Lensing of 21-cm Fluctuations by Primordial Gravitational Waves

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Weak-gravitational-lensing distortions to the intensity pattern of 21-cm radiation from the dark ages can be decomposed geometrically into curl and curl-free components. Lensing by primordial gravitational waves induces a curl component, while the contribution from lensing by density fluctuations is strongly suppressed. Angular fluctuations in the 21-cm background extend to very small angular scales, and measurements at different frequencies probe different shells in redshift space. There is thus a huge trove of information with which to reconstruct the curl component of the lensing field, allowing tensor-to-scalar ratios conceivably as small as $r \sim 10^{-9}$—far smaller than those currently accessible—to be probed.

One of the principal aims of early-Universe cosmology is detection of the inflationary gravitational-wave (IGW) background [1] via measurement of the curl pattern [2] that it induces in the cosmic microwave background (CMB) polarization. Likewise, a principal aim of physical cosmology is the measurement of the distribution of atomic hydrogen during the "dark ages," the epoch after recombination and before the formation of the first stars and galaxies, via detection of hydrogen's 21-cm line [3–5]. Several experiments are poised to soon detect the 21-cm signal from the epoch of reionization [6], and there are longer-term prospects to delve into the dark ages [7]. In this Letter, we show that angular fluctuations of the 21-cm intensity may ultimately provide an IGW probe that extends to amplitudes smaller than those currently accessible with the CMB.

The weak-gravitational-lensing of galaxies by large-scale density perturbations [8] was detected in 2000 [9] and is now a chief aim of a number of ongoing and future galaxy surveys. These efforts seek the lensing-induced distortions of galaxy shapes. Weak lensing of the CMB by density perturbations was detected recently [10]. The observational signatures here are lensing-induced position-dependent departures from statistical isotropy in the two-point CMB correlation functions, or equivalently, the four-point correlation functions induced by lensing [11].

Primordial gravitational waves can likewise lens both galaxies and the CMB [12–14]. The most general lensing pattern can, like the CMB polarization, be decomposed into curl and curl-free parts [15]. Since density perturbations produce (to linear order in the deflection angle) no curl in the lensing pattern, the curl component provides an IGW probe. The problem, however, is that the curl signal, even with the most optimistic assumptions about IGWs, is well below the noise for both current galaxy surveys and even for optimistic next-generation CMB experiments.

Here we consider lensing of intensity fluctuations in the 21-cm signal from atomic hydrogen in the dark ages. Atomic hydrogen in the redshift range $30 \lesssim z \lesssim 200$ can absorb radiation deep in the Rayleigh-Jeans region of the CMB [3]. The measurement of this absorption, over some narrow frequency range (corresponding to a narrow redshift range), over the sky, thus maps the spatial distribution of hydrogen at that redshift. The angular power spectrum of these 21-cm fluctuations extends to multipole moments $l \sim 10^7$ (limited only by the baryonic Jeans mass) [3], far larger than those, $l \sim 3000$, to which the CMB power spectrum extends (beyond which fluctuations are suppressed by Silk damping). The signatures of gravitational lensing of these 21-cm angular correlations are precisely the same as those of the lensing of the CMB temperature map—local departures from statistical isotropy. We can therefore adopt, unchanged, the mathematical formalism for lensing of the CMB.

Our work resembles, in spirit, that in Ref. [16], which argued that the huge number of Fourier modes available in 21-cm maps of the dark-age hydrogen distribution would provide considerable statistical significance in detecting the IGW distortion to matter fluctuations. However, they consider the *intrinsic* distortion to matter fluctuations by IGWs. On the other hand, we consider the distortion to the *images* of the matter distribution by lensing by IGWs. Our work is related to that of Ref. [17], which considered the reconstruction of the lensing field due to density perturbations with 21-cm fluctuations.

The most general deflection field $\vec{\Delta}$ can be written as a function of position $\hat{n}$ on the sky as [15]

$$\vec{\Delta} = \vec{\nabla}_{\tilde{\theta}}\phi(\hat{n}) + \vec{\nabla}_{\tilde{\theta}} \times \Omega(\hat{n}), \qquad (1)$$

in terms of curl-free ($\vec{\nabla}_{\tilde{\theta}}\phi$) and curl ($\vec{\nabla}_{\tilde{\theta}} \times \Omega$) components. The angular power spectrum for the curl field $\Omega(\hat{n})$ due to the lensing of sources at redshift z by IGWs with power spectrum $P_T(k)$ is

$$C_L^{\Omega} = 2 \int \frac{d^3k}{(2\pi)^3} P_T(k)[F_L^X(k)]^2, \qquad (2)$$

where

$$F_L^\Omega(k) = -\sqrt{\frac{2\pi(L+2)!}{(L-2)!}} \int_{k\eta(z)}^{k\eta_0} \frac{T(w)}{L(L+1)}$$

$$\times \frac{j_L(k\eta_0 - w)}{(k\eta_0 - w)^2} dw, \qquad (3)$$

and η_0 and $\eta(z)$ are the conformal time today and at redshift z, respectively. Here $T(w) \simeq 3j_1(w)/w$ is the gravitational-wave transfer function, and $j_n(x)$ are the spherical Bessel functions. The angular power spectra for the lensing of sources at several redshifts are shown in Fig. 1; for $L \lesssim 6$, the source-redshift dependence is weak for a scale-invariant gravitational-wave background.

We now review how this power spectrum is measured following the treatment of lensing of the CMB in Ref. [14], focusing on a single redshift slice first. Given a map $I(\hat{n})$ of the 21-cm intensity as a function of position $\hat{n}$ on the sky, the minimum-variance estimator for the spherical-harmonic coefficients for the curl component of lensing is

$$\widehat{\Omega}_{LM} = \frac{\sum_{ll'} Q_{ll'}^{\ominus L*} \widehat{A_{ll'}^{\ominus LM}} / (C_l^{\mathrm{map}} C_{l'}^{\mathrm{map}})}{\sum_{ll'} |Q_{ll'}^{\ominus L}|^2 / (C_l^{\mathrm{map}} C_{l'}^{\mathrm{map}})}, \qquad (4)$$

where $C_l^{\mathrm{map}} = C_l + C_l^n$ is the angular power spectrum of the map with C_l the power spectrum of the 21-cm intensity
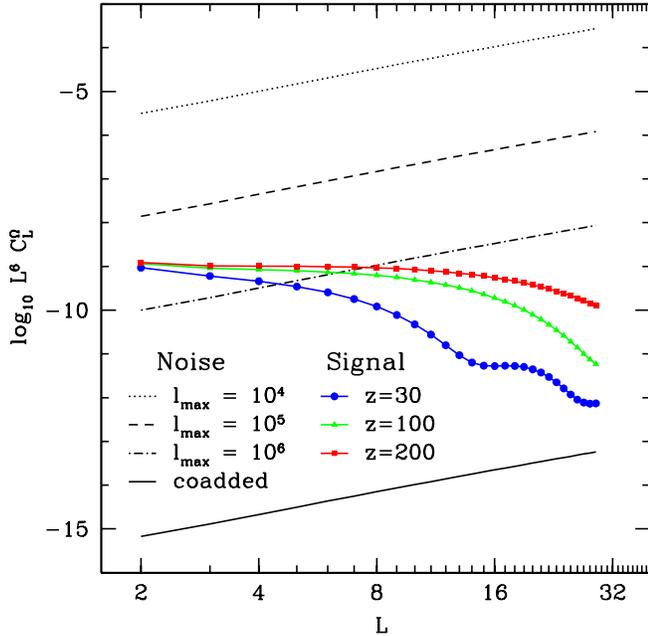

FIG. 1 (color online). The power spectrum for the deflection-field curl component for the lensing of sources at various redshifts by a scale-invariant spectrum of IGWs of the largest amplitude ($r = 0.2$) consistent with current measurements. We also superimpose noise power spectra for lensing reconstruction carried out to various values of $l_{\max}$. Also shown is the noise power spectrum we estimate from coadding the signals from all possible redshifts, assuming $l_{\max} = 10^6$.

and C_l^n the noise power spectrum, and the sums are only over $l + l' + L = $ odd. We use lower-case l for CMB fluctuations and upper-case L for the lensing deflection field. Here,

$$Q_{ll'}^{\ominus L} = \frac{i}{\sqrt{2L+1}} \left[\frac{C_l G_{l'l}^L}{\sqrt{l'(l'+1)}} - \frac{C_{l'} G_{ll'}^L}{\sqrt{l(l+1)}} \right],$$

$$G_{ll'}^L \equiv \sqrt{\frac{L(L+1)l(l+1)l'(l'+1)(2l+1)(2l'+1)}{4\pi}} C_{l0l'1}^{L1},$$

$$\widehat{A_{ll'}^{\ominus LM}} = \sum_{mm'} a_{lm}^{\mathrm{map}} a_{l'm'}^{*\mathrm{map}} (-1)^{m'} \mathcal{C}_{lml',-m'}^{LM}, \qquad (5)$$

where $\widehat{A_{ll'}^{\ominus LM}}$ are estimators for odd-parity bipolar-spherical-harmonic coefficients [18] in terms of the spherical-harmonic coefficients a_{lm}^{map} of the 21-cm map and Clebsch-Gordan coefficients $\mathcal{C}_{lml',-m'}^{LM}$. The estimator for the power spectrum of the curl component of the deflection field is then $\widehat{C_L^\Omega} = \sum_m |\widehat{\Omega}_{LM}|^2/(2L+1)$. The variance of $\widehat{\Omega}_{LM}$ under the null hypothesis is given by

$$(\sigma_L^\Omega)^2 \equiv \langle |\widehat{\Omega}_{LM}|^2 \rangle = 2 \left[\sum_{ll'} |Q_{ll'}^{\ominus L}|^2 / (C_l^{\mathrm{map}} C_{l'}^{\mathrm{map}}) \right]^{-1}. \qquad (6)$$

This noise power spectrum is plotted in Fig. 1 using the 21-cm power spectra from Ref. [3] and taking the noise power spectrum $C_l^n = 0$ for $l < l_{\max}$ and $C_l^n = \infty$ for $l > l_{\max}$. We show results for several $l_{\max}$ which are, roughly speaking, the maximum value of l with which the 21-cm power spectrum can be measured with high signal to noise. The signal to noise (squared) with which IGWs can be detected is then

$$(S/N)^2 = \sum_L (L + 1/2)(C_L^\Omega)^2 / (\sigma_L^\Omega)^4. \qquad (7)$$

Before reviewing the numerical results, it is instructive to consider an analytic estimate of the noise power spectrum $(\sigma_L^\Omega)^2$. To do so, we use the flat-sky approximation [13],

$$(\sigma_L^\Omega)^{-2} = \int \frac{d^2l}{(2\pi)^2} \frac{(\vec{L} \times \vec{l})^2 (C_l - C_{|\vec{L}-\vec{l}|})^2}{2C_l^{\mathrm{map}} C_{|\vec{L}-\vec{l}|}^{\mathrm{map}}}. \qquad (8)$$

For $L \ll l$, we approximate $|\vec{L} - \vec{l}| \simeq l - L\cos\alpha$, where $\cos\alpha \equiv \hat{L} \cdot \hat{l}$, and $C_{|\vec{L}-\vec{l}|} \simeq C_l - L(\cos\alpha)(\partial C_l/\partial l)$. If $C_l \propto l^n$, then

$$(\sigma_L^\Omega)^{-2} = \int \frac{l\, dl}{4\pi^2} \int_0^{2\pi} d\alpha \frac{1}{2} L^4 \sin^2\alpha \cos^2\alpha \left(\frac{\partial \ln C_l}{\partial \ln l} \right)^2$$

$$\simeq L^4 n^2 l_{\max}^2 / (64\pi). \qquad (9)$$

The flat-sky calculation is accurate for $L \gtrsim 20$ and *over-estimates* the noise by up to 30% at smaller L. As shown in Fig. 2 in Ref. [3], the 21-cm power spectrum extends without suppression out to $l \gtrsim 10^6$, and values of $l_{\max} \sim 10^7$ are perhaps achievable with a bit more effort. However,

given the rapid suppression of the 21-cm power spectrum at higher l, the return on the investment of noise reduction in terms of higher l_{max} will probably be small above $l_{max} \simeq 10^7$.

We now approximate the Ω power spectrum (for $r = 0.2$) as $C_L^\Omega \simeq 10^{-11}(L/2)^{-6}$. Although this approximation differs from the numerical results for different redshifts z at $L \simeq 30$, it is quite accurate for all $30 \lesssim z \lesssim 200$ for the smallest L where most of the signal arises. From Eq. (7), the signal-to-noise with which the gravitational-wave background can be detected is

$$(S/N) \simeq 4.5(l_{max}/10^6)^2(n/2)^2(L_{min}/2)^{-1}, \quad (10)$$

where L_{min} is the minimum L that can be measured.

There are several things to note about this result: (1) The signal to noise obtained with the adopted fiducial values for l_{max}, L, and n is significant. (2) The scaling of the signal-to-noise with l_{max} is very rapid, and greater than what might have been expected ($\propto l_{max}$) naively. The origin of this rapid scaling is similar to that for detection of the local-model trispectrum [19] (as the signal we are measuring here is, strictly speaking, an intensity trispectrum). Thus, the sensitivity to a gravitational-wave background increases very rapidly as the angular resolution of the map is improved. (3) The sensitivity decreases as L_{min} is increased, so good sky coverage is important for gravitational-wave detection.

While a signal-to-noise of 4.5 is respectable, and could be improved with even larger l_{max}, we can go much further: By changing the frequency at which the 21-cm map is made, we look at spherical shells of atomic hydrogen at different redshifts. Suppose, then, that we have 21-cm maps at two different frequencies that correspond to spherical shells separated along the line of sight by a comoving distance δR. Those two maps are statistically independent at the highest l (where the vast majority of the signal-to-noise for IGW detection arises) if $(\delta R/R) \gtrsim l^{-1}$. If ΔR is the separation in comoving radius corresponding to the entire frequency range covered by the observations (say, redshifts $z \simeq 30 - 200$), then the total number of statistically independent maps that can be obtained is $N_z \simeq (\Delta R/\delta R) \simeq l(\Delta R/R) \simeq 0.15l$. If so, then each map contributes an independent upper limit to the GW background amplitude, and the signal-to-noise from these redshift ranges can be added in quadrature, increasing the total signal-to-noise by a factor $N_z^{1/2}$. But there may be room for even more improvement: If most of the lensing occurs at redshifts $z \lesssim 30$ (as is the case for the lowest L), then the lensing pattern is the same for all redshift shells, in which case every redshift shell contributes coherently to an estimator for Ω_{LM}. In this case, $(\sigma_L^\Omega)^2$ is decreased by a factor of N_z^{-1}, and the signal to noise is increased by a factor N_z relative to the single-z estimate. Since most of the signal comes from the lowest L, we estimate that the signal-to-noise for IGW detection obtained by coadding redshift shells will be

$$(S/N)_{tot} \simeq 6.8 \times 10^5 (l_{max}/10^6)^3(n/2)^2(L_{min}/2)^{-1}, \quad (11)$$

assuming (as above) the largest currently allowed IGW amplitude $r \simeq 0.2$. Put another way, the smallest tensor-to-scalar ratio that can be detected at the 3σ level is

$$r \simeq 10^{-6}(L_{min}/2)(l_{max}/10^6)^{-3}(n/2)^{-2}. \quad (12)$$

Note that the dependence on l_{max} is very steep, and including all the information to $l_{max} = 10^7$ could yield a detection threshold of $r \simeq 10^{-9}$. Note also that a more sophisticated analysis, including the full structure of cross correlations between redshift maps, may be able to improve upon the $N_z^{1/2}$ scaling even in the case where the lensing signal is incoherent at different redshifts. The full-sky calculation, including a more realistic shape of C_l, yields a result consistent with this estimate (Fig. 1).

To put this result in perspective, we note that the current upper bound $r \lesssim 0.22$ comes from WMAP measurements of temperature-polarization correlations, although not from B-mode null searches. The forthcoming generation of suborbital B-mode experiments are targeting $r \lesssim 0.1$, and a dedicated CMB-polarization satellite might then get to $r \sim 10^{-2}$ [20].

Measurement of gravitational-wave amplitudes $r \lesssim 0.01$ with CMB polarization will have to contend with the additional contribution to B-mode polarization from gravitational lensing (by density perturbations) of primordial E modes [21]. The two contributions (IGW and lensing) to B modes can be distinguished if the lensing deflection angle can be reconstructed with small-scale CMB fluctuations [22,23]. This may allow values $r \sim 10^{-3}$ to be probed, although it requires a far more sophisticated CMB experiment (with far better angular resolution) than simple detection of B modes would require.

Further progress in the separation of lensing and IGW contributions to B modes can be obtained with 21-cm measurements [17] of precisely the type we discuss here but of the curl-free lensing component (due to density perturbations) rather than the curl component from IGWs. Such measurements, when combined with a precise CMB polarization experiment, can, in principle, get to IGW amplitudes comparable to those we have discussed here. Measurement of the 21-cm curl component may therefore ultimately be competitive for the most sensitive probe of IGWs, even if a sensitive CMB-polarization experiment is done. Furthermore, if both 21-cm observations and a CMB-polarization map are available, then the measurement of the 21-cm curl component can be used as a cross-check and to complement a measurement from the combination of B-mode polarization with 21-cm lensing subtraction.

While we have focused here on the dark ages, similar measurements can be performed with 21-cm fluctuations from the epoch of reionization and with galaxy surveys; the critical issue will be how high l_{max} can get. While the 21-cm curl component induced by lensing by density

perturbations at second order is too small to be an issue [13], a curl component may conceivably arise since the atomic-hydrogen distribution is not perfectly Gaussian due to nonlinear gravitational collapse and baryonic effects. We speculate that this curl component will be small for the small-L modes at which the IGW signal peaks. We also imagine that the information from multiple redshifts may be combined to separate the IGW and any bias-induced signal.

To close, we note that the measurements we describe will be challenging and are very futuristic compared to what current and next-generation experiments will accomplish. Following Ref. [24], an $l_{max} \sim 10^6$ at frequency 21 cm $(1 + z)$ for $z = 200$ requires a baseline of 10 000 km, most likely from above the ionosphere. Still, 21-cm cosmology is an exciting and rapidly developing experimental arena, for a good number of scientific reasons [4], and we hope that the idea presented here provides one additional motivation to carry such work forward.

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