Possible Fermi Liquid in the Lightly Doped Kitaev Spin Liquid

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We study the lightly doped Kitaev spin liquid (LDKSL) and find it to be the Fermi liquid. The LDKSL satisfies the two key properties of the standard Landau Fermi liquid: the low-energy quasiparticles are well defined and the Fermi sea has the quantized volume determined by Luttinger's theorem. These features can be observed in angle-resolved photoemission spectroscopy measurements. Meanwhile, the LDKSL has the topological Kitaev spin liquid surrounding the Fermi sea. So the LDKSL violates the Wiedemann-Franz law and has a large Wilson ratio. These results have the potential experimental verifiability in iridates upon doping.

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Doping the spin liquid brings new phases in strongly correlated electron systems and is believed to be the key physics in high T_c cuprates [1]. Searching the parent spin liquid in the quantum-frustrated Mott insulators gets a lot of attention [2]. In Ref. [3], Kitaev proposed an exactly solvable spin model on a honeycomb lattice that has the spin-liquid ground state. Recently, the Kitaev model has become potentially realizable in strong spin-orbit coupling magnets, such as layered iridates A_2IrO_3 ($A =$ Na, Li) [4,5]. In these materials, the spin interactions contain both the isotropic Heisenberg term and the anisotropic Kitaev term [6,7]. Numerical results showed that the Kitaev spin liquid (KSL) is topologically robust against small Heisenberg-type perturbation and magnetic fields [7,8]. The exact parent spin model of the KSL and its experimental realization provide us the opportunity to test the physics of the doped spin liquid. The KSL is a resonating valence-bond (RVB) state [9,10]. So the doped Kitaev spin liquid (DKSL) is in the broader context of the doped RVB state, which is related to the physics of high T_c cuprates [1,9].

The DKSL was studied in Refs. [11,12] by using slave-boson methods. Both of them obtained p-wave superconductors. Motivated by these studies, in this Letter we mainly focus on the lightly doped Kitaev spin liquid (LDKSL). We find that the LDKSL has the possibility to be the Fermi liquid before turning into a p-wave superconductor in the large doping regime. Without doping, the KSL has topological order; it has fourfold degeneracy on the torus. The topological order is protected by Z_2 gauge symmetry and is robust against local perturbations [7,8,13]. The Fermi liquid of the LDKSL is protected by the topological order of the KSL. In Refs. [11,12], however, the topological robustness of the KSL broke down upon doping immediately, because the slave-boson decompositions of the physical electron operator are not invariant under the Z_2 gauge transformation. To protect the topological robustness of the KSL even in the doped case, we will use the dopon representation of electron operators [14,15] to study the LDKSL. The dopon theory is a full fermionic decomposition and the physical electron is Z_2-gauge invariant. The electron is in terms of two different fermionic components: "dopons," the well-defined quasiparticle excitations with the charge -e and spin $1/2$; and"spinons," the neutral, spin-$1/2$ excitations of the KSL. The dopon sector forms the Fermi liquid surrounded by the KSL of the spinons. Hybridizing between the dopons and spinons vanishes on the mean-field level. The LDKSL has well-defined, low-energy quasiparticles on the Fermi surface. The Fermi sea has the quantized volume satisfying Luttinger's theorem [16,17]. It belongs to the class of a Landau Fermi liquid. The Fermi surface is electron-like, regardless of whether we dope the holes or electrons into the KSL. The Landau-Fermi liquid properties can be verified in angle-resolved photoemission spectroscopy (ARPES) experiments. The background neutral spinons also have physical, observable contributions. The Fermi liquid of the LDKSL has a temperature-dependent specific heat coefficient and violates the Wiedemann-Franz law. In the $T = 0$ K limit, the LDKSL has a large Wilson ratio.

The KSL has the parent Hamiltonian of a spin $S = 1/2$ system on a honeycomb lattice [3]:

$$H_K = -J \sum_{\alpha\text{-links}} S_m^\alpha S_n^\alpha, \qquad \alpha = x, y, z, \tag{1}$$

where S_m^α is α-component of the $S = 1/2$ spin operator on mth site and the summation runs over all the nearest-neighbor α-links ($\alpha = x, y, z$) joining A and B sublattices oriented in the αth direction, as shown in Fig. 1. The primitive vectors of the honeycomb lattice can be chosen as $\mathbf{l}_1 = (\sqrt{3}/2)\hat{\mathbf{e}}_1 + \frac{1}{2}\hat{\mathbf{e}}_2$ and $\mathbf{l}_2 = \hat{\mathbf{e}}_2$. To solve the spin model (1) exactly, Kitaev introduced four Majorana fermions b_m^x, b_m^y, b_m^z, and c_m to rewrite the spin operators as $S_m^\alpha = i b_m^\alpha c_m$ ($\alpha = x, y, z$) with the constraint $D_m = b_m^x b_m^y b_m^z c_m = \frac{1}{4}$. In this Letter, the Majorana fermions are normalized such as $\{\gamma_m^\alpha, \gamma_n^\beta\} = \delta_{mn}\delta_{\alpha\beta}$ with $\gamma_m^{x,y,z} = b_m^{x,y,z}$, $\gamma_m^0 = c_m$. In terms of the Majorana fermions, the Kitaev model (1) now has the form

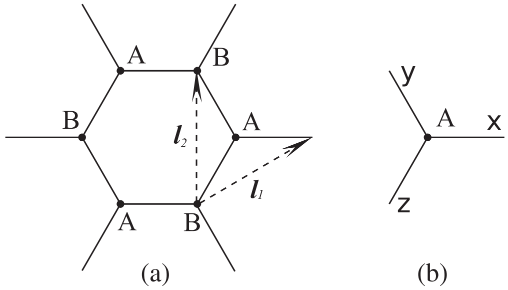

FIG. 1. (a) A honeycomb lattice and (b) the x, y, and z links joining A and B sublattices. $\mathbf{l}_1 = (\sqrt{3}/2)\hat{\mathbf{e}}_1 + \frac{1}{2}\hat{\mathbf{e}}_2$ and $\mathbf{l}_2 = \hat{\mathbf{e}}_2$ are the primitive vectors.

$$H_K = J \sum_{\alpha\text{-links}} ic_m c_n \hat{u}^{\alpha}_{mn}, \qquad \alpha = x, y, z, \tag{2}$$

with $\hat{u}^{\alpha}_{mn} = ic^{\alpha}_m c^{\alpha}_n$ and $\hat{u}^{\alpha}_{mn} = -\hat{u}^{\alpha}_{nm}$. Without loss of generality, we assume that m is in A sublattice.

From the four Majorana fermions, we can construct a fermionic Schwinger representation for the spin operators [10]:

$$S^{\alpha}_m = -\tfrac{1}{4}\mathrm{tr}[(F_m F^{\dagger}_m - I)\sigma^{\alpha}], \qquad \alpha = x, y, z, \tag{3}$$

with the definition

$$F_m \equiv \begin{pmatrix} f_{m\uparrow} & f^{\dagger}_{m\downarrow} \\ f_{m\downarrow} & -f^{\dagger}_{m\uparrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \sum_{\alpha=x,y,z,0} \gamma^{\alpha}_m \sigma^{\alpha}, \tag{4}$$

where $\sigma^{\alpha}(\alpha = x, y, z)$ are Pauli matrices and $\sigma^0 = iI$ (I is the identity matrix). The single-occupancy constraint is $G^{\alpha}_m = -\frac{1}{4}\,\mathrm{tr}[(F^{\dagger}_m F_m - I)\sigma^{\alpha}] = 0$. The mapping between the Majorana fermions and the Schwinger fermions is not unique and all such mappings are equivalent under SU(2) gauge transformations. The key point of the exact solution is $[H, \hat{u}^{\alpha}_{mn}] = 0$, and then $\hat{u}^{\alpha}_{mn}$ are the constants of motion with the eigenvalues $U^{\alpha}_{mn} = \pm 1/2$. The KSL is the RVB state with the parent mean-field Hamiltonian [10]

$$\begin{aligned} H_K = J \sum_{\alpha\text{-links}} U^{\alpha}_{mn}\hat{u}^0_{mn} - 2E_{\mathrm{vp}} \sum_{x\text{-links}} U^{x}_{mn}\hat{u}^{x}_{mn} \\ - 2E_{\mathrm{vp}} \sum_{y\text{-links}} U^{y}_{mn}\hat{u}^{y}_{mn} - 2E_{\mathrm{vp}} \sum_{z\text{-links}} U^{z}_{mn}\hat{u}^{z}_{mn}, \end{aligned} \tag{5}$$

with $\hat{u}^0_{mn} = \frac{i}{2}(f^{\dagger}_{m\uparrow}f_{n\uparrow} - f^{\dagger}_{m\uparrow}f^{\dagger}_{n\uparrow}) + \mathrm{H.c.}$, $\hat{u}^x_{mn} = i(f^{\dagger}_{m\downarrow}f_{n\downarrow} + f^{\dagger}_{m\downarrow}f^{\dagger}_{n\downarrow}) + \mathrm{H.c.}$, $\hat{u}^y_{mn} = \frac{i}{2}(f^{\dagger}_{m\downarrow}f_{n\downarrow} - f^{\dagger}_{m\downarrow}f^{\dagger}_{n\downarrow}) + \mathrm{H.c.}$, and $\hat{u}^z_{mn} = \frac{i}{2}(f^{\dagger}_{m\uparrow}f_{n\uparrow} + f^{\dagger}_{m\uparrow}f^{\dagger}_{n\uparrow}) + \mathrm{H.c.}$. Here, E_{vp} is the energy of the nearest-neighbor pair of the vortices on the honeycomb lattice [18]. The Hamiltonian (5) has Z_2 gauge symmetry and is invariant under the local Z_2 transformation

$$f_{m\sigma} \to \tilde{f}_{m\sigma} = G_m f_{m\sigma}, \qquad U^{\alpha}_{mn} \to \tilde{U}^{\alpha}_{mn} = G_m U^{\alpha}_{mn} G_n, \tag{6}$$

where G_m is an arbitrary function with only the two values ± 1. The Z_2 gauge symmetry leads to the topological order with fourfold degeneracy for the KSL on the torus. The projected ground state of the parent Hamiltonian (5) belongs to the RVB type, $|\mathrm{KSL}\rangle = \mathcal{P}|\Psi\rangle_{\mathrm{MF}}$ ($\mathcal{P}$ removes the double occupancy). It is the exact ground-state wave function of the Kitaev model (1).

The projected construction of KSL (5) is a good starting point for the understanding of the doped RVB state. In this Letter, we study the doped Kitaev model $H = H_t + H_K$. Here, H_K is the bilinear parent Hamiltonian (5) of the KSL and H_t is the hopping Hamiltonian in the form

$$H_t = -t \sum_{\langle mn\rangle\sigma} \mathcal{P} c^{\dagger}_{m\sigma} c_{n\sigma} \mathcal{P}. \tag{7}$$

We will employ the dopon representation for the electron operator [14,15]

$$c^{\dagger}_{m\sigma} = \frac{1}{\sqrt{2}} \mathcal{P}_d f^{\dagger}_{m\sigma} \Big(\sum_{\sigma'} \sigma' f_{m\sigma'} d_{m-\sigma'} \Big) \mathcal{P}_d. \tag{8}$$

Here, the spinon $f_{m\sigma}$ and the dopon $d_{m\sigma}$ are both fermionic operators. On every site, the states $|\uparrow 0\rangle$, $|\downarrow 0\rangle$, and the local singlet state $\frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ in terms of $f_{m\sigma}$ and $d_{m\sigma}$ are physical states mapping onto the states $|\uparrow\rangle$, $|\downarrow\rangle$, and the vacancy state $|0\rangle$ in terms of $c_{m\sigma}$, respectively. The operator $\mathcal{P}_d$ is to project out the unphysical triplet states of the spinon and dopon. The spinon sector is always half-filled on a honeycomb lattice, $\sum_{\sigma} f^{\dagger}_{m\sigma} f_{m\sigma} = 1$.

Here we give some comments on dopon decomposition (8). The dopon theory starts from the Mott insulator at half-filling and describes the KSL in terms of the spinons $f_{m\sigma}$. The spinon sector is still at half-filling in the doped case. The dopant holes are described in terms of the dopons $d_{m\sigma}$. The LDKSL is a doped Mott insulator. The physical electron is bilinear in the spinon operators and, thus, is Z_2-gauge invariant. The topological robustness of KSL is not lost upon doping in the dopon theory, in contrast to slave-boson methods in which the electron operator $c_{m\sigma} \propto b_m f^{\dagger}_{m\sigma}$ (b_m is the holon) is no longer invariant under Z_2 gauge transformation (6) and topological order is broken.

Slave-boson methods describe the hole components in term of bosonic holons. In a dilute heavy limit, the holons may be treated as nearly immobile spin vacancies [19,20]. Numerical results indicate that holons may not be stable low-energy excitations. The vacancy combined with a π Z_2 flux has a negative binding energy [19]. The low-energy excitations of the dopant holes have fermionic statistics [11,19]. The π Z_2 flux in the KSL can be constructed from $\hat{u}^{\alpha}_{mn}$ in terms of spinons [18]. It has the spin degree of freedom. Therefore, low-energy excitations of the dopant holes are fermions with the charge -e and spin $1/2$. We describe them as dopons in decomposition (8).

Honestly, there is no rigorous analytic proof for the validity of the dopon theory; nevertheless, it quite makes sense for us to use it in LDKSL if we have the simple belief in the robustness of the topological order. We can rewrite the dopon representation in matrix form:

$$C_m = \frac{1}{2\sqrt{2}}\sigma^z \mathcal{P}_d D_m^\dagger \mathcal{P}_d \sigma^z - \frac{1}{2\sqrt{2}}\mathcal{P}_d (F_m F_m^\dagger - I)\sigma^z D_m^\dagger \mathcal{P}_d,$$

with the definition

$$C_m \equiv \begin{pmatrix} c_{m\uparrow} & d_{m\downarrow}^\dagger \\ d_{m\downarrow} & -d_{m\uparrow}^\dagger \end{pmatrix}$$

and

$$D_m \equiv \begin{pmatrix} d_{m\uparrow} & d_{m\downarrow} \\ d_{m\downarrow}^\dagger & -d_{m\uparrow}^\dagger \end{pmatrix}$$

Then the hopping Hamiltonian (7) has the form in the dopon representation

$$\begin{aligned} H_t = -\frac{t}{8}\sum_{\langle mn\rangle} \mathcal{P}_d\{&\mathrm{tr}(\sigma^z D_m D_n^\dagger) - \mathrm{tr}[D_m \sigma^z (F_m F_m^\dagger - I)\sigma^z D_n^\dagger] \\ &- \mathrm{tr}[D_m \sigma^z (F_n F_n^\dagger - I)\sigma^z D_n^\dagger] \\ &+ \mathrm{tr}[D_m \sigma^z (F_m F_m^\dagger - I)(F_n F_n^\dagger - I)\sigma^z D_n^\dagger \sigma^z]\}\mathcal{P}_d. \end{aligned} \quad (9)$$

In the lightly doped regime close to the KSL, the dopon can move freely in the local ferromagnetic environment of the KSL. The motion of dopons introduces extra frustrations that stabilize the KSL. The most likely spontaneous instability comes from the hybridizing of two fermionic degrees of freedom. Hybridizing between the spinons and dopons, $B_m \propto \langle \mathrm{tr}(D_m F_m)\rangle \neq 0$, breaks both Z_2 gauge symmetry and physical U(1) electromagnetic symmetry [14,15]. Then we rediscover the p-wave superconductor obtained in Refs. [11,12]. This is true in the large doping regime, but questionable in the LDKSL. The KSL has a gap E_{vp} in the spectrum. Spontaneous mixing B_m competes with gap energy E_{vp} and vanishes in the lightly doped case, $x < \frac{E_{\mathrm{vp}}}{t/2} \simeq 0.134$ (x is the doping level per site). In this Letter, we set $t = J = 1$ and $E_{\mathrm{vp}} = 0.0668J$ [3]. Without mixing among dopons and spinons, $B_m = 0$, the four-operator and six-operator terms in Eq. (9) can be decomposed into bilinear terms such as $D_m^\dagger D_n$ and $F_m F_n^\dagger$. Spinons and dopons are decoupled on the mean-field level. Therefore, the LDKSL is the Fermi liquid formed by dopons surrounded by the KSL formed by spinons.

The full, effective mean-field Hamiltonian of the LDKSL is

$$H_{\mathrm{MF}} = H_d + H_f. \quad (10)$$

From now on we will ignore the effect of the projection $\mathcal{P}_d$ on the mean-field level. Here, the dopon Hamiltonian reads out

$$H_d = \sum_{\mathcal{A}\mathbf{k}\sigma} \epsilon_{\mathcal{A}\mathbf{k}}^d d_{\mathcal{A}\mathbf{k}\sigma}^\dagger d_{\mathcal{A}\mathbf{k}\sigma} + 2xN\mu_d, \quad (11)$$

with $\mathcal{A} = 1, 2$ and $\epsilon_{(1/2)\mathbf{k}}^d = \pm\frac{t}{2}|e^{i\mathbf{k}\cdot\mathbf{l}_1} + e^{i\mathbf{k}\cdot\mathbf{l}_2} + 1| - \mu_d$. N is the number of unit cells. The chemical potential μ_d is determined by the particle number of dopant holes x. For $x = 0.1$, $\mu_d = -1.324$. The spinon Hamiltonian is now ($U_{mn}^\alpha = \frac{1}{2}$) [21]:

$$H_f = \sum_{\mathbf{k}}{}' (\psi_{\mathbf{k}\uparrow}^\dagger M_{\mathbf{k}\uparrow}\psi_{\mathbf{k}\uparrow} + \psi_{\mathbf{k}\downarrow}^\dagger M_{\mathbf{k}\downarrow}\psi_{\mathbf{k}\downarrow}). \quad (12)$$

Here, $\sum'$ takes the summation over half the Brillouin zone. $\psi_{\mathbf{k}\sigma}^\dagger = (f_{1\mathbf{k}\sigma}^\dagger f_{2\mathbf{k}\sigma}^\dagger f_{1-\mathbf{k}\sigma} f_{2-\mathbf{k}\sigma})$, $\sigma = \uparrow/\downarrow$ is the Nambu representation of the spinons. The Hamiltonian matrices $M_{\mathbf{k}\sigma}$ are given as

$$M_{\mathbf{k}\uparrow} = \begin{pmatrix} 0 & ig_-(\mathbf{k}) & 0 & -ig_+(\mathbf{k}) \\ -ig_-(-\mathbf{k}) & 0 & ig_+(-\mathbf{k}) & 0 \\ 0 & -ig_+(\mathbf{k}) & 0 & ig_-(\mathbf{k}) \\ ig_+(-\mathbf{k}) & 0 & -ig_-(-\mathbf{k}) & 0 \end{pmatrix}$$

and

$$M_{\mathbf{k}\downarrow} = \begin{pmatrix} 0 & -ih_+(\mathbf{k}) & 0 & -ih_-(\mathbf{k}) \\ ih_+(-\mathbf{k}) & 0 & ih_-(-\mathbf{k}) & 0 \\ 0 & -ih_-(\mathbf{k}) & 0 & -ih_+(\mathbf{k}) \\ ih_-(-\mathbf{k}) & 0 & ih_+(-\mathbf{k}) & 0 \end{pmatrix}.$$

Here, $g_\pm(\mathbf{k}) = \frac{J_{\mathrm{eff}}}{4}(e^{-i\mathbf{k}\cdot\mathbf{l}_1} + e^{-i\mathbf{k}\cdot\mathbf{l}_2} + 1) \pm \frac{E_{\mathrm{vp}}^{\mathrm{eff}}}{2}$ and $h_\pm(\mathbf{k}) = \frac{E_{\mathrm{vp}}^{\mathrm{eff}}}{2}(e^{-i\mathbf{k}\cdot\mathbf{l}_1} \pm e^{-i\mathbf{k}\cdot\mathbf{l}_2})$. The effective constants are $J_{\mathrm{eff}} = J[(1-x)^2 - xt/2]$ and $E_{\mathrm{vp}}^{\mathrm{eff}} = E_{\mathrm{vp}}[(1-x)^2 - xt/2]$. The spinon Hamiltonian has the eigenvalues $\pm\epsilon_{(1/2)\mathbf{k}(\uparrow/\downarrow)}^f$ with $\epsilon_{1\mathbf{k}\uparrow}^f = \frac{J_{\mathrm{eff}}}{2}|e^{i\mathbf{k}\cdot\mathbf{l}_1} + e^{i\mathbf{k}\cdot\mathbf{l}_2} + 1|$ and $\epsilon_{2\mathbf{k}\uparrow}^f = \epsilon_{1\mathbf{k}\downarrow}^f = \epsilon_{2\mathbf{k}\downarrow}^f = E_{\mathrm{vp}}^{\mathrm{eff}}$.

The LDKSL has well-defined low-energy excitations and satisfies Luttinger's theorem. It belongs to the class of a Landau Fermi liquid. On the mean-field level, we can write down the electron Green function G_c in the LDKSL

$$G_c(\mathbf{k}, \omega) \propto G_d(\mathbf{k}, \omega) + G_{\mathrm{inc}} = \sum_{\mathcal{A}} \frac{1}{\omega + i\gamma - \epsilon_{\mathcal{A}\mathbf{k}}^d} + \cdots, \quad (13)$$

where $G_d(\mathbf{k}, \omega)$ is the Green function for the dopon sector and G_{inc} is the incoherent part. The real part of the electron Green function $\mathrm{Re}G_c$ satisfies Luttinger's theorem [16,17]:

$$\frac{1}{N}\sum_{\mathbf{k}\sigma} \mathrm{Re}G_c(\mathbf{k}, 0) > 0 = 2x \qquad \mathrm{mod}2. \quad (14)$$

The imaginary part of the electron Green function is proportional to the intensity $I(\mathbf{k}, \omega) \propto \mathrm{Im}G_c(\mathbf{k}, \omega)$ in ARPES measurements. The intensity at Fermi energy is shown in Fig. 2(a). The dispersions are shown in Fig. 2(b). Regardless of whether we dope the holes or electrons

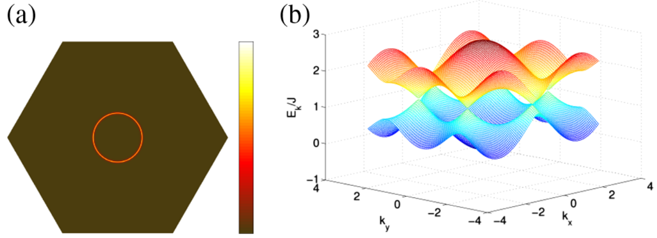

FIG. 2 (color online). (a) The ARPES intensity $I(\mathbf{k}, 0)$ near the Fermi energy for the LDKSL at $x = 0.1$. (b) The dispersions in ARPES measurements.

into the KSL, the Fermi surface is always electron-like around the Brillouin zone center.

The LDKSL is the Fermi liquid; however, it has some unusual properties due to the existence of the KSL of neutral spinons in the background. The LDKSL has the specific heat coefficient $\gamma = \gamma_d + \gamma_f$, with the dopon specific heat coefficient

$$\gamma_d = \frac{1}{4}\sum_{\mathcal{A}\mathbf{k}\sigma}(\beta\epsilon^d_{\mathcal{A}\mathbf{k}})^2\beta\mathrm{sech}^2(\beta\epsilon^d_{\mathcal{A}\mathbf{k}}/2), \tag{15}$$

and the spinon specific heat coefficient

$$\gamma_f = \frac{1}{2}\sum_{\mathcal{A}\mathbf{k}\sigma}{}'(\beta\epsilon^f_{\mathcal{A}\mathbf{k}\sigma})^2\beta\mathrm{sech}^2(\beta\epsilon^f_{\mathcal{A}\mathbf{k}\sigma}/2). \tag{16}$$

The specific heat coefficient γ in the LDKSL is no longer temperature independent at low temperatures. In transport measurements, the spinon sector makes a contribution to the thermal conductivity, but not to the electric conductivity, due to its charge neutrality. Therefore, the Wiedemann-Franz law breaks down. A similar violation due to the neutral mode was also found in disordered metallic systems [22,23]. We can estimate the Lorentz number of the LDKSL as follows:

$$L_m = \frac{\kappa}{T\sigma} \sim \frac{\gamma_d + \gamma_f}{\gamma_d}L_0, \tag{17}$$

with $L_0 \equiv \frac{\gamma_d}{T\sigma} = \frac{\pi^2}{3}(\frac{k_B}{e})^2$. Here, we assume that the mean free paths for dopons and spinons are close to each other. For $x = 0.1$, the temperature-dependent Lorentz number is shown in Fig. 3. There is a maximum at the temperature around $T^* = 0.31E^{\mathrm{eff}}_{\mathrm{vp}}$.

The spin susceptibility of the LDKSL is given as $\chi = \chi_d + \chi_f$, with the dopon spin susceptibility

$$\chi_d = \frac{1}{16}\sum_{\mathcal{A}\mathbf{k}\sigma}\beta\mathrm{sech}^2(\beta\epsilon^d_{\mathcal{A}\mathbf{k}}/2), \tag{18}$$

which has a temperature-independent Pauli behavior at low temperatures and is proportional to the density of states of the dopon sector. The spinon sector is a p wave-type spin liquid and has a vanishing density of states at zero temperature. However, the magnetic spin response is finite

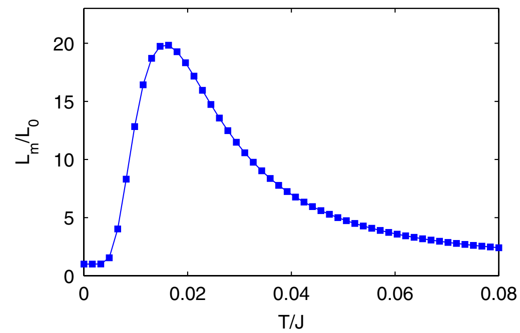

FIG. 3 (color online). The estimation of the temperature-dependent Lorentz number for the LDKSL at $x = 0.1$.

under magnetic fields along the z direction. The mean-field spin susceptibility of the spinon sector is given as [24]

$$\chi_f = \frac{\alpha_\uparrow\alpha_\downarrow}{\alpha_\uparrow + \alpha_\downarrow}, \tag{19}$$

with $\alpha_\sigma = \sum'_{\mathbf{k}}\alpha_{\mathbf{k}\sigma}$ and

$$\alpha_{\mathbf{k}\uparrow} = \frac{1}{\epsilon^f_{1\mathbf{k}\uparrow}\epsilon^f_{2\mathbf{k}\uparrow}}\left[\epsilon^f_{1\mathbf{k}\uparrow} + \epsilon^f_{2\mathbf{k}\uparrow} - \frac{4g_-(\mathbf{k})g_-(-\mathbf{k})}{\epsilon^f_{1\mathbf{k}\uparrow} + \epsilon^f_{2\mathbf{k}\uparrow}}\right], \tag{20}$$

$$\alpha_{\mathbf{k}\downarrow} = \frac{1}{\epsilon^f_{1\mathbf{k}\downarrow}\epsilon^f_{2\mathbf{k}\downarrow}}\left[\epsilon^f_{1\mathbf{k}\downarrow} + \epsilon^f_{2\mathbf{k}\downarrow} - \frac{4h_+(\mathbf{k})h_+(-\mathbf{k})}{\epsilon^f_{1\mathbf{k}\downarrow} + \epsilon^f_{2\mathbf{k}\downarrow}}\right]. \tag{21}$$

At the finite doping, χ_f is given as

$$\chi_f(x) = \frac{J}{J_{\mathrm{eff}}}\chi_f(0). \tag{22}$$

The effective exchange constant J_{eff} is reduced upon doping; however, the spinon magnetic response will be enhanced, consistent with numerical results [20]. At zero temperature, the spinon sector has the vanishing specific heat coefficient, but finite spin susceptibility. The LDKSL has the Wilson ratio

$$R = \frac{\chi_f/\chi_d + 1}{\gamma_f/\gamma_d + 1}R_d, \tag{23}$$

where $R_d = 1$ is the dopon Wilson ratio. At zero temperature for $x = 0.1$, the LDKSL has the Wilson ratio $R \simeq 8$, whichis much larger than 1 on the mean-field level.

In conclusion, we study the lightly doped Kitaev spin liquid, which has a potential experimental realization in doped, layered iridates. We calculate the electron Green function and thermodynamic properties for the LDKSL, which can be measured in further experiments. The LDKSL is the Fermi liquid with well-defined quasiparticles and quantized Luttinger's volume, satisfying Luttinger's theorem. The LDKSL has a temperature-dependent Lorentz number and a large Wilson ratio.

It should be emphasized that the Fermi liquid is possible only in the *lightly* DKSL in which the kinetic energy ($xt \ll J$) is small enough to leave the background KSL unchanged. In the large doping regime, we rediscover p-wave superconductors. The Fermi liquid in the LDKSL is reliable based on two main arguments. First, we believe the KSL is topologically robust even against doping. The Fermi liquid in the LDKSL does not break the topological order. Second, the KSL has a gap E_{vp} in the spectrum. Spontaneous mixing between the doped holes and the background spinons, B_m, competes with the gap energy E_{vp} and vanishes in the lightly doped case [25,28].

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