

Nonlocal Memory Effects in the Dynamics of Open Quantum Systems

Elsi-Mari Laine,¹ Heinz-Peter Breuer,^{2,*} Jyrki Piilo,^{1,†} Chuan-Feng Li,^{3,‡} and Guang-Can Guo³

¹*Turku Centre for Quantum Physics, Department of Physics and Astronomy, University of Turku, FI-20014 Turun yliopisto, Finland*

²*Physikalisches Institut, Universität Freiburg, Hermann-Herder-Strasse 3, D-79104 Freiburg, Germany*

³*Key Laboratory of Quantum Information, University of Science and Technology of China, CAS, Hefei 230026, China*

(Received 18 November 2011; revised manuscript received 6 March 2012; published 22 May 2012)

We explore the possibility to generate nonlocal dynamical maps of an open quantum system through local system-environment interactions. Employing a generic decoherence process induced by a local interaction Hamiltonian, we show that initial correlations in a composite environment can lead to nonlocal open system dynamics which exhibit strong memory effects, although the local dynamics is Markovian. In a model of two entangled photons interacting with two dephasing environments, we find a direct connection between the degree of memory effects and the amount of correlation in the initial environmental state. The results demonstrate that, contrary to conventional wisdom, enlarging an open system can change the dynamics from Markovian to non-Markovian.

DOI: [10.1103/PhysRevLett.108.210402](https://doi.org/10.1103/PhysRevLett.108.210402)

PACS numbers: 03.65.Yz, 03.67.-a, 42.50.-p

Coupling a quantum mechanical system to an external environment causes the system to lose information to its surroundings. Since nearly all realistic quantum systems are open, understanding and controlling the dynamics arising from the presence of the environment is of central importance in present-day research [1,2]. The standard approach to the dynamics of open quantum systems employs the concept of a quantum Markov process that is given by a semigroup of completely positive dynamical maps and a corresponding quantum master equation with a generator in Lindblad form [3,4]. However, many quantum systems exhibit non-Markovian behavior in which there is a flow of information from the environment back to the open system, signifying the presence of quantum memory effects [5–10].

For many processes occurring in nature, approximations allowing a simple Markovian description are not applicable. It is known, for example, that strong system-environment couplings, structured and finite reservoirs, low temperatures, and the presence of large initial system-environment correlations can give rise to memory effects in the open system dynamics. The recognition of the importance of non-Markovian processes has initiated many essential steps toward the development of a general consistent theory of non-Markovian quantum dynamics [11–17] as well as achievements in the experimental detection and control of memory effects [18,19].

In this Letter, we introduce a hitherto unexplored source for quantum memory effects, namely, the presence of initial correlations between the subsystems of a composite environment that interact locally with the subsystems of a composite open system. It is demonstrated that correlations between the environmental subsystems can generate a nonlocal quantum process from a perfectly local interaction Hamiltonian. We will show further that a nonlocal decoherence process can lead to non-Markovian behavior, although the local dynamics of both subsystems is

Markovian. These features are discussed by employing two theoretical models, namely, a generic decoherence model of two qubits interacting with correlated multimode fields and an experimentally realizable model of entangled down-converted photons traveling through birefringent media. We thus find a new, experimentally controllable source for memory effects in a quantum dynamical process. Besides the practical importance of the result in the physical realization and control of dynamical processes, it also reveals an unexpected feature about the nature of non-Markovian dynamics of composite quantum systems: Enlarging the open system can actually turn the dynamics from a Markovian to a non-Markovian regime.

We consider an open system S consisting of two subsystems labeled by an index $i = 1, 2$ and an environment E that is also composed of two subsystems. We assume that there are only local system-environment interactions, i.e., that subsystem i of S interacts only with its environment i of E . The local interactions are described by unitaries $U_i(t)$, and S and E are supposed to be uncorrelated at the initial time. The open system state at time t is thus given by

$$\begin{aligned} \rho_S^{12}(t) &= \Phi_{12}(t)[\rho_S^{12}(0)] \\ &= \text{tr}_E\{[U_1(t) \otimes U_2(t)]\rho_S^{12}(0) \otimes \rho_E^{12}(0)[U_1^\dagger(t) \otimes U_2^\dagger(t)]\}, \end{aligned}$$

where $\Phi_{12}(t)$ represents the quantum dynamical map describing the time evolution of S . If the two environments are initially uncorrelated, $\rho_E^{12}(0) = \rho_E^1(0) \otimes \rho_E^2(0)$, this map factorizes, and the dynamics of S is given by a product of local maps, $\Phi_{12}(t) = \Phi_1(t) \otimes \Phi_2(t)$. However, when $\rho_E^{12}(0)$ exhibits correlations $\Phi_{12}(t)$ does not in general factorize and the environmental correlations may give rise to a nonlocal process even when the interaction Hamiltonian is purely local. For a local map the dynamical properties of the subsystems completely determine the global system dynamics, but when the map is nonlocal the global system

can exhibit features which are not present in the dynamics of the individual subsystems. Here we explore especially quantum memory effects arising from a nonlocal dynamics.

We consider a dephasing map for two qubits of the general form

$$\rho_S^{12}(t) = \begin{pmatrix} |a|^2 & ab^* \kappa_2(t) & ac^* \kappa_1(t) & ad^* \kappa_{12}(t) \\ ba^* \kappa_2^*(t) & |b|^2 & bc^* \Lambda_{12}(t) & bd^* \kappa_1(t) \\ ca^* \kappa_1^*(t) & cb^* \Lambda_{12}^*(t) & |c|^2 & cd^* \kappa_2(t) \\ da^* \kappa_{12}^*(t) & db^* \kappa_1^*(t) & dc^* \kappa_2^*(t) & |d|^2 \end{pmatrix}, \quad (1)$$

where the initial state of the two qubit system is a pure state given by

$$|\psi_{12}\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle. \quad (2)$$

The corresponding dynamics for subsystems 1 and 2 are given by $\rho_S^1(t) = \text{tr}_2[\rho_S^{12}(t)]$ and $\rho_S^2(t) = \text{tr}_1[\rho_S^{12}(t)]$. The states $\rho_S^1(t)$ and $\rho_S^2(t)$ are fully determined by the functions $\kappa_1(t)$ and $\kappa_2(t)$, depending neither on $\kappa_{12}(t)$ nor on $\Lambda_{12}(t)$. The interaction Hamiltonian is assumed to be local; i.e., we have

$$H_{\text{int}}(t) = \chi_1(t)H_1 + \chi_2(t)H_2, \quad (3)$$

where the function $\chi_i(t)$ is 1 for $t_i^s \leq t \leq t_i^f$ and zero otherwise. Here, t_i^s and t_i^f denote the times the interaction is switched on and switched off in system i , respectively. Since the local Hamiltonians H_i commute, the time evolution of the total system is given by $|\Psi(t)\rangle = \exp[-i \int_0^t dt' H_{\text{int}}(t')] |\Psi(0)\rangle$. We will further denote the local interaction times as $t_i(t) = \int_0^t \chi_i(t') dt'$, and for convenience we will not explicitly write the time dependence of t_i .

Before turning to the details of the physical systems under study, let us briefly discuss the concept of memory effects. Memory effects are quantified in Ref. [13] by employing the trace distance $D(\rho_A, \rho_B) = \frac{1}{2} \text{tr}|\rho_A - \rho_B|$ between two quantum states ρ_A and ρ_B . This quantity can be interpreted as a measure for the distinguishability of the two states [20–22]. In view of this interpretation, the characteristic feature of a non-Markovian quantum process is the increase of the distinguishability, i.e., a reversed flow of information from the environment back to the open system. Through this recycling of information the earlier states of the open system influence its later states [8], which expresses the emergence of memory effects in the open system dynamics. The measure for non-Markovianity is written as

$$\mathcal{N}(\Phi) = \max_{\rho_{A,B}(0)} \int_{\sigma>0} dt \sigma(t, \rho_{A,B}(0)), \quad (4)$$

where $\sigma(t, \rho_{A,B}(0)) = \frac{d}{dt} D(\rho_A(t), \rho_B(t))$. Here, the time integration is extended over all subintervals of time in which the rate of change of the trace distance σ is positive,

and the maximum is taken over all pairs of initial states. The quantity in Eq. (4) thus measures the maximal total amount of information which flows from the environment back to the open system over the whole time evolution.

First, we study a generic model of two qubits interacting with correlated multimode fields. The local interaction Hamiltonians of Eq. (3) are $H_i = \sum_k \sigma_z^i (g_k b_k^{i\dagger} + g_k^* b_k^i)$. We assume that the interaction strengths in both systems are identical, $g_k^1 = g_k^2$. The local time evolution of the systems is then given by the unitary

$$U_i(t) = \exp\left\{ \sigma_z^i \sum_k [b_k^{i\dagger} \xi_k(t_i) - b_k^i \xi_k^*(t_i)] \right\}, \quad (5)$$

where $\xi_k(t_i) = g_k(1 - e^{i\omega_k t_i})/\omega_k$. The local unitary of Eq. (5) acts in the following way:

$$U_i(t)|0\rangle \otimes |\eta\rangle = |0\rangle \otimes \bigotimes_k D(-\xi_k(t_i))|\eta\rangle,$$

$$U_i(t)|1\rangle \otimes |\eta\rangle = |1\rangle \otimes \bigotimes_k D(\xi_k(t_i))|\eta\rangle,$$

where $D(\xi_k)$ is the displacement operator for the k th mode. Let us take as the initial state $|\Psi(0)\rangle = |\psi_{12}\rangle \otimes |\eta_{12}\rangle$, where $|\psi_{12}\rangle$ is given by Eq. (2) and $|\eta_{12}\rangle = \bigotimes_k |\eta_{12}^k\rangle$. The decoherence process is then given by Eq. (1), where $\kappa_1(t) = \langle \eta_{12}^{10} | \eta_{12}^{00} \rangle$, $\kappa_2(t) = \langle \eta_{12}^{01} | \eta_{12}^{00} \rangle$, $\kappa_{12}(t) = \langle \eta_{12}^{11} | \eta_{12}^{00} \rangle$, and $\Lambda_{12}(t) = \langle \eta_{12}^{10} | \eta_{12}^{01} \rangle$ with

$$|\eta_{12}^{nm}(t)\rangle = \bigotimes_k [D((-1)^{n+1} \xi_k(t_1)) \otimes D((-1)^{m+1} \xi_k(t_2))] |\eta_{12}\rangle.$$

After some algebra, one finds

$$\kappa_1(t) = \prod_k \chi_k(-2\xi_k(t_1), 0),$$

$$\kappa_2(t) = \prod_k \chi_k(0, -2\xi_k(t_2)),$$

$$\kappa_{12}(t) = \prod_k \chi_k(-2\xi_k(t_1), -2\xi_k(t_2)),$$

$$\Lambda_{12}(t) = \prod_k \chi_k(-2\xi_k(t_1), +2\xi_k(t_2)),$$

where $\chi_k(x, y)$ is the characteristic function of $|\eta_{12}^k\rangle$.

Let us consider a two-mode Gaussian state with the characteristic function $\chi_k(x, y) = \chi_k(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \exp(-\frac{1}{2} \vec{\lambda}^T \boldsymbol{\sigma} \vec{\lambda})$, where $\lambda_1 = \Re[x]$, $\lambda_2 = \Im[x]$, $\lambda_3 = \Re[y]$, $\lambda_4 = \Im[y]$ and

$$\boldsymbol{\sigma} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$

is the covariance matrix of the state. Let us take $A = B = \mathbb{1}$ and $C = c\mathbb{1}$. Now the state is uncorrelated if and only if $c = 0$. We can write

$$\chi_k(x, y) = \exp[-\frac{1}{2}\{|x|^2 + |y|^2 + c(xy^* + x^*y)\}].$$

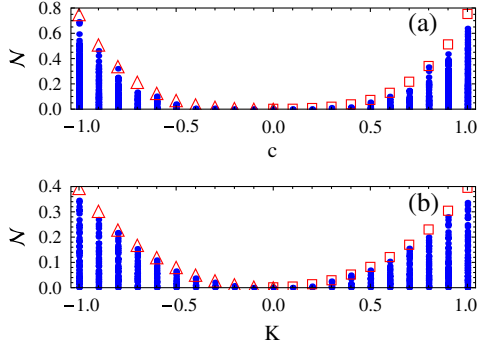


FIG. 1 (color online). Maximization over the pairs of initial states. The (blue) dots represent the increase of the trace distance for 1000 randomly drawn initial pairs of states. (a) Two qubits interacting with multimode fields. The measure for non-Markovianity for different values of c and for $\alpha = 1$. The (red) triangles represent the measure for the maximizing pair $(1/\sqrt{2})(|00\rangle \pm |11\rangle)$, and the (red) squares represent the measure for the maximizing pair $(1/\sqrt{2})(|01\rangle \pm |10\rangle)$. (b) Two photons moving through quartz plates. The measure for non-Markovianity for different values of the correlation coefficient K , and a fixed $C_{11}^{1/2}T\Delta n = 1$. The (red) triangles represent the measure for the maximizing pair $(1/\sqrt{2})(|HH\rangle \pm |VV\rangle)$, and the (red) squares represent the measure for the maximizing pair $(1/\sqrt{2})(|HV\rangle \pm |VH\rangle)$.

For $c = -1$, we get $\chi_k(x, y) = \exp[-\frac{1}{2}|x - y|^2]$ and $\kappa_{12}(t) = \exp[-2\sum_k |\xi_k(t_1) - \xi_k(t_2)|^2]$. Performing the continuum limit, we obtain

$$\kappa_{12}(t) = \exp\left\{-4 \int_0^\infty d\omega J(\omega) \frac{1 - \cos[\omega|t_1(t) - t_2(t)|]}{\omega^2}\right\},$$

where $J(\omega)$ is the spectral density of the reservoir. For an Ohmic spectral density $J(\omega) = \alpha\omega \exp(-\omega/\omega_c)$ with coupling constant α and frequency cutoff ω_c , we have

$$\begin{aligned} \kappa_1(t) &= [1 + \omega_c^2 t_1^2(t)]^{-2\alpha}, & \kappa_2(t) &= [1 + \omega_c^2 t_2^2(t)]^{-2\alpha}, \\ \kappa_{12}(t) &= [1 + \omega_c^2 |t_1(t) - t_2(t)|^2]^{-2\alpha}, \\ \Lambda_{12}(t) &= \kappa_1^2(t)\kappa_2^2(t)/\kappa_{12}(t). \end{aligned}$$

The maximization over the initial states in Eq. (4) for different values of c is presented in Fig. 1(a). The trace distance dynamics of the subsystems 1 and 2 as well as the global dynamics are presented in Fig. 2. We clearly see that the trace distance in the subsystems 1 and 2 continuously decreases, but for the total system the trace distance does indeed increase: We obtain a dynamics which is locally Markovian but globally exhibits nonlocal memory effects.

As our second example, we examine an experimentally realizable model of a pair of entangled photons subjected to local birefringent environments [23,24]. The photon pair is created in a spontaneous parametric down-conversion process after which the photons separate, traveling along different arms $i = 1, 2$, and move through different quartz plates. When a photon enters a quartz plate

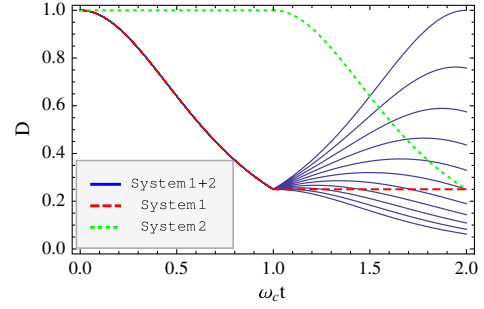


FIG. 2 (color online). Trace distance dynamics for the two qubits interacting with correlated multimode fields. We take $\alpha = 1$, $t_1^s = 0$, $t_1^f = 1 = t_2^s$, and $t_2^f = 2$ in units of ω_c^{-1} . The solid (blue) lines represent the trace distance with different values of c for the global dynamics of the two qubits for the maximizing pair of initial states. The dashed (red) line and the dotted (green) line give the trace distance evolution for the initial states $1/\sqrt{2}(|0\rangle \pm |1\rangle)$ in systems 1 and 2, respectively.

a local interaction between the polarization degrees of freedom (forming the open system) and the frequency degrees of freedom (forming the environment) is switched on. The Hamiltonian describing the local interaction in Eq. (3) of a photon induced by the corresponding quartz plate is given by

$$H_i = - \int d\omega_i \omega_i [n_V |V\rangle\langle V| + n_H |H\rangle\langle H|] \otimes |\omega_i\rangle\langle\omega_i|,$$

where $|\lambda\rangle \otimes |\omega_i\rangle$ denotes the state of a photon in arm i with polarization $\lambda = H, V$ (horizontal or vertical) and frequency ω_i . The refraction index of the polarization state λ is denoted by n_λ . The total initial state is given by $|\Psi(0)\rangle = |\psi_{12}\rangle \otimes \int d\omega_1 d\omega_2 g(\omega_1, \omega_2) |\omega_1, \omega_2\rangle$, where $|\psi_{12}\rangle = a|HH\rangle + b|HV\rangle + c|VH\rangle + d|VV\rangle$. Initially, the environment formed by the mode degrees of freedom is thus in a correlated state, with $g(\omega_1, \omega_2)$ denoting the amplitude of finding a photon with frequency ω_1 in arm 1 and a photon with frequency ω_2 in arm 2. The corresponding joint probability distribution will be denoted by $P(\omega_1, \omega_2) = |g(\omega_1, \omega_2)|^2$.

The state of the open system (polarization states) at time t is of the form of Eq. (1) with the functions $\kappa_1(t) = G(\Delta n t_1, 0)$, $\kappa_2(t) = G(0, \Delta n t_2)$, $\kappa_{12}(t) = G(\Delta n t_1, \Delta n t_2)$, and $\Lambda_{12}(t) = G(\Delta n t_1, -\Delta n t_2)$, where

$$G(\tau_1, \tau_2) = \int d\omega_1 d\omega_2 P(\omega_1, \omega_2) e^{-i(\omega_1 \tau_1 + \omega_2 \tau_2)}$$

is the Fourier transform of $P(\omega_1, \omega_2)$ and $\Delta n = n_V - n_H$ is the birefringence. Note that although the Hamiltonian of (3) is a sum of local interaction terms, the corresponding dynamical map $\Phi_{12}(t)$ is a product of local dynamical maps if and only if $\kappa_{12}(t) = \kappa_1(t)\kappa_2(t)$ and $\Lambda_{12}(t) = \kappa_1(t)\kappa_2^*(t)$. This is the case only when the joint frequency distribution $P(\omega_1, \omega_2)$ factorizes, i.e., when the frequencies ω_1 and ω_2 are uncorrelated.

To characterize the correlations in the initial environmental state we introduce the covariance matrix $C = (C_{ij})$ with elements $C_{ij} = \langle \omega_i \omega_j \rangle - \langle \omega_i \rangle \langle \omega_j \rangle$. We will assume that both the means and the variances of ω_1 and ω_2 are equal; i.e., $\langle \omega_1 \rangle = \langle \omega_2 \rangle = \omega_0/2$ and $C_{11} = C_{22} = \langle \omega_i^2 \rangle - \langle \omega_i \rangle^2$. To quantify the frequency correlations we use the correlation coefficient $K = C_{12}/\sqrt{C_{11}C_{22}} = C_{12}/C_{11}$. We have $|K| \leq 1$, where the equality sign holds; i.e., $K = \pm 1$, if and only if ω_1 and ω_2 are linearly related.

Let us take a Gaussian frequency distribution

$$P(\omega_1, \omega_2) = \frac{1}{2\pi\sqrt{\det C}} e^{-(1/2)(\vec{\omega} - \langle \vec{\omega} \rangle)^T C^{-1} (\vec{\omega} - \langle \vec{\omega} \rangle)}, \quad (6)$$

where $\vec{\omega} = (\omega_1, \omega_2)^T$ and $\langle \vec{\omega} \rangle = (\langle \omega_1 \rangle, \langle \omega_2 \rangle)^T$. One can easily find the Fourier transform of this distribution,

$$G(\tau_1, \tau_2) = e^{i\omega_0(\tau_1 + \tau_2)/2 - C_{11}(\tau_1^2 + \tau_2^2 + 2K\tau_1\tau_2)/2}. \quad (7)$$

We assume for simplicity that the total interaction times for both photons are equal, denoting it by $T = t_1^f - t_1^s = t_2^f - t_2^s$, and that the quartz plates are mounted one after another, i.e., $t_1^f = t_2^s$. We can then derive an analytic expression for the measure (4) of the non-Markovianity of the process. The maximization over the pair of initial states in Eq. (4) is illustrated in Fig. 1(b). Using Eqs. (1) and (7), we obtain the time dependence of the trace distance for the maximizing initial pairs,

$$D(t) = \exp\left[-\frac{\Delta n^2}{2} C_{11}(t_1^2 + t_2^2 - 2|K|t_1t_2)\right].$$

During the interaction of the photon in arm 1, the trace distance first decreases from the initial value 1 to the value $D_1 = \exp[-\frac{\Delta n^2}{2} C_{11}T^2]$. The subsequent interaction of the photon in arm 2 depends on the function $f(t_2) = t_2^2 - 2|K|Tt_2$ for $t_2 \in [0, T]$. This function decreases monotonically in the interval $[0, |K|T]$ from the value $f(0) = 0$ to the value $f(|K|T) = -(KT)^2$, which means that the trace distance increases over this interval to the value $D_2 = \exp\{-\frac{\Delta n^2}{2} C_{11}[T^2 - (KT)^2]\}$. It follows that the non-Markovianity measure is given by

$$\mathcal{N} = D_2 - D_1 = e^{-(1/2)C_{11}(\Delta n T)^2} [e^{(1/2)C_{11}(\Delta n T)^2 K^2} - 1]. \quad (8)$$

This equation establishes a direct connection between the measure for non-Markovianity and the degree of correlations in the initial environmental state as quantified by the correlation coefficient K . We also see that the process is Markovian if and only if $K = 0$. The relation Eq. (8) is further illustrated in Fig. 3, where we have plotted the frequency distribution $P(\omega_1, \omega_2)$ and the dynamics of the trace distance for three different values of the correlation coefficient, $K = 0.0, -0.5, -1.0$ (anticorrelation). One clearly observes that when the frequencies ω_1 and ω_2 become more anticorrelated the dynamics becomes more non-Markovian. In general, we conclude that the reduced dynamics of the two-photon polarization state is non-Markovian whenever the frequency distribution $P(\omega_1, \omega_2)$

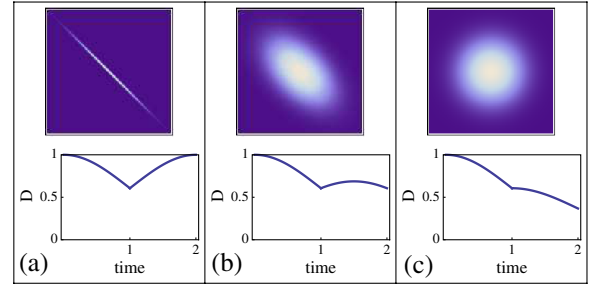


FIG. 3 (color online). Connection between the frequency distribution $P(\omega_1, \omega_2)$ (top) and the dynamics of the trace distance (bottom) for different values of K . (a) $K = -1$, (b) $K = -0.5$, (c) $K = 0$. The unit of time is $\sqrt{C_{11}}T\Delta n$.

exhibits correlation. This behavior occurs globally, i.e., when we study the dynamics of the composite state of both photons. However, if one observes the local dynamics of either of the photons, the process is always Markovian.

Summarizing, we have found a new source for memory effects in the dynamics of open quantum systems. We studied a generic dephasing model as well as a theoretical scheme that is experimentally realizable with current technology. The initial correlations within the environment induce a nonlocal map which gives rise to memory effects in the open system dynamics. Locally, each subsystem follows Markovian dynamics, but globally they are exposed to memory effects even though the interaction Hamiltonian is local. Since for classical stochastic processes a non-Markovian process can be embedded in a Markovian one by a suitable enlargement of the number of relevant variables, the general view has been that enlarging the open quantum system under study tunes the dynamics towards a Markovian behavior. This can be done for certain non-Markovian quantum processes as well [25–28], but we see that also the exactly opposite behavior can occur; i.e., enlarging the subsystem can bring the dynamics from a Markovian to a non-Markovian regime.

This work was supported by the National Fundamental Research Program, National Natural Science Foundation of China (Grants No. 60921091, No. 10874162, and No. 10734060), the Academy of Finland (Project 259827), the Jenny and Antti Wihuri Foundation, the Graduate School of Modern Optics and Photonics, and the German Academic Exchange Service (DAAD). E.-M.L., H.-P.B., and J.P. thank the USTC group for the hospitality during a research visit at the Key Laboratory of Quantum Information.

*breuer@physik.uni-freiburg.de

†jyrki.piilo@utu.fi

‡cfli@ustc.edu.cn

- [1] J. T. Barreiro, M. Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller, and R. Blatt, *Nature (London)* **470**, 486 (2011).

- [2] C. J. Myatt, B. E. King, Q. A. Turchette, C. A. Sackett, D. Kielpinski, W. M. Itano, C. Monroe, and D. J. Wineland, *Nature (London)* **403**, 269 (2000).
- [3] G. Lindblad, *Commun. Math. Phys.* **48**, 119 (1976).
- [4] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, *J. Math. Phys. (N.Y.)* **17**, 821 (1976).
- [5] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University, Oxford, 2007).
- [6] H. Lee, Y.-C. Cheng, and G. R. Fleming, *Science* **316**, 1462 (2007).
- [7] L. S. Cederbaum, E. Gindensperger, and I. Burghardt, *Phys. Rev. Lett.* **94**, 113003 (2005).
- [8] J. Piilo, S. Maniscalco, K. Härkönen, and K.-A. Suominen, *Phys. Rev. Lett.* **100**, 180402 (2008); J. Piilo, K. Härkönen, S. Maniscalco, and K.-A. Suominen, *Phys. Rev. A* **79**, 062112 (2009).
- [9] P. Rebentrost and A. Aspuru-Guzik, *J. Chem. Phys.* **134**, 101103 (2011).
- [10] T. J. G. Apollaro, C. Di Franco, F. Plastina, and M. Paternostro, *Phys. Rev. A* **83**, 032103 (2011).
- [11] M. M. Wolf, J. Eisert, T. S. Cubitt, and J. I. Cirac, *Phys. Rev. Lett.* **101**, 150402 (2008).
- [12] A. Shabani and D. A. Lidar, *Phys. Rev. Lett.* **102**, 100402 (2009).
- [13] H.-P. Breuer, E.-M. Laine, and J. Piilo, *Phys. Rev. Lett.* **103**, 210401 (2009).
- [14] E.-M. Laine, J. Piilo, and H.-P. Breuer, *Phys. Rev. A* **81**, 062115 (2010).
- [15] A. Rivas, S. F. Huelga, and M. B. Plenio, *Phys. Rev. Lett.* **105**, 050403 (2010).
- [16] D. Chruściński and A. Kossakowski, *Phys. Rev. Lett.* **104**, 070406 (2010).
- [17] B. Vacchini, A. Smirne, E.-M. Laine, J. Piilo, and H.-P. Breuer, *New J. Phys.* **13**, 093004 (2011).
- [18] J.-S. Tang, C.-F. Li, Y.-L. Li, X.-B. Zou, G.-C. Guo, H.-P. Breuer, E.-M. Laine, and J. Piilo, *Europhys. Lett.* **97**, 10002 (2012).
- [19] B.-H. Liu, L. Li, Y.-F. Huang, C.-F. Li, G.-C. Guo, E.-M. Laine, H.-P. Breuer, and J. Piilo, *Nature Phys.* **7**, 931 (2011).
- [20] C. W. Helstrom, *Quantum Detection and Estimation Theory* (Academic, New York, 1976).
- [21] A. S. Holevo, *Trans. Moscow Math. Soc.* **26**, 133 (1972).
- [22] M. Hayashi, *Quantum Information* (Springer-Verlag, Berlin, 2006).
- [23] P. G. Kwiat, A. J. Berglund, J. B. Altepeter, and A. G. White, *Science* **290**, 498 (2000).
- [24] J.-S. Xu, X.-Y. Xu, C.-F. Li, C.-J. Zhang, X.-B. Zou, and G.-C. Guo, *Nature Commun.* **1**, 7 (2010).
- [25] K. H. Hughes, C. D. Christ, and I. Burghardt, *J. Chem. Phys.* **131**, 024109 (2009).
- [26] J. Prior, A. W. Chin, S. F. Huelga, and M. B. Plenio, *Phys. Rev. Lett.* **105**, 050404 (2010).
- [27] P. Siegle, I. Goychuk, P. Talkner, and P. Hänggi, *Phys. Rev. E* **81**, 011136 (2010).
- [28] R. Martinazzo, B. Vacchini, K. H. Hughes, and I. Burghardt, *J. Chem. Phys.* **134**, 011101 (2011).