Observation of the Intrinsic Abraham Force in Time-Varying Magnetic and Electric Fields

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The Abraham force exerted by a time-dependent electromagnetic field on neutral, polarizable matter has two contributions. The one induced by a time-varying magnetic field and a static electric field is reported here for the first time. We discuss our results in the context of the radiative momentum in matter. Our observations are consistent with Abraham's and Nelson's versions for radiative momentum.

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The Abraham force is the force exerted by a time-dependent electromagnetic field on neutral, polarizable matter, and has been debated for over a hundred years. The macroscopic Maxwell's equations provide a continuity equation for electromagnetic momentum that takes the general form

$$\partial_t \mathbf{G} + \nabla \cdot \mathbf{T} = \mathbf{f}_1, \quad (1)$$

with $\mathbf{G}$ "some" electromagnetic momentum density, $\mathbf{f}_1$ some force density exerted on the radiation, and $\mathbf{T}$ "some" stress tensor. The apparent arbitrariness in assigning expressions for $\mathbf{G}$, $\mathbf{T}$ and $\mathbf{f}_1$ is known as the Abraham-Minkowski (AM) controversy. (For reviews see, e.g., [1,2]). In the Minkowski version one adopts $\mathbf{G} = \mathbf{G}_M = \mathbf{D} \times \mathbf{B}$ and Maxwell's equations lead to $\mathbf{f}_1(M) = \varepsilon_0(\nabla\varepsilon_r)\mathbf{E}^2 + \mu_0^{-1}(\nabla\mu_r^{-1})\mathbf{B}^2$; i.e., there is no force proportional to both the electric and magnetic fields. The Abraham version insists on a momentum proportional to the energy flow so that $\mathbf{G} = \mathbf{G}_A = \varepsilon_0\mu_0\mathbf{E} \times \mathbf{H}$, in which case the Abraham force density takes the form $\mathbf{f}_1(A) = \mathbf{f}_1(M) - \varepsilon_0(\varepsilon_r - 1/\mu_r)\partial_t(\mathbf{E} \times \mathbf{B})$. The second term on the right hand side will be referred to as the "Abraham force density". Other versions can be found in the literature, such as Peierls' proposition $\mathbf{f}_1(P) = \varepsilon_0(\varepsilon_r - 1)(\partial_t\mathbf{E} \times \mathbf{B} - \frac{1}{5}\mathbf{E} \times \partial_t\mathbf{B})$, in which case the full time derivative of the Abraham version has disappeared [3]. In the Einstein-Laub version the force $\mathbf{f}_1$ achieves a gradient, i.e., a part of $\nabla \cdot \mathbf{T}$. For the homogenous case, which we will consider for the remainder, the Einstein-Laub version is equivalent to the Abraham version.

In this work we will measure explicitly the force exerted on matter by a combination of a time-dependent electric field $\mathbf{E}(t)$ and a time dependent magnetic field $\mathbf{B}(t)$. The Newton-Lorentz force on an object with mass density ρ is unambiguously given by $\rho\partial_t\mathbf{v} = \rho_q\mathbf{E} + \mathbf{J}_q \times \mathbf{B}$, with ρ_q the charge density, $\mathbf{J}_q$ the charge current density, and $\mathbf{v}$ the velocity. Under the assumption of macroscopic fields and in the absence of free charges and currents [4], it can be cast in the form [5]

$$\rho\partial_t\mathbf{v} + \nabla \cdot \mathbf{W} = \mathbf{f}_2, \quad (2)$$

with $\mathbf{W}$ another stress tensor and $\mathbf{f}_2 = -\mathbf{f}_1(M) + \varepsilon_0(\varepsilon_r - 1)\partial_t(\mathbf{E} \times \mathbf{B})$. Since $\mathbf{W}$ vanishes outside the object one identifies $\mathbf{F} = \int \mathbf{f}_2 dV$ as the force exerted by the electromagnetic field on the matter. From Newton's third law one expects that $\mathbf{f}_1 = -\mathbf{f}_2$. This is not valid for neither the Minkowski nor the Peierls version, and is true only for the Abraham version if $\mu_r = 1$. The version for which Newton's third law strictly holds is the Nelson version [6], in which the radiative momentum density is chosen as $\mathbf{G} = \mathbf{G}_N = \varepsilon_0\mathbf{E} \times \mathbf{B}$, i.e., equal to the expression in vacuum.

When taking a quantum-mechanical approach to the problem, new controversies seem to appear. It is well known that the presence of magnetic fields makes the kinetic momentum $\mathbf{P}_{\text{kin}} = \sum_i m_i\dot{\mathbf{r}}_i$ different from the conjugated momentum $\mathbf{P} = \mathbf{P}_{\text{kin}} + \frac{1}{2}\sum_i q_i\mathbf{B} \times \mathbf{r}_i$. This difference was recently put forward as a solution to the AM controversy [7]. However, in quantum mechanics a third "pseudo-momentum" $\mathbf{K} = \mathbf{P}_{\text{kin}} + \sum_i q_i\mathbf{B} \times \mathbf{r}_i$ appears that is conserved in time [8], and is clearly different from the other two. It is easy to verify that the validity of the Nelson version is directly related to the conservation of $\mathbf{K}$. A second controversy was initiated by Feigel [9] who argued that the quantum vacuum provides an additional contribution to the Abraham force. This proposition was refuted theoretically [8] and experimentally [10].

The experimental observation of the Abraham force induced by an *oscillating* electric field and a *static* magnetic field was reported by James [11] and by Walker *et al.* [12,13] in solid dielectrics, and recently by Rikken and van Tiggelen in gases [10]. These observations clearly invalidated the Minkowski version for $\mathbf{f}_1$, although modifications of the Minkowsi energy-momentum tensor have been proposed to make it consistent with these results [14,15]. However, the Abraham force due to an *oscillating* magnetic field and a *static* electric field has so far never been observed and was even reported *unobservable* in a specifically designed experiment [16,17]. The two cases clearly correspond to physically different situations. A time-dependent electric field creates moving charges ($\mathbf{J}_q = \partial_t \mathbf{P}$) that are subject to the Lorentz force. A time-varying magnetic field induces a rotational electric field that acts on the polarization charges. Walker *et al.* suggested an explanation for their failure to observe the $\mathbf{E} \times \partial_t \mathbf{B}$ component [18] that involves the compensation of this bulk component by a surface contribution. The current experimental situation surrounding the AM controversy for low-frequency electromagnetic fields is therefore unsatisfactory as not a single prediction for the Abraham force has been experimentally confirmed. At optical frequencies the situation is even less clear. Here, the Abraham force cannot be observed directly, as it averages to zero over one cycle, and one has to resort to the observation of momentum transfer from light to matter, with the aforementioned conceptual difficulty of which type of momentum to consider. For recent discussions, see the two comments [19] on the work of She *et al.* [20].

In this Letter, we report the first observation of the *intrinsic* Abraham force on a dielectric induced by a time-varying magnetic field. Our observations reveal the symmetry between electric and magnetic variations; i.e., we confirm $\mathbf{f} \propto \partial_t(\mathbf{E} \times \mathbf{B})$, excluding the Peierls' version of radiative momentum, and in particular confirming the Abraham and Nelson versions. We cannot discriminate between the latter two, as $\mu = 1$ in our case. On the theoretical side, our finding supports the dominant role of pseudo-momentum in the controversy on radiative momentum, and not the one of kinetic or conjugated momentum as was suggested by other work [7].

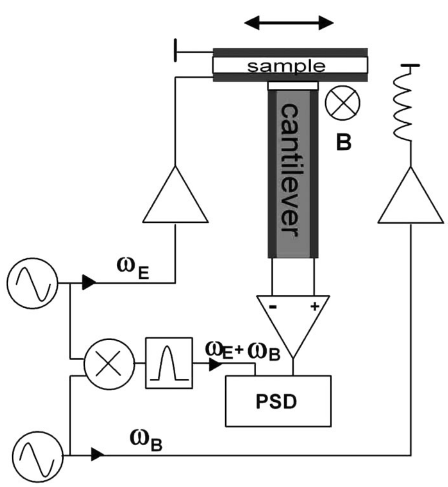

FIG. 1. Schematic setup of the experiment. A magnetic field B is applied perpendicular to the drawing at frequency ω_B, and high voltage at frequency ω_E is applied to the electrodes of the sample. The cantilever signal is phase sensitively detected at the sum frequency $\omega_B + \omega_E$. Typical sample size is $9 \times 2 \times 0.35$ mm^3. PSD stands for phase sensitive detector.

A schematic view of our experiment is shown in Fig. 1. The sample consists of a slab of Y5V dielectric recovered from a ceramic capacitor [21] (measured $\varepsilon_r = 1.7 \times 10^5$). It is covered on both faces by a silver-paint electrode and mounted by means of an insulating spacer on a piezoelectric bimorph cantilever [22]. The sensitivity of the cantilever was determined by applying a known force to the sample and measuring the charge generated by the cantilever with an electrometer. An oscillating magnetic field $B \cos \omega_B t$ is applied perpendicular to the electric field $E \cos \omega_E t$ inside the slab. The cantilever signal is phase sensitively detected at $\omega_B + \omega_E$ and corresponds therefore to a force on the sample proportional to $\mathbf{E}(t) \times \mathbf{B}(t)$. All other possible forces that would result from field gradients give a net zero contribution because of the symmetry of our experimental geometry and appear at other frequencies [see Eq. (1)].

A typical result is shown in Fig. 2. A clearly linear electric and magnetic field dependence of this force is observed, which we therefore identify as the Abraham force. The linear dependence of $\mathbf{F}_A = \mathbf{f}_A V$, where V is the sample volume, on the frequency of the electric field, crossed with a static magnetic field, was explicitly verified by Rikken and van Tiggelen [10], i.e., $\mathbf{F}_A \propto \partial_t \mathbf{E} \times \mathbf{B}$. To investigate the dependence of $\mathbf{F}_A$ on the magnetic field variation in time, we have varied ω_B whilst keeping

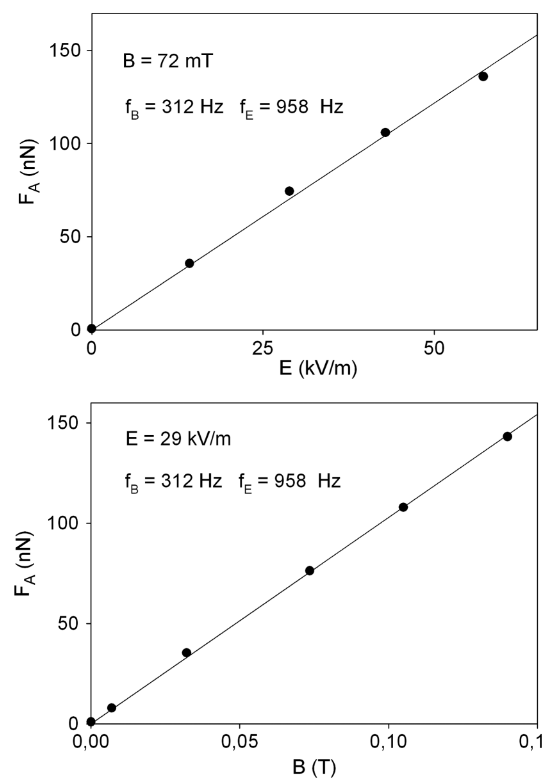

FIG. 2. Abraham force exerted on a slab of Y5V dielectric by crossed oscillating electric and magnetic fields ($f_i = \omega_i / 2\pi$). Straight lines are linear fits to the data.

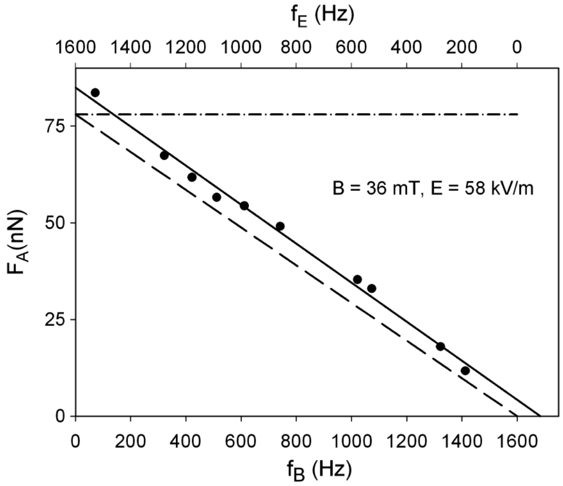

FIG. 3. Dependence of the Abraham force on a solidly contacted Y5V slab on the frequencies of the electric and magnetic fields, while the sum frequency and the amplitudes are kept constant. The solid line is a linear fit to the data. Note that the intercept with the ordinate of the theoretical lines (dashed and dot-dashed, see text) is subject to a systematic uncertainty of 5%.

$\omega_B + \omega_E$ constant. This guarantees that the sensitivity of our cantilever remains constant. The result is shown in Fig. 3, and shows clearly a strictly linear dependence on the electric field frequency. So we find quantitative agreement with a prediction $\mathbf{f}_A = \varepsilon_0(\varepsilon_r - 1)\partial_t\mathbf{E} \times \mathbf{B}$ (dashed line) and not at all with the predicted $\mathbf{f}_A = \varepsilon_0(\varepsilon_r - 1)\partial_t(\mathbf{E} \times \mathbf{B})$ (dot-dashed line) ($\mu = 1$ for Y5V). From this, one could infer the absence of a contribution to $\mathbf{F}_A$ of the form $\mathbf{E} \times \partial_t\mathbf{B}$. This concurs with the findings by Walker *et al.* [16]. The explanation for the apparent absence of such a contribution is the almost complete compensation of this contribution by an additional force on the electrodes. Walker *et al.* [18] have proposed a description for such a compensation in terms of the Maxwell stress tensor. We propose a simpler explanation, based on the different space charges present in the sample-electrode system, as illustrated in Fig. 4. It can be easily shown for the surface charge densities $\sigma_1 = \epsilon_0\varepsilon V/d = -\sigma_4$ and $\sigma_2 = -\epsilon_0(\varepsilon - 1)V/d = -\sigma_3$. The time varying magnetic field induces an electric field $\mathbf{E}_{\mathrm{ind}}$ that obeys

$$\int_S \mathbf{E}_{\mathrm{ind}} \cdot dl = \int \partial_t\mathbf{B} \cdot dS. \tag{3}$$

The Abraham force on the dielectric due to a time varying magnetic field corresponds to the force exerted by $\mathbf{E}_{\mathrm{ind}}$ on σ_2 and σ_3 but it will be almost completely compensated by the force exerted by $\mathbf{E}_{\mathrm{ind}}$ on σ_1 and σ_4, i.e., on the electrodes. It follows that the force due to a time varying magnetic field on the ensemble of electrodes plus dielectric is only $1/(\varepsilon - 1)$ of this force on the dielectric alone. It is therefore very difficult to observe the $\mathbf{E} \times \partial_t\mathbf{B}$ contribution to the Abraham force when the electric field is supplied by electrodes that are fixed to the dielectric.

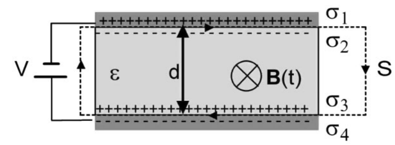

FIG. 4. Section of the sample + contacts, showing the different surface charges.

In order to observe the intrinsic Abraham force on the dielectric, including the $\mathbf{E} \times \partial_t\mathbf{B}$ contribution, we have used a configuration where the electrodes are no longer rigidly connected to the dielectric (Fig. 5). The electrodes are fixed in the laboratory frame, and the sample is fixed to the cantilever but otherwise free to move. The electrical contact between sample and electrodes is provided by an ionic liquid [23]. Now the ionic surface charges on the liquid side of the liquid-sample interface (i.e., the equivalents of σ_1 and σ_4) that almost balance the surface charges on the sample can move freely in the liquid along the interface under the influence of $\mathbf{E}_{\mathrm{ind}}$. They would therefore in the ideal case not exert any force on the sample. In practice such ionic movement will partially be transferred to the sample by the inevitable viscous drag of the liquid on the sample. Therefore, partial compensation of the Abraham force on the sample may still occur in this configuration, but it should be much weaker than for the case of fixed electrodes. Such a drag could also transfer a part of the Lorentz force $\mathbf{F}_L$, experienced by the ionic current in the liquid, to the sample. It can be easily shown that this Lorentz force is given by $F_L = IBl = 2\pi f_E CVBl$, where I is the current passing through the liquid, l is the total thickness of the two liquid layers, C the capacitance and V the applied voltage.

The result of the measurement of the Abraham force with the liquid contacts is shown in Fig. 6 as a function of the electric field frequency f_E, where $f_E + f_B$ is kept constant. We clearly observe an Abraham force that does not vanish for low f_E as in Fig. 3. The full symbols are the raw data, and the open symbols are the data when corrected for the measured resistive losses in the ionic liquid, which somewhat reduces the electric field on the Y5V slab. Currently we do not have a conclusive explanation for the small remaining negative slope. Combined with the earlier result that $F_A \propto f_E$ for a constant magnetic field [10], Fig. 6 proves that $F_A \propto f_E + f_B \propto \partial_t(\mathbf{E} \times \mathbf{B})$. For

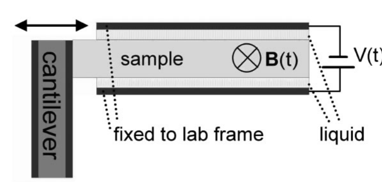

FIG. 5. Modified cantilever setup. Electrodes are fixed to the lab frame, and electrical contact to the sample is provided by an ionic liquid.

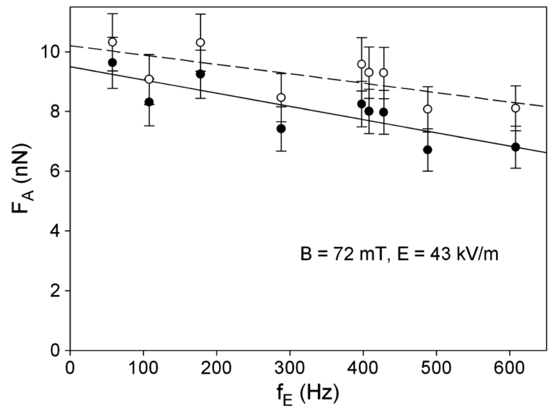

FIG. 6. Abraham force measured on a Y5V slab, contacted by an ionic liquid as illustrated in Fig. 5. $f_E + f_B = 690$ Hz. The solid line is a linear fit to the raw data (closed symbols). The dashed line is a linear fit to the corrected data (see text).

our parameters, the Lorentz force on the liquid is $F_L = 1.5$ nN at $f_E = 100$ Hz and it has the same sign as F_A. Its contribution to the observed force on the sample can therefore be neglected at low f_E In the liquid contact configuration, it is difficult to accurately determine the size of the contacted area of the sample and thereby the absolute value of the driving Abraham force density. From the measured capacitance value we deduce a contacted area that would result in an Abraham force of 15 nN, in reasonable agreement with the observed value.

In conclusion, we have confirmed the suggestion by Walker *et al.* [16,17] that in solidly contacted dielectrics, the only measurable contribution to the Abraham force is of the form $\partial_t \mathbf{E} \times \mathbf{B}$. We have provided a simple explanation for this apparent absence of a $\mathbf{E} \times \partial_t \mathbf{B}$ contribution in terms of the compensating surface charge in the electrodes. For a dielectric that is contacted by means of a conducting liquid, we have observed the intrinsic Abraham force, which we find to be proportional to $\partial_t(\mathbf{E} \times \mathbf{B})$, thereby explicitly verifying the Abraham and Nelson predictions of the mechanical force density of electromagnetic fields in dielectrics. This result greatly limits the arbitrariness in Eq. (1), leaving only the possibility to assign terms derived from Maxwell's equations to either the momentum or the stress tensor, but not to the force. In particular, our results invalidate the Minkowski and Peierls versions of electromagnetic momentum. In the solution proposed by Barnett [7] the Abraham version was associated with kinetic momentum, and the Minkowski version with canonical momentum. We have argued that the Nelson version is intimately related to a third pseudo momentum. To finally discriminate between the two remaining candidates—the Abraham and Nelson versions—one would have to discriminate between $(\varepsilon_r - 1/\mu_r)$ and $(\varepsilon_r - 1)$ as a prefactor, which will be the subject of future work.

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