Spatial Power Combination for Omnidirectional Radiation via Anisotropic Metamaterials

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We present an efficient approach to realize the spatial power combination for omnidirectional radiation via metamaterials in the two-dimensional case. We propose a radially anisotropic zero-index metamaterial which can always produce ominidirectional radiation, independent of the number and position of the sources inside the metamaterial. When the radial component of the permeability tensor is approaching zero and the wave impedance is equal to that of free space, waves emitted from all sources inside the metamaterial are transformed into perfectly cylindrical waves without any reflection, and powers from different sources can be combined together to enhance the omnidirectional radiation. We have designed and fabricated such a radially anisotropic metamaterial, and both numerical and experimental results demonstrate the spatial power combination with high efficiency. The proposed idea can be extended to the three-dimensional case to generate perfectly coherent isotropic radiation in nature, which does not exist now. Metamaterial is a unique approach to obtain such a high-efficiency spatial power combination for omnidirectional radiation and isotropic radiation.

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In a three-dimensional (3D) world, the isotropic electromagnetic (EM) radiation can be conceptually produced by a point source, which radiates the same intensity of power in all directions. The isotropic radiation is usually used as a reference of directional radiation for practical antennas. In reality, however, coherent 3D isotropic radiation cannot exist since they violate Maxwell's equations [1]. Radiation from the Sun and other stars is isotropic and satisfies Maxwell's equations, but it is incoherent. Although 3D coherent isotropic radiation does not exist, antennas can generate omnidirectional radiation, which has the same power distributions in all directions in one plane but decreasing powers along the vertical angle. Omnidirectional antennas have found wide applications in wireless communications and broadcasting [1], such as cell phones, WiFi, base stations, and FM radios, in which the radiation powers are required as high as possible to transmit EM waves to long distances. To obtain high powers, spatial power combination is often adopted.

The spatial power combination is a commonly used technique in microwave and millimeter wave frequencies, due to the limited output power from individual solid state devices [2–4], which generally includes radiation units and a power-combining network. At high frequencies, it is rather difficult to realize a high-efficiency power combining network. The traditional combination circuits, such as Wilkinson power dividers and Lange couplers, suffer from large transmission-line losses, and hence the combination efficiency is greatly deteriorated. To solve the problem, the quasioptical and waveguide based spatial power-combination method has been proposed [4]. However, the used configuration is very complicated, requiring high fabrication precision and complicated architecture, and the number of input channels are limited. Hence, up until now, scientists and engineers are still finding efficient and inexpensive ways to realize spatial-power combinations.

Besides the efficiency concern, the conventional spatial power-combination techniques are invalid to omnidirectional radiation since an array of radiation units will always produce directive radiation. Metamaterials provides a way to solve the problem. Composed of periodic or nonperiodic subwavelength particles, metamaterials can realize unusual permittivity and permeability which do not exist in nature [5,6] and generate tailored material properties [7]. Hence, they can be used to control EM waves to achieve some fantastic phenomena that had only been in the human imagination, such as invisibility cloaks [8–10] and transformation-optics devices [11–13]. Among the family of metamaterials, more attention has recently been paid to zero-index materials, in which the phase velocity of EM waves approach infinity. Such materials are proved to be useful to enhance the directivity of antennas [14–25].

In this Letter, we propose a new feature of metamaterials which has not been explored before: to realize the spatial power combination for omnidirectional radiation. In the two-dimensional (2D) case, the isotropic radiation is the same as omnidirectional radiation, which can be generated by a line source in free space and does not violate Maxwell's equations. However, it is impossible to make spatial power combination for omnidirectional radiation using several line sources since they always produce directional radiation. Here, we present a radially anisotropic zero-index metamaterial (RAZIM) to solve the problem.

Consider the 2D problem shown in Fig. 1: in free space, a circular region or a circular-ring region is filled with RAZIM. Under the cylindrical coordinate system, the

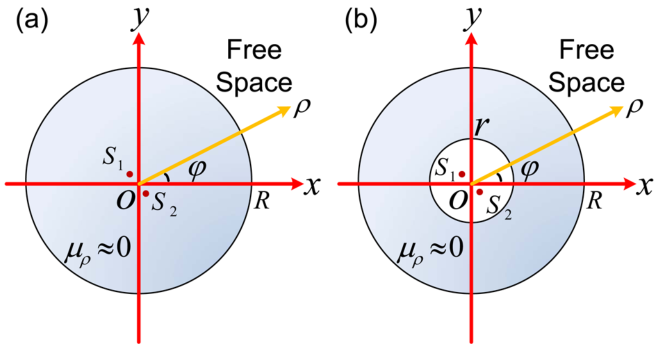

FIG. 1 (color online). 2D RAZIM structures for omnidirectional radiations and power combination. (a) A circular disk with radius R, in which S_1 and S_2 are positions of line sources 1 and 2. (b) A circular ring with outer radius R and inner radius r, in which S_1 and S_2 are positions of line sources 1 and 2.

material is described by permittivity and permeability tensors as

$$\bar{\bar{\varepsilon}} = \hat{\rho}\,\hat{\rho}\,\varepsilon_\rho + \hat{\varphi}\,\hat{\varphi}\,\varepsilon_\varphi + \hat{z}\,\hat{z}\,\varepsilon_z \tag{1}$$

$$\bar{\bar{\mu}} = \hat{\rho}\,\hat{\rho}\,\mu_\rho + \hat{\varphi}\,\hat{\varphi}\,\mu_\varphi + \hat{z}\,\hat{z}\,\mu_z \tag{2}$$

in which the component μ_ρ approaches zero. Next, consider the case of transverse electric (TE) polarization where only the z component of the electric field, E_z, and φ and ρ components of the magnetic field, H_φ and H_ρ, exist. After mathematical derivations [26,27], we get a differential equation of E_z as

$$\frac{1}{\mu_\varphi}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_z}{\partial\rho}\right) + \frac{1}{\rho\mu_\rho}\frac{\partial^2 E_z}{\partial\varphi^2} + \omega^2\varepsilon_z\rho E_z = 0. \tag{3}$$

Let $E_z = \Psi(\rho)\Theta(\varphi)$, then

$$\frac{1}{\rho^2}\frac{d^2\Psi}{d\rho^2} + \frac{1}{\rho}\frac{d\Psi}{d\rho} + \left(\omega^2\mu_\varphi\varepsilon_z\rho^2 - \frac{n^2}{\mu_\rho/\mu_\varphi}\right)\Psi = 0 \tag{4}$$

and $\Theta(\varphi)$ has the wave form of $e^{in\varphi}$. Next, consider boundary conditions on the air-RAZIM interface with $\varepsilon_z = \mu_\varphi = 1$, the solution Eq. (4) is expressed as

$$E_z = \sum_{n=-\infty}^{\infty} i^n (A_n H_n^{(1)}(k_s\rho))e^{in\varphi} \tag{5}$$

in which $k_s = \omega\sqrt{\mu_\varphi\varepsilon_z} = k_0$, and A_n is the amplitude. Since $\mu_\rho \to 0$, we have $n \to 0$, and then

$$E_z = A_0 H_0^{(1)}(k_0\rho). \tag{6}$$

It is clear that the EM waves in RAZIM are only propagating along the radial direction and the fields have no variation in terms of ϕ, supporting purely cylindrical waves no matter where the source is located.

To demonstrate the omnidirectional radiation properties of sources governed by RAZIM, numerical simulations are conducted when two line sources are located at different positions, as shown in Fig. 1. In simulations, we adopt the same geometric sizes as those in later experiments: the radius of the circular region $R = 33.33$ mm, and the inner radius of the circular ring $r = 16.65$ mm. Both the RAZIM circular disk shown in Fig. 1(a) and the circular ring shown in Fig. 1(b) are simulated, in which two line sources with unit electric current, (1) located at (0, 1.5 mm) and (2) located at (0, −4.0 mm), are considered. The operating frequency is selected as 10.4 GHz.

Numerical simulations demonstrate that perfect cylindrical waves are generated no matter where the source is located inside the circular RAZIM in Fig. 1(a) or in free space within the circular RAZIM ring in Fig. 1(b), enabling omnidirectional radiation [27]. However, the cylindrical waves excited by both sources may be in-phase or out-phase, depending on their relative positions. If we want the two sources to enhance the radiation power of cylindrical waves, i.e., to make power combination, the in-phase or nearly in-phase condition must be satisfied.

Because of similar behaviors to control cylindrical-wave radiation using the circular-disk and circular-ring RAZIM, we take the circular ring as an example to derive the power-combination condition. When only source 1 exists, the electric field in the air region inside the ring is written as

$$E_{z1}^{(1)}(\vec{\rho}) = \frac{i}{4}\omega\mu_0 I_1 H_0^{(1)}(k_0|\vec{\rho} - \vec{\rho}_1|) \tag{7}$$

in which I_1 and $\vec{\rho}_1$ are the electric current and position of source 1. From boundary conditions, the radiation electric field of source 1 in region 3 out of the ring can be written as [27]

$$E_{z1}^{(3)}(\vec{\rho}) = \frac{i}{4}\omega\mu_0 I_1 J_0(k_0\rho_1) H_0^{(1)}(k_0\rho). \tag{8}$$

When sources 1 and 2 exist simultaneously, the total radiation electric field in the free space is given by

$$E_z^{(3)}(\vec{\rho}) = \frac{i}{4}\omega\mu_0[I_1 J_0(k_0\rho_1) + I_2 J_0(k_0\rho_2)]H_0^{(1)}(k_0\rho) \tag{9}$$

in which I_1 and I_2 are electric currents and $\vec{\rho}_1$ and $\vec{\rho}_2$ are positions of sources 1 and 2, and $J_0(\cdot)$ is the zero-order Bessel's function.

From the property of the Bessel function, we notice that the total electric field radiated by both sources will be larger than the electric field by any single source when $I_1 J_0(k_0\rho_1)$ and $I_2 J_0(k_0\rho_2)$ have the same sign, and the combined power will be larger than the single power. For example, when two sources are close to the center of the circular ring so that $k_0\rho_1$ and $k_0\rho_2$ are much less than the first zero point of Bessel function, both $J_0(k_0\rho_1)$ and $J_0(k_0\rho_2)$ are close to 1, making a significant power combination.

The above conclusion has been validated by numerical simulations. Since $\rho_1 = 1.5$ mm and $\rho_2 = 4$ mm are close to center, the radiation fields by two sources are nearly in-phase. As a result, the total field becomes much stronger, and the combined power is nearly 4 times larger than single power, as shown in Fig. 2. Such simulation results show

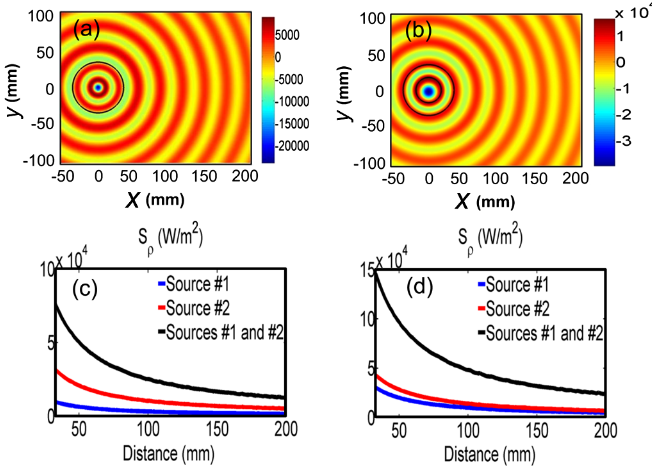

FIG. 2 (color online). The numerical simulation results of RAZIM circular disk and circular ring at 10.4 GHz when both sources exist, in which $R = 33.33$ mm, $r = 16.65$ mm, and the source current is 1 A. (a) Real parts of radiated electric fields for the circular disk. (b) Real parts of radiated electric fields for the circular ring. (c) Power density along the line $y = 0$ for the circular disk. (d) Power density along the line $y = 0$ for the circular ring.

high-efficiency power combination for perfect omnidirectional radiations through both circular-ring and circular-disk RAZIM.

To verify the predicted peculiar phenomena experimentally, we design RAZIM using split-ring resonators (SRR), which have strong magnetic-resonance responses [5]. A single SRR unit is illustrated in Fig. 3(a), in which the vertical direction is oriented to $\hat{z}$, the horizontal direction is $\hat{\varphi}$, and the normal direction is faced to $\hat{\rho}$. The geometric parameters of a SRR unit are designed as $p = 2.3$ mm, $w = 0.3$ mm, $f = 1.5$ mm, $s = 0.475$ mm, and $d = 0.15$ mm. The corresponding plasma frequency is close to 10.4 GHz. Since the circular-ring RAZIM is more convenient in actual applications, we only fabricate this structure in experiments based on the designed SRR [Fig. 3(b)], which has the same geometric parameters as in simulations. The sample is composed of six concentric layers, and each layer is a thin printed circuit board (relative permittivity: 2.65, loss tangent: 0.001, thickness: 0.25 mm) etched with a number of SRR. The gap between the different layers is 3.33 mm.

When the incident field impinges from different directions ($\hat{\rho}$, $\hat{\varphi}$, and $\hat{z}$), the unit cell (SRR) has different EM responses, which leads to equivalent material of radial anisotropy. From the parameter retrieval method [7], we get the effective permeability and permittivity components, as illustrated in Fig. 3(c). It is observed that the radial component of permeability is nearly zero with tiny loss ($\mu_\rho = 0.026 + i0.03$) at the plasma frequency (10.4 GHz), while the transverse components are $\mu_\varphi = 0.95 - i0.033$ and $\varepsilon_z = 3.125 + i0.04$, realizing RAZIM.

In experiments, the sample is placed in a near-field scanning system (microwave planar waveguide) [28], to

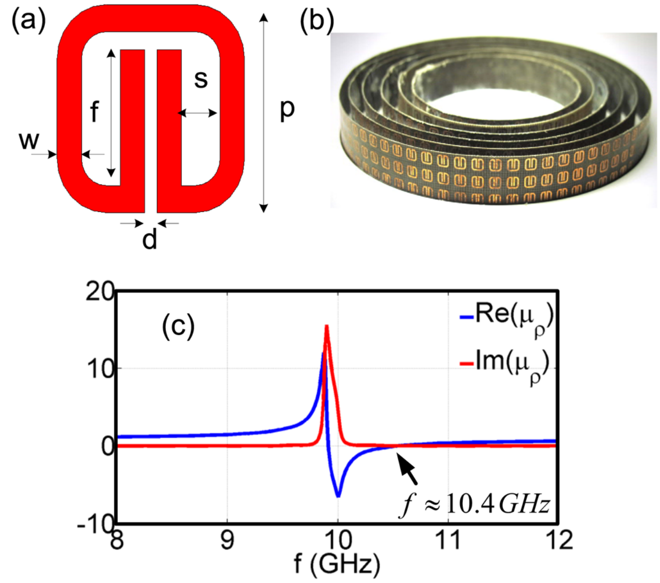

FIG. 3 (color online). Design of RAZIM. (a) Single SRR. (b) Fabricated circular-ring sample, which is composed of six PCB layers. (c) Retrieved radial component of the permeability (μ_ρ), which is nearly zero at the magnetic plasma frequency (10.4 GHz).

measure the 2D electric-field distributions, as shown in Fig. 4. The upper and lower metal plates form the planar waveguide, which ensures transverse electric and magnetic mode as the dominant mode. A detecting probe is mounted in the upper plate and two feeding probes in the lower plate. Similar to numerical simulations, the two sources are

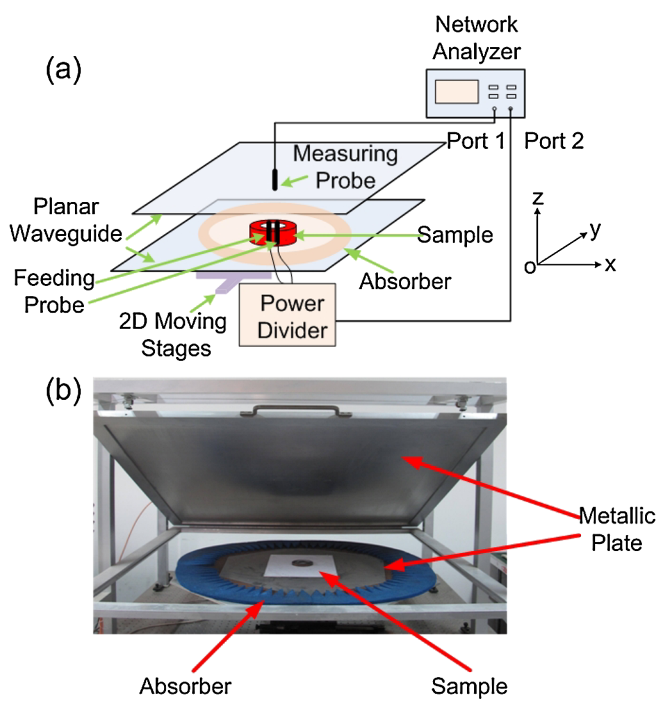

FIG. 4 (color online). Experimental setup (a) and photo (b) of the measurement system.

located at (10, 1.5 mm) and (10, −4.0 mm), respectively. The two feeding probes are connected to two output ports of a power divider. The power difference of two ports at 10.4 GHz is nearly 0.15 dB and the phase difference is nearly 5 degrees. 2D moving stages have been used to carry the lower metal plate and move in x and y directions, so that we can measure EM field distributions within an area. The step resolution in both x and y directions during scanning is selected as 1 mm.

Three separate experiments are conducted to verify the omnidirectional radiation and power-combination properties. In the first two experiments, sources 1 and 2 are excited, respectively, and the measured electric fields (real parts and phases) are demonstrated in Figs. 5(a)–5(d), from which we observe that the RAZIM ring can always guide EM waves to radiate omnidirectionally with nearly the same phase. Because of this important feature, when two sources are excited simultaneously in the third experiment, a significant enhancement of omnidirectional radiation is obtained, as illustrated in Figs. 5(e) and 5(f). The field enhancement will directly result in high-efficiency power combination.

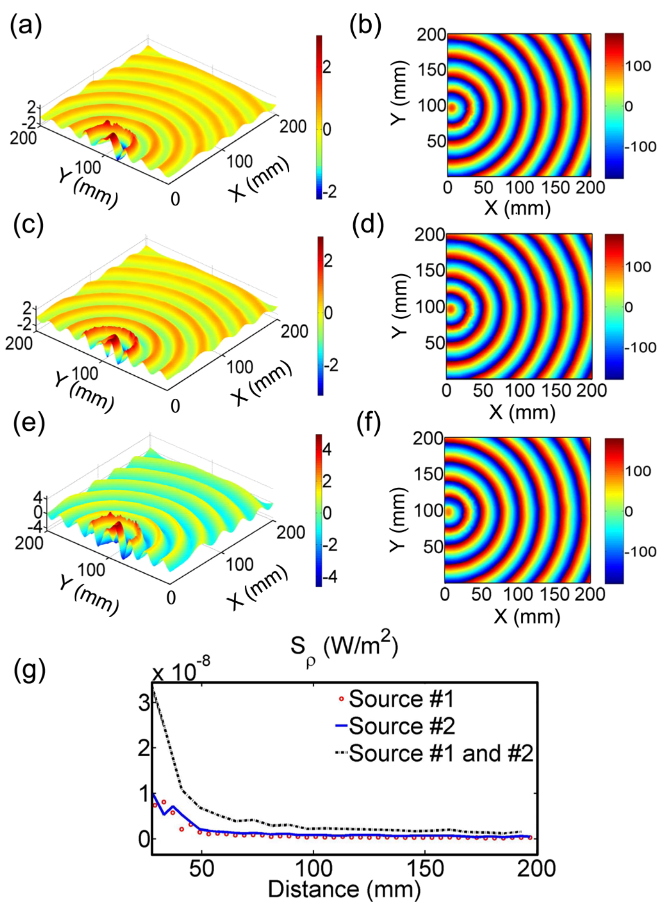

FIG. 5 (color online). Measurement results of the circular-ring RAZIM at 10.4 GHz, in which $R = 33.33$ mm and $r = 16.65$ mm. (a, b) Real part and phase of an electric field under the excitation of source 1. (c, d) Real part and phase of an electric field under the excitation of source 2. (e, f) Real part and phase of an electric field under the excitations of both sources. (g) Power densities under the excitations of sources 1, 2, and both along the line $y = 100$ mm.

Figure 5(g) gives the measured powers, which show that the combined power (34 nW/m^2) is nearly 4 times larger than the power of single source (source 1: 8 nW/m^2; source 2: 10 nW/m^2), demonstrating a highly efficient spatial-power combination. People may be confused by the nearly 4 times of single power, instead of 2 times. In physics, this is due to the interactions of two sources or antennas. Suppose that the current of source 1 is I and its self-radiation impedance is R_{11}, then its radiation power is $P_1 = I^2 R_{11}$. Similarly, for source 2, we have $P_2 = I^2 R_{22}$. When sources 1 and 2 exist simultaneously, the total radiation power will be $P = I^2(R_{11} + R_{12} + R_{21} + R_{22})$, in which R_{12} and R_{21} are mutual impedances of two sources. The role of RAZIM may result in a nearly equal mutual impedance to self-impedance, making the total radiation power nearly 4 times larger than the single power.

The above idea is also valid for multiple line sources and an array of continuous antennas [27]. In conclusion, we have proposed a method to realize the spatial power combination for omnidirectional radiations, which is a new feature of metamaterials and cannot be fulfilled by conventional approaches. We show that a circular ring (or circular disk) of RAZIM can make multiple sources (or antennas) inside radiate EM waves omnidirectional and combine the powers of each source (or antenna) with very high efficiency. The proposed method can be extended to a quasi-3D case by generating a cylindrical RAZIM pipe with certain length to produce omnidirectional radiations on different planes along the pipe with a high-efficiency spatial power combination. The idea can be further extended to a fully 3D case. A spherical RAZIM shell will generate perfectly isotropic radiations to all 3D directions, which cannot be realized using the conventional technologies.

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