}\to 0$ is the cluster state Hamiltonian H_{cl} of the logical qubits.

Fidelity.—The usability of H for quantum computations depends on the question of how well the logical cluster state is approximated by $|\psi_H\rangle$. This can be quantified by the fidelity $F = |\langle\psi_H|\psi_{\text{CS}}\rangle|^2$ of two states and its generalization for density operators $F = \langle\psi_{\text{CS}}|\rho|\psi_{\text{CS}}\rangle$ [7]. The fidelity translates to the "success probability" of a MBQC using $|\psi_H\rangle$ as a resource state [5].

As shown in Ref. [8], for a Hamiltonian $H_N(\lambda)$ with control parameter λ, size N, and two ground states $\psi_N(\lambda)$, $\psi_N(\lambda')$, one finds $\lim_{N\to\infty} F(\psi_N(\lambda), \psi_N(\lambda')) = d^N$ with $d\in(0,1)$ being constant. The "fidelity per site" $d := \lim_{N\to\infty}\sqrt[N]{F(\psi_N(\lambda), \psi_N(\lambda'))}$ is therefore intensive. Since $d<1$ for $\lambda_{xz}>0$, the fidelity of the logical cluster state with $|\psi_H\rangle$ vanishes for $N\to\infty$. This questions the usability for large systems. However, the concept can still be applied by quantum error correction techniques [5]. In this context the value d translates to the success probability per measurement, which must be large enough to fulfill the threshold theorem [9].

We calculate the fidelity $F(|\psi_{\text{CS}}\rangle, \rho) = \langle\psi_{\text{CS}}|\rho|\psi_{\text{CS}}\rangle$ of the logical cluster state $|\psi_{\text{CS}}\rangle$ with the canonical density operator $\rho := \frac{1}{Z}e^{-\beta H} = \frac{1}{Z}\sum_i e^{-\beta E_{|\psi_i\rangle}} |\psi_i\rangle\langle\psi_i|$. Here $Z = \text{Tr}(e^{-\beta H})$ denotes the partition function, $\beta = \frac{1}{k_B T}$, and one finds

$$F(|\psi_{\text{CS}}\rangle, \rho) = F(|+\rangle_{\mathcal{L}}, \rho^{\text{loc}}_{\mathcal{L}}) = F(|+\rangle_\mu, \rho^{\text{loc}}_\mu)^N =: d^N,$$

with $|+\rangle_\mu := 1/\sqrt{2}(|0_{\log}\rangle_\mu + |1_{\log}\rangle_\mu)$. Consequently, it is sufficient to study a single lattice site (we omit index μ)

$$d = \langle +|R^\dagger \rho^{\text{diag}} R|+\rangle = \frac{1}{Z}\sum_{i=0}^{15} e^{-\beta E_{|i\rangle}} |\langle i|R|+\rangle|^2,$$

with $Z = \sum_i e^{-\beta E_{|i\rangle}}$. Next, we approximate $e^{-\beta E_{|i\rangle}}|\langle i|R|+\rangle|^2 \approx 0$, $\forall\ i\neq 0$ and $Z \approx e^{-\beta E_{|0\rangle}} + e^{-\beta E_{|1\rangle}}$, which is justified by the following observations: (a) For MBQC, we have to choose the temperature low enough so that even the first excited state plays a minor role. Due to the exponential scaling factor we can omit all contributions of high-energy states. (b) Due to the orthogonality of the vectors $|i\rangle$, one has $|\langle i|R|+\rangle|^2 \le 1 - |\langle 0|R|+\rangle|^2$ for all $i\in\{1,\ldots,15\}$. For not too large λ_{xz}, we expect $|\langle 0|R|+\rangle|^2 \lesssim 1$, so the contributions of the other states are small. (c) For the first excited state, the relation $|\langle 1|R|+\rangle|^2 = 0$ holds. This is proven by the fact that $R|+\rangle$ and $|1\rangle$ do not have the same conserved eigenvalue of the K operator.

The resulting fidelity per site d,

$$d \approx \frac{1}{1+e^{-\beta\Delta E}} |\langle 0|R|+\rangle|^2, \quad (5)$$

is shown in Fig. 2 for different T. For finite T and for small λ_{xz}, the fidelity is dominated by thermal fluctuations. For large λ_{xz}, the curve follows the zero-temperature ground-state fidelity. In between, there is a trade-off between both effects. If one assumes an error correction algorithm for a 1-WQC with a simple error model of Pauli errors [10] as it is given in [11] with an error threshold of 1.4% ($d>0.986$), then the maximum $T_{\max}$ where this threshold holds is $T_{\max} = 2.18\times 10^{-4} g/k_B$. It is reached for $\lambda^{\text{opt}}_{xz} = 0.222g$.

The Hamiltonian H can therefore be used as a 1-WQC under conditions that in principle could be prepared in a laboratory. Next, we determine the robustness of such an effective cluster state against external perturbations.

Z field.—First, we consider the presence of an external field in Z direction:

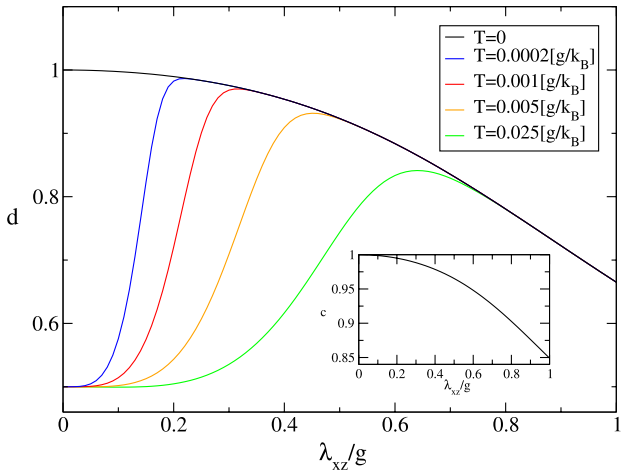

FIG. 2 (color online). Fidelity per site d [Eq. (5)] in dependence of λ_{xz} for different temperatures T. Inset: Contribution $c(\lambda_{xz}/g)$ of a physical Z operator to the effective $\tilde{X}$ operator.

$$H_z := gH_0 + \lambda_{xz}V - h_z \sum_{\mu\in\mathcal{L}} \sum_{i=1}^{4} Z_{(\mu,i)}.$$

Let us formulate H_z using the basis Σ^{diag} limited to $\mathcal{S}$. Since the perturbation commutes with $CZ_{\mathcal{M}}$, one has

$$H_z^{\text{diag}}|_{\mathcal{S}} = -\frac{\Delta E}{2} \sum_{\mu\in\mathcal{L}} \tilde{Z}_\mu - 4h_z c(\lambda_{xz}/g) \sum_{\mu\in\mathcal{L}} \tilde{X}_\mu,$$

where $c(\lambda_{xz}/g) \in \mathbb{R}$ can be read easily from the matrix $R_\mu Z_{(\mu,i)} R_\mu^\dagger$. One finds $\lim_{\lambda_{xz}\to 0} c = 1$ and the space $\mathcal{S}$ is decoupled from the high-energy space for this limit. The value $c(\lambda_{xz}/g)$ is plotted in Fig. 2. We stress that the scale ΔE is of order 4 in λ_{xz}, while the scale of the $\tilde{X}$ field is proportional to h_z. One therefore expects a polarization of the ground state for very small ratios h_z/λ_{xz}.

This is confirmed by solving the Hamiltonian H_z exactly in the basis Σ^{loc}:

$$H_z^{\text{loc}} = -\sum_{\mu\in\mathcal{L}} \left(H_\mu^{\text{loc}} + h_z \sum_{i=1}^{4} Z_{(\mu,i)} \right) =: \sum_{\mu\in\mathcal{L}} H_{z,\mu}^{\text{loc}},$$

which is still a sum of local terms $H_{z,\mu}^{\text{loc}}$. The fidelity per site of the ground state is calculated as in the unperturbed case using the eigenvectors of $H_{z,\mu}^{\text{loc}}$. Figure 3(a) shows that very small ratios h_z/λ_{xz} already have a significant impact on d. For the above example, one finds the upper bound $h_z^{\max} = 1.52 \times 10^{-5} g$, satisfying the threshold $d \geq 0.986$ at $T = 0$. Thermal fluctuations play only a minor role, because the gap is strongly increased by the external field [6].

ZZ coupling.—Second, we consider the effect of additional Ising ZZ couplings on the bonds $m \in \mathcal{M}$:

$$H_{zz} := gH_0 + \lambda_{xz}V - \lambda_{zz} \sum_{m\in\mathcal{M}} Z_{m(1)}Z_{m(2)}.$$

Now we formulate the Hamiltonian using the basis Σ^{diag} limited to $\mathcal{S}$:

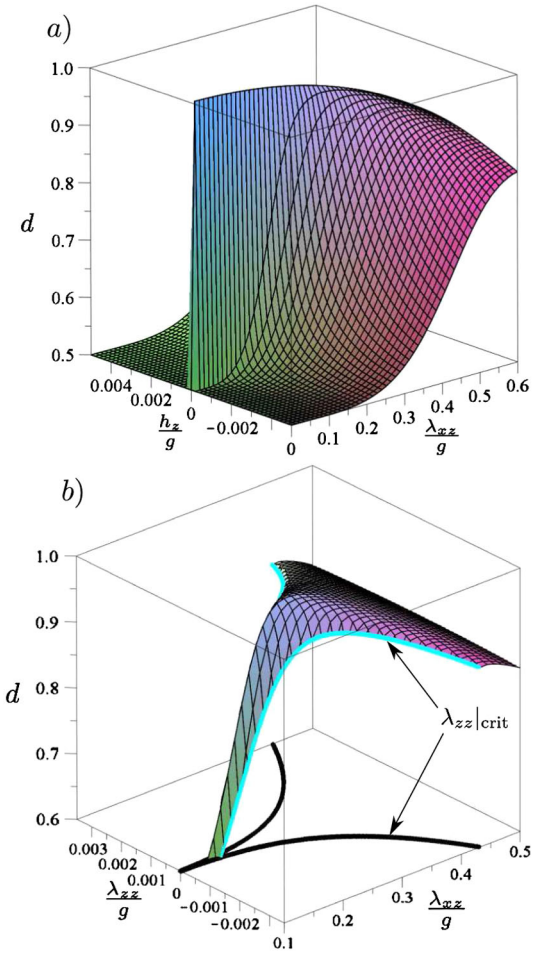

FIG. 3 (color online). (a) Ground-state fidelity per site $d = |\langle 0_z|R_z|+\rangle|^2$ of H_z as a function of λ_{xz} and h_z for $T = 0$. (b) Fidelity per site d [Eq. (8)] of H_{zz} in dependence of λ_{xz} and λ_{zz} for $T = 0.001 g/k_B$.

$$H_{zz}^{\text{diag}}|_{\mathcal{S}} = -\frac{\Delta E}{2} \sum_{\mu\in\mathcal{L}} \tilde{Z}_\mu - \lambda_{zz} c^2(\lambda_{xz}) \sum_{m\in M} \tilde{X}_{m(1)}\tilde{X}_{m(2)},$$

where again the space $\mathcal{S}$ is decoupled from the high-energy space for the limit $\lambda_{xz} \to 0$. This Hamiltonian represents a TFIM on the square lattice. For this model, a quantum phase transition takes place at $\frac{2\lambda_{zz}|_{\text{crit}} c^2}{\Delta E} = 0.3285$, separating an ordered phase from a disordered phase [12]. The gap closes at the critical point, changing the ground state significantly, so the system is not useful for MBQC anymore. The energy ΔE is a fourth-order term in λ_{xz}, while the Ising part is of the order λ_{zz}. Very small values $\lambda_{zz}/\lambda_{xz}$ are therefore sufficient to destroy the cluster phase. The estimated critical line $\lambda_{zz}|_{\text{crit}}$ is shown in Fig. 3(b) as a function of λ_{xz}.

To approximately calculate the fidelity per site for finite temperatures, we use the analog of Eq. (5):

$$d \approx \frac{1}{1 + \frac{1}{4\pi^2} \int_{\vec{k}} e^{-\beta\omega(\vec{k})} d\vec{k}} d(|\psi_{H_{zz}|_{\mathcal{S}}}\rangle, |\psi_{\text{CS}}|_{\mathcal{S}}\rangle). \quad (6)$$

We note that point (c) is no longer valid, since K is no longer conserved, but the use of Eq. (5) is still justified by points (a) and (b). The dispersion $\omega(\vec{k}) \approx \sqrt{(\Delta E_{zz})^2 + (v \cdot |\vec{k}|)^2}$ of the first excited mode is taken into account by this equation, while correlated excitations are neglected [13]. The energy gap ΔE_{zz} of the TFIM is calculated by a dlogPadé [6,6] approximation of its order-13 series expansion [12] and $v = 0.99\Delta E/2$ [14] is the spin wave velocity at the critical point.

We now transform $|\psi_{\mathrm{CS}}\rangle$ into the effective basis

$$|\psi_{\mathrm{CS}}^{\mathrm{diag}}\rangle|_{\mathcal{S}} = \bigotimes_{\mu\in\mathcal{L}} R_\mu|+_\mu\rangle|_{\mathcal{S}} = \bigotimes_{\mu\in\mathcal{L}} |0_\mu\rangle\langle 0_\mu|R_\mu|+_\mu\rangle, \quad (7)$$

(using $\langle 1|R_\mu|+_\mu\rangle = 0$) such that Eq. (6) reads

$$d \approx \frac{|\langle 0|R|+\rangle|^2}{1 + \frac{1}{2\pi}\int_0^{2\sqrt{\pi}} e^{-\beta\omega(r)} r dr}\, d_{\mathrm{TFIM}}, \quad (8)$$

where $d_{\mathrm{TFIM}} := d(|\psi_{H_{zz}|_{\mathcal{S}}}\rangle, \otimes_{\mu\in\mathcal{L}}|0_\mu\rangle)$ corresponds to the ground-state fidelity per site of a TFIM with the polarized state. We have calculated it as a high-order series expansion about the high-field limit

$$d_{\mathrm{TFIM}} = 1 - \frac{1}{8}\lambda^2 - \frac{93}{256}\lambda^4 - \frac{2961}{2048}\lambda^6 - \frac{243\,005}{32\,768}\lambda^8 - \frac{812\,949\,139}{18\,874\,368}\lambda^{10} - \frac{17\,716\,040\,461\,601}{65\,229\,815\,808}\lambda^{12},$$

where $\lambda := \frac{2\lambda_{zz}c^2}{\Delta E}$ [6].

The fidelity per site of $H_{zz}|_{\mathrm{eff}}$ is plotted in Fig. 3(b). Close to the critical point, the fidelity drops due to quantum and thermal fluctuations. For the above example, one finds the upper bound $\lambda_{zz}^{\max} = 8.33 \times 10^{-5} g$, satisfying the threshold $d \geq 0.986$ at $T = 0$.

We additionally calculated the energy gap (order 5) and the ground-state fidelity (order 4) of the full Hamiltonian H_{zz} as series expansions. The high-energy contributions turned out to be negligible corrections to the low-energy results [6]. Consequently, all conclusions drawn above for the low-energy space are in quantitative agreement with the properties of the full model. The cluster phase breaks down by a second-order phase transition in the 3D Ising universality class already for very weak external perturbations.

Conclusions.—We have seen that the effective cluster state of H could be used as a 1-WQC under conditions that, in principle, can be prepared in a laboratory. However, to be of practical use, effective cluster states must also be robust against additional perturbations. Let us stress that we have not discussed the effects of additional noise in the system that originates from other external sources, giving local and time-dependent perturbations. It is very well possible that such noise can be defeated by quantum error correction techniques. Here, we have focused on global external perturbations that are typically present in any experimental realization as subleading terms in the Hamiltonian. The consequences of such additional operators cannot be resolved by existing error-correction procedures.

We have shown that very small external perturbations can already have a significant impact on effective cluster states. Typically, the effective multisite interactions yielding the effective cluster state arise in a high order in perturbation theory (here, order 4). Therefore, any external perturbation acting in the effective low-energy model in a lower order (here, order 1) represents a strong constraint for the effective implementation of a 1-WQC. This effect is present on any lattice and in any dimension for the problem studied in this work.

The physical mechanism leading to the dramatic loss of fidelity is actually very different for the two perturbations we have considered. The external Z field leads to a polarization of the ground state and therefore to a reduction of entanglement. The additional Ising coupling induces thermal and quantum fluctuations due to a quantum phase transition.

The qualitative aspects of our work are relevant for a much broader class of problems: any effective low-energy model that is derived perturbatively and that contains dominant multisite interactions is expected to be affected by external perturbations. The physical reason is that effective n-site interactions arise typically in order n, while it is likely that external perturbations exist that act nontrivially on the effective low-energy degrees of freedom already in a lower order. Consequently, no generic solution for a protection of effectively implemented multisite interactions can be formulated.

A prominent example is Kitaev's honeycomb model, which contains the so-called toric code as an effective low-energy model perturbatively in order 4 [15]. The toric code is a topological stabilizer code consisting solely of four-spin interactions. One can easily show that exactly the same kind of external perturbations studied in this work again give rise to operators in the effective model in order-1 perturbation theory, causing a breakdown of the topological phase for very small external perturbations. The experimental realization of the Abelian phase of Kitaev's honeycomb model is therefore a very hard task.

In light of the severe constraints found in this work for the realization of effective cluster states, let us finally mention concepts for MBQC using elementary entities with larger spins, which represent a promising route for future research [16–19].

K. P. S. acknowledges ESF and EuroHorcs for funding through his EURYI.

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