Electrostatic Response of a Two-Component Plasma with Coulomb Collisions

A. V. Brantov and V. Yu. Bychenkov
P. N. Lebedev Physics Institute, Russian Academy of Science, Leninskii Prospect 53, Moscow 119991, Russia

W. Rozmus
Theoretical Physics Institute, University of Alberta, Edmonton T6G 2E1, Alberta, Canada
(Received 14 September 2011; published 14 May 2012)

A rigorous procedure is proposed for finding a solution to kinetic equations with the Landau electron-electron, electron-ion, ion-electron, and ion-ion collision integrals in fully ionized plasma. The linear plasma response to the perturbation in the electrostatic field is described in terms of plasma dielectric permittivity. Solutions of the dispersion relation for electron plasma waves, ion-acoustic waves, and entropy modes are found in the entire range of frequencies, wave vectors, and particle collisionality. Several fits are obtained to enable practical applications of these results.

DOI: 10.1103/PhysRevLett.108.205001 PACS numbers: 52.25.Dg, 52.25.Mq, 52.35.Fp

Recent advances in the study of laboratory and astrophysical plasmas and the broad scope of this research, covering plasmas from gas discharges to inertial confinement fusion targets, have underscored the need for a basic theoretical framework that is applicable to the entire range of temporal and spatial scales and for all ion and electron collisional times. Nonetheless, even a basic plasma description within the context of linear response theory has yet to be produced, a general and practical expression for electron-ion susceptibility function that is valid for the entire range of frequencies, ω, wave numbers, k, and for all values of the collisionality parameter $k\lambda_{ab}$ ($a, b = e, i$), where λ_{ab} is the mean free path for collisions between species a and b. The main difficulty in obtaining such an expression lies in the solution of the coupled integrodifferential kinetic equations of the Fokker-Planck (FP) type [1] for the electron distribution function (DF) and ion DF. This Letter provides a derivation of the plasma dielectric permittivity for longitudinal perturbations and discusses solutions of the dispersion relation for electron plasma waves (EPW), ion-acoustic waves (IAW), and entropy waves in two-component weakly coupled plasmas.

One way to calculate the susceptibility of collisional plasmas for specific parameters is by finding numerical solutions to the coupled FP kinetic equations with the Landau collision integrals. For example, Tracy *et al.* [2] solved numerically an eigenvalue problem for the linearized FP equation. This work [2] is an improvement on the studies by Ono and Kulsrud [3] and Randall [4] who used the alternating-direction-implicit technique. However, in all of these numerical studies the linearized ion FP equation with the complete Landau collisional term was solved without coupling to electrons which were treated in the collisionless approximation. Also, the broad range and large number of plasma parameters such as, ω, k, Z–the ion charge number, A–atomic number, T_a–the particle temperatures ($a = e$, i), and ν_{ai}–the collision frequencies that are involved in the theoretical description of collisional plasmas make analytical and practical solutions far more desirable than numerical simulations.

Approximate methods for finding the susceptibility functions [5–8] have been developed before using simplified procedures, e.g., moment expansions [9,10], to solve the complete collision integrals. They are valid in limited domains of frequency, wavelength, and plasma parameters. In a different approach, the simplified kinetic models such as the Bhatnagar-Gross-Krook collision integral [11,12], the Lenard-Bernstein collision term [13,14], or the Lorentz model [15] have been used to derive the linear response functions of collisional plasma. The two approaches lead to approximate results with a limited range of applicability.

Consider linear, small amplitude, periodic, $\sim e^{-i\omega t + ikx}$, perturbations of the background plasma variables. The background state corresponds to homogeneous Maxwellian particle DFs, F_M^a ($a = e$, i), with densities n_a ($Zn_i = n_e$) and temperatures T_a. The Fourier (ω, k) transformed perturbations (we dropped the subscripts ω and k for simplicity) of the DF $f_a(v, \mu) = \Sigma_0^\infty f_l^a(v) P_l(\mu)$ are expanded in a series of Legendre polynomials $P_l(\mu)$, where μ is the cosine of the angle between $\mathbf{v}$ and $\mathbf{k}$. With these expansions the kinetic equations with Landau collisional terms are decomposed into two infinite hierarchies ($a = e, i$) of equations for the harmonics $f_l^a(v)$ of the electron and ion DFs coupled through Coulomb terms:

$$\hat{L} f_l^a - C_{aa}^l - C_{ab}^l = (e_a E/T_a) v f_M^a \delta_{l1},$$
$$\hat{L} f_l^a \equiv -i\omega f_l^a + ikv \frac{l}{2l-1} f_{l-1}^a + ikv \frac{l+1}{2l+3} f_{l+1}^a. \qquad (1)$$

The collision operators, C_{ab}^l, are defined by the Rosenbluth potentials. Because of the small mass ratio $m_e/m_i \ll 1$, collisions between electrons and ions can be described in a simplified form [6,16,17],

$$C_{ei}^l = -\frac{l(l+1)}{2}\nu_{ei} f_l^e + \delta_{l1}\nu_{ei}\frac{v}{v_{Te}^2}F_M^e u_i,$$
$$C_{ie}^l = \delta_{l1}\left(\frac{v}{v_{Ti}}F_M^i \frac{\mathcal{R}_{ie}}{n_i m_i v_{Ti}} + \nu_{ei}^T \frac{m_e n_e}{m_i n_i} C_{ie}^{(1)}\right), \quad (2)$$
$$C_{ie}^{(1)} = \frac{1}{v^2}\frac{\partial}{\partial v}v^3 f_l^i + \frac{T_e}{m_i v^2}\left(\frac{\partial}{\partial v}v^2\frac{\partial}{\partial v} - l^2 - l\right)f_l^i,$$

where $\delta_{gh} = 1$ or 0 if $g = h$ or $g \neq h$, correspondingly, $\nu_{ab}(v) = 4\pi n_b (e_a e_b)^2 \Lambda_{ab}/m_a^2 v^3$ are the velocity dependent particle collision frequencies, Λ_{ab} is the Coulomb logarithm, $u_a = 4\pi \int dv v^3 f_1^a/3n_a$ is the mean particle velocity, $v_{Ta} = \sqrt{T_a/m_a}$ is the particle thermal velocity, and $\mathcal{R}_{ie} = 4\pi m_e \int dv v^3 \nu_{ei}(v) f_1^e/3$ is the friction force. The term $C_{ie}^{(1)}$ in the expression for C_{ie}^l in Eq. (2) is small and can be neglected for the calculation of an ion DF. At the same time, $C_{ie}^{(1)}$ is important to ensure ion momentum conservation. For like particle collisions, C_{aa}^l, we use the general form of the collisional operator [6,16].

We seek a general solution for the harmonics of the DFs in the form $f_l^e = [(i\delta_{l0} - \omega\psi_l^{eN})eE/kT_e - iku_i\psi_l^{eR}]F_M^e$ and $f_l^i = (\omega\psi_l^{iN} - i\delta_{l0})(e_iE/kT_i + \mathcal{R}_{ie}/kn_iT_i)F_M^i$, where the basis functions ψ_l^{bA} satisfy Eqs. (1) with different sources on the right-hand side,

$$\hat{L}\psi_l^{bA} + \delta_{be}\frac{l(l+1)}{2}\nu_{ei}\psi_l^{bA} - \frac{1}{F_M^b}C_{bb}^l[F_M^b\psi_l^{bA}] = S_l^A, \quad (3)$$

where $S_l^A = \delta_{l0}\delta_{AN}S^N + 3\delta_{l1}\delta_{AR}S^R$, $S^N = 1$, $S^R = iv\nu_{ei}/3kv_{Te}^2$. It is convenient to rewrite the ion DF using u_i as $f_l^i = iku_iF_M^i(\psi_l^{iN} - i\delta_{l0}/\omega)/(1 + i\omega J_i)$, where $J_i = 4\pi\int v^2 dv \psi_0^{iN} S^N F_M^i/n_i$. We will use a simplified form of C_{aa}^l for $l \gg 1$ in Eq. (3) in order to close this infinite system of equations [18]. Because of this simplification, starting from the order $l = l_{\max}$, all the equations for the harmonics of the basis functions take on the following simple form ($l > l_{\max}$):

$$2\hat{L}\psi_l^{bA} = -l(l+1)\nu_b^*\psi_l^{bA}, \quad (4)$$

where $\nu_a^* = \nu_{ei}\delta_{ae} + \nu_{aa}(I_0^a + 2J_{-1}^a/3 - I_2^a/3)$ and $\{I, J\}_m^a = 4\pi/(n_a v^m)\int_{\{0,v\}}^{\{v,\infty\}} dv v^{m+2} F_M^a$. The infinite system of Eqs. (4) has been solved following the summation procedure [16,18,19] where one evaluates the renormalized effective collision frequencies ν_l^a from the following recurrence formula

$$\nu_l^a = -i\omega + \frac{1}{2}l(l+1)\nu_a^* + \frac{(l+1)^2}{4(l+1)^2 - 1}\frac{k^2v^2}{\nu_{l+1}^a}. \quad (5)$$

Equation (5) can also be represented in terms of continuous fractions. In practice, finding $\nu_{l_{\max}}^a$ with high accuracy requires no more than 20–30 iterations. After that, it is sufficient to solve a finite number of Eqs. (3) to find basis functions ψ_l^{bA} for $l \leq l_{\max}$ given that $\psi_{l_{\max}+1}^{bA} = i[(l_{\max}+1)/(2l_{\max}+3)](kv/\nu_{l_{\max}+1}^b)\psi_{l_{\max}}^{bA}$. We solve this system of equations, expanding the basis functions ψ_l^{bA} in Sonine-Laguerre polynomials: $\psi_{2l}^{bA} = \frac{\lambda_{bi}}{v_{Tb}}\sum_{n=0}^{\infty} c_{2l,n}^{bA} L_n^{1/2}(\frac{v^2}{2v_{Tb}^2})$ and $\psi_{2l+1}^{bA} = \frac{\lambda_{bi}v}{v_{Tb}^2}\sum_{n=0}^{\infty} c_{2l+1,n}^{bA} L_n^{1/2}(\frac{v^2}{2v_{Tb}^2})$, where $\lambda_{ei} = 3\sqrt{\pi/2}v_{Te}/\nu_{ei}(v_{Te}) = v_{Te}/\nu_{ei}^T$ and $\lambda_{aa} = 3\sqrt{\pi}v_{Ta}/\nu_{aa}(v_{Ta}) = v_{Ta}/\nu_{aa}^T$ are the e-i and i-i (e-e) mean free paths. Substitution of this expansion into Eq. (3) gives a system of linear algebraic equations for the coefficients c_{ln}^{bA}. This system was solved with the MATHEMATICA software package. The calculations were performed for $l_{\max} = 8$ resulting in an error related to the closure procedure that does not exceed 1%–2%. By finding $\psi_{0,1}^{eA}$ and ψ_0^{iA} from the above approximation and using the definition of electron DF, one obtains an electric current $j = -en_e(u_e - u_i)$ (cf. [16]) in the following form

$$j = -\frac{ie^2 n_e}{k^2 T_e}\omega(1 + i\omega J_N^N)E + en_eu_i(1 + i\omega J_N^R). \quad (6)$$

We have derived an ion mean velocity from the momentum conservation equation $-i\omega m_i n_i u_i + ik\hat{P}^i = Zen_iE + \mathcal{R}_{ie} - m_e n_e \nu_{ei}^T u_i$, where $\hat{P}^i = 4\pi m_i/3\int dv v^4(f_0^i + 2/5 f_2^i)$ and we use previously introduced definitions for the velocity momenta of the electron basis functions ψ_0^{eA} and ψ_1^{eA} [16]: $\{J_N^A, J_R^A\} = 4\pi\int v^2 dv\{\psi_0^{eA}S^N, \psi_1^{eA}S^R\}F_M^e/n_e$, correspondingly, $\mathcal{R}_{ie} = i\omega J_R^N en_eE - k^2u_in_eT_eJ_R^R$ and $J_R^N = J_N^R$ (Onsager's symmetry). From the definition of the ion DF and first two equations in (3) one finds $ik\hat{P}^i = i\omega m_in_iu_i + ik^2n_iT_iu_i/\omega(1 + i\omega J_i)$. Using these relations and after expressing the ion velocity, u_i, in terms of the electric field, E, the dielectric permittivity of collisional plasma $\epsilon = 1 + i4\pi j/\omega E$ reads

$$\epsilon = 1 + \frac{1 + i\omega J_N^N}{k^2\lambda_{De}^2} + \frac{1 + i\omega J_i}{k^2\lambda_{Di}^2}\frac{(1 + i\omega J_N^R)^2}{1 - ig\omega(1 + i\omega J_i)\tilde{J}_R^R}, \quad (7)$$

where λ_{Da} is the Debye length, $g = ZT_e/T_i$, and $\tilde{J}_R^R = J_R^R + \nu_{ei}^T/k^2v_{Te}^2$. In the collisionless kinetic limit one has $J_R^A = 0$ and $\{J_i, J_N^N\} = i\{J_+(\omega/kv_{Ti}), J_+(\omega/kv_{Te})\}/\omega$, so that Eq. (7) reduces to the well-known expression (cf., e.g., [12]): $\epsilon = 1 + \sum_a[1 - \omega J_+(\omega/kv_{Ta})]/k^2\lambda_{Da}^2$. In the strongly collisional limit, $k\lambda_{ai} \to \infty$, Eq. (7) corresponds to the collisional hydrodynamic plasma response derived from classical transport theory [6,17]. In the limit of cold ions, i.e., $1 + i\omega J_i = -k^2v_{Ti}^2/\omega^2$, Eq. (7) reads

$$\epsilon = 1 + \frac{1 + i\omega J_N^N}{k^2\lambda_{De}^2} - \frac{\omega_{pi}^2}{\omega^2}\frac{(1 + i\omega J_N^R)^2}{1 + i\tilde{J}_R^R k^2 c_s^2/\omega}, \quad (8)$$

where $c_s = \sqrt{ZT_e/m_i}$ is the ion-acoustic velocity. Equation (8) has the same form as the dielectric permittivity from Ref. [16]. However, we use here a definition of J_R^R that involves $f_1^a(v)$ and is valid for all Z while in Ref. [16] we employed the approximation $Z \gg 1$ to calculate J_R^R. The dielectric permittivity of collisional plasma, (7) or (8) is not a simple sum of separate electron and ion contributions, but contains an interaction term due to the friction force between electrons and ions. In particular, in the cold ion limit (8) this term corresponds to the last factor multiplying the standard ion contribution: $-\omega_{pi}^2/\omega^2$.

The dielectric permittivity ϵ (7) is the main result of this Letter. It describes the plasma response over the entire range of frequencies, wavelengths, and for arbitrary electron and ion collisionality. The dispersion relation (DR) $\epsilon = 0$ describes the high-frequency, $\omega \gtrsim \omega_{pe}$, EPW, the low-frequency, $\omega < \omega_{pi}$, IAW and the aperiodic perturbation, $\omega = -i\gamma$, entropy waves. In the limit $\omega \gg k v_{Te}, \nu_{ei}$, solution of the DR results in the well-known EPW mode with $\omega_{\rm EPW} = \omega_{pe}\sqrt{1+3\tilde{k}^2}$ where $\tilde{k} \equiv k\lambda_{De}$ and damping rate $\gamma_{\rm EPW}$

$$\gamma_{\rm EPW} = \gamma_L + (\frac{1}{2} - 2\tilde{k}^2)\nu_{ei}^T + \frac{4}{5}\tilde{k}^2\nu_{ee}^T. \tag{9}$$

The two first terms in Eq. (9) are the standard Landau, γ_L [12], and e-i collisional damping rates while the last term describes the e-e collisional contribution [20], which is important for pure electron plasmas (e.g., a Penning trap).

In the limit $\omega \ll (k v_{Te}, \nu_{ei}^T)$ one may neglect the frequency dependence of the functions J_A^B, i.e., in the quasistatic approximation for electrons [16]. In this limit $|\,\omega J_A^B\,| \ll 1$ and the DR reduces to

$$\begin{aligned} &1 + \frac{1+\tilde{k}^2}{g(1+i\omega J_i)} + i\omega\left[2J_R^N - \frac{J_N^N}{1+\tilde{k}^2} - \tilde{J}_R^R(1+\tilde{k}^2)\right] \\ &\equiv 1 + \frac{1+\tilde{k}^2}{g}D_i(\omega,k) + iD_e(\omega,k) = 0, \end{aligned} \tag{10}$$

where the electron term D_e is small and contributes only to the wave dissipation. For weakly damped IAW, the solution of Eq. (10) is $\omega = \omega_s - i\gamma_s$ where the ion-acoustic frequency is defined by the ion contribution D_i from the solution of the equation $D_i'(\omega_s, k) = -r \equiv -g/(1+\tilde{k}^2)$ and the small damping rate (that requires $g \gtrsim 2$), $\gamma_s = \gamma_i + \gamma_e$, is given by the relations $\gamma_i = D_i''(\partial D_i'/\partial\omega)^{-1}|_{\omega=\omega_s}$ and $\gamma_e = rD_e'(\partial D_i'/\partial\omega)^{-1}|_{\omega=\omega_s}$. In the simplest case of a strongly nonisothermal plasma, $g \gg 1$ and in the quasineutral approximation, $\tilde{k} \ll 1$, when $\omega_s = kc_s$ one has $\gamma_e = k^2c_s^2(J_N^N + \tilde{J}_R^R - 2J_R^N)/2$ [21]. In general our theory depends on three variables describing: collisionality, $k\lambda_{ii}$ or $k\lambda_{ei}$, the e-i temperature ratio, g, and charge separation effects, $\tilde{k}$. However, we have found that the function r, which combines g and $\tilde{k}$ is the correct variable to characterize the dependence of IAW properties on g and $\tilde{k}$. This is reflected in the discussion of numerical results below. In order to avoid numerical calculations for each set of plasma parameters, we propose fitting formulas below for both the IAW frequency and damping rate for the most interesting case of $r \geq 1$. For ω_s this reads ($k_{e,i} \equiv k\lambda_{ei,ii}$)

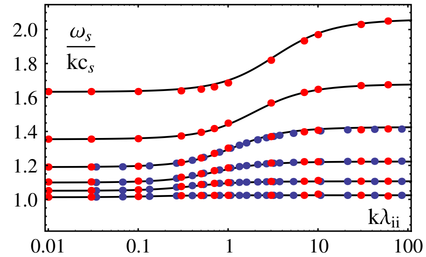

FIG. 1 (color online). Ion-acoustic frequency ω_s/kc_s as function of $k\lambda_{ii}$ [gray dots (red dots online)] for $r = 1, 2, 4, 8, 16, 64$ (from top to bottom, respectively) in comparison with results of Ref. [2] [black dots (blue dots online)]. The soild lines correspond to the fitting expression (11).

$$\begin{aligned} \omega_s &= k v_{\rm Ti}\sqrt{r + 5/3 + Q(r,k_i)(G(r) - 5/3)}, \\ Q(r,x) &= \frac{r^{3/2}x^2 + x\sqrt{r}}{r^{3/2}x^2 + 3x\sqrt{r} + 10}, \\ G &= \frac{3r^3 + 11r^2 + 12}{r^3 + 7r}. \end{aligned} \tag{11}$$

It describes the smooth transition from $\omega_s = kv_{Ti}\sqrt{r+5/3}$ in a plasma with strongly collisional ions, $k_i\sqrt{r} \ll 1$, to $\omega_s = kv_{\rm Ti}\sqrt{r+G(r)}$ in collisionless plasma, $k_i\sqrt{r} \gg 1$ with an accuracy of better than 3%. Figure 1 compares the IAW frequency with Eq. (11) and numerical results [2]. Similarly, we propose the following fit for the electron IAW damping rate for $g \gtrsim 2$ over the entire range of electron collisionality, k_e, and $\tilde{k}$ ($\Gamma_e = \gamma_e\omega_{pe}/kc_s\omega_{pi}$),

$$\begin{aligned} \Gamma_e\frac{2^{10}}{75\pi} &= \frac{\gamma_N + 1.7\gamma_N^{0.5}Z^{0.45}k_e^{0.95Z^{-0.04}} + 0.083Z^{0.6}k_e^{1+0.5\gamma_N}}{k_e(1+0.05Z^{0.6}k_e^{0.5\gamma_N})(1+\tilde{k}^2)^2/1.64} \\ &\quad + \frac{\gamma_R + 0.86Z^{0.35}k_e^{0.7}}{k_e + 0.4Z^{0.35}k_e^{1.7}} \\ &\quad - \frac{2.48(\gamma_{NR} + \gamma_{NR}^{0.25}Z^{0.45}k_e^{0.75})}{(k_e + 0.24Z^{0.42}k_e^{1.65})(1+\tilde{k}^2)} \\ \gamma_N &= \frac{Z+2.74}{Z+1}, \quad \gamma_{NR} = \frac{Z+2.72}{Z+1.4}, \quad \gamma_R = \frac{Z+2.42}{Z+1.38}, \end{aligned} \tag{12}$$

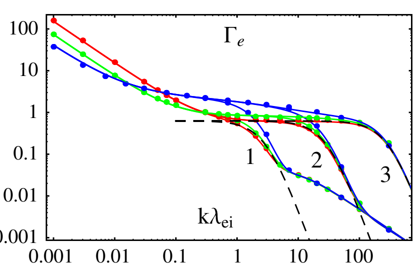

FIG. 2 (color online). Dependence of the electron damping rate Γ_e on $k\lambda_{ei}$ for $Z = 1$ [gray dots (red dots online)], 4 [light gray dots (green dots online)], and 64 [black dots (blue dots online)], and for $\lambda_{ei}/\lambda_{De} = 3$ (1), 30 (2), and 300 (3). The soild lines correspond to the fitting expression (12). The dashed lines correspond to the collisionless limit [12].

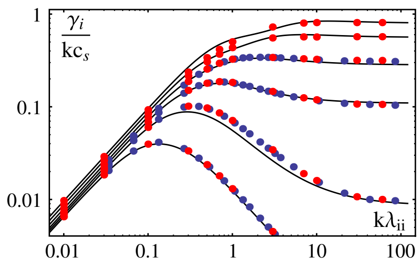

FIG. 3 (color online). Dependence of the ion damping rate γ_i/kc_s on $k\lambda_{ii}$ [gray dots (red dots online)] for $r = 1, 2, 4, 8, 16, 64$ (from top to bottom, respectively) in comparison with results of Ref. [2] [black dots (blue dots online)]. The soild lines correspond to the fitting expressions (13) and (14).

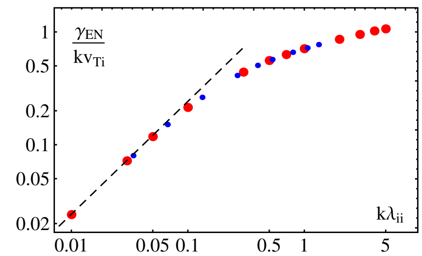

FIG. 4 (color online). Damping rate of the entropy mode as a function of $k\lambda_{ii}$ [gray dots (red dots online)] for $r = 8$ in comparison with Ref. [2] [black dots (blue dots online)]. Dashed line corresponds to the classical limit [17].

which reproduces the exact electron dissipative contribution with an accuracy of better than 10% for all Z in the range $1 < Z < 100$. This is illustrated by Fig. 2 for $\lambda_{ei}/\lambda_{De} = (3, 30, 300)$.

Ion contributions to IAW damping can be written in the form $\gamma_i = \gamma_i^H + R(k_i, r)\gamma_i^L$ where γ_i^H and γ_i^L are the ion collisional and collisionless (Landau) damping rates, respectively. For $k_i \ll F(r)$, where $F(r) = (4 + 0.3e^{r/2})/(7 + r^{2.5})$ our theory reproduces hydrodynamic ion damping [9] with excellent accuracy. Our fitting expression for γ_i^H is a generalization of the result from Ref. [9] for quasineutral IAW ($\tilde{k} \ll 1$) to arbitrary $\tilde{k}$ (c.f. Eq. (18) from Ref. [9])

$$\frac{\gamma_i^H}{kv_{Ti}} = k_i \frac{r + 3.02}{r + 1.67} \frac{0.80rk_i^2 + 1.49}{r^2k_i^4 + 4.05rk_i^2 + 2.33}. \qquad (13)$$

On the other hand, γ_i^L is the well-known damping rate due to the ion Cherenkov effect, which follows from a solution of the dispersion equation for collisionless plasma. It can be interpolated with an accuracy of $\leq 8\%$ by

$$\frac{\gamma_i^L}{kv_{Ti}} = \sqrt{\frac{\pi}{8}} r^2 \exp\left[-\frac{r}{2} - \frac{G(r)}{2}\right] \frac{10 + 21r + r^3}{2r^2 + r^3}. \qquad (14)$$

From our numerical calculations the phenomenological function $R(r, k_i)$, which describes the smooth transition between collisional and collisionless ion damping rates can be represented as $R^{-1} = 1 + [rk_i^2(0.05r + 0.04)]^{-1}$. This expression provides an accuracy of better than 20% for any $r \geq 1$. Figure 3 shows the IAW ion damping as a function of $k\lambda_{ii}$ for different r in comparison with both numerical result [2] and our fit given by Eqs. (13) and (14).

The DR also yields a zero frequency, entropy mode $\omega = -i\gamma_{\mathrm{EN}}$ at $|\omega| \lesssim kv_{Ti}$ corresponding to ion temperature perturbations [2]. The damping rate from Fig. 4 is defined by ion thermal conductivity, κ_i. The form of γ_{EN} illustrates the transition from strongly collisional transport, where $\gamma_{\mathrm{EN}} = 2(1 + g)k^2\kappa_i/n_i(5 + 3g)$ [2] and $\kappa_i = 3.9n_i v_{Ti}^2/\nu_{ii}^T$, to nonlocal [22] regimes of ion thermal transport.

In conclusion, we have developed a theory of the dielectric permittivity of collisional, fully ionized plasmas applicable for arbitrary frequencies, wave numbers, collision frequencies, and Debye lengths of both electrons and ions. It reproduces all known limits, and describes the continuous transition between them over the entire (ω, k) space. Easy to use fitting formulas are derived for IAW with potential applications to Thomson scattering, stimulated Brillouin scattering, and return current IAWs instability in inertial confinement fusion plasma.

The authors acknowledge the financial support by the Russian Foundation for Basic Research and the Natural Sciences and Engineering Research Council of Canada.

[1] L. D. Landau, Zh. Eksp. Teor. Fiz. **7**, 203 (1937); [J. Phys. (Moscow) **10**, 25 (1946)].
[2] M. D. Tracy, E. A. Williams, K. G. Estabrook, J. S. De Groot, and S. M. Cameron, Phys. Fluids B **5**, 1430 (1993).
[3] M. Ono and M. R. Kulsrud, Phys. Fluids **18**, 1287 (1975).
[4] C. J. Randall, Phys. Fluids **25**, 2231 (1982).
[5] G. G. Comisar, Phys. Fluids **6**, 76 (1963).
[6] I. P. Shkarofsky, T. W. Johnston, and M. P. Bachynski, *The Particle Kinetics of Plasmas* (AddisonWesley, Reading, 1966).
[7] B. Buti, Phys. Rev. **165**, 195 (1968).
[8] E. M. Epperlein, R. W. Short, and A. Simon, Phys. Rev. Lett. **69**, 1765 (1992).
[9] V. Yu. Bychenkov, J. Myatt, W. Rozmus, and V. T. Tikhonchuk, Phys. Plasmas **1**, 2419 (1994).
[10] J. Zheng and C. X. Yu, Plasma Phys. Controlled Fusion **42**, 435 (2000).
[11] D. Bhadra and R. K. Varma, Phys. Fluids **7**, 1091 (1964).
[12] A. F. Aleksandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodinamics* (Springer, Berlin, 1984).
[13] A. Lenard and I. B. Bernstein, Phys. Rev. **112**, 1456 (1958).
[14] R. W. Short and A. Simon, Phys. Plasmas **9**, 3245 (2002).
[15] V. Yu. Bychenkov, Plasma Phys. Rep. **24**, 801 (1998).
[16] A. V. Brantov, V. Y. Bychenkov, W. Rozmus, and C. E. Capjack, Phys. Rev. Lett. **93**, 125002 (2004); JETP **100**, 1159 (2005).
[17] S. I. Braginskii, Rev. Plasma Phys. **1**, 205 (1965).

[18] V. Yu. Bychenkov, V. N. Novikov and V. T. Tikhonchuk, JETP **87**, 916 (1998).

[19] J. F. Luciani, P. Mora, and J. Virmont, Phys. Rev. Lett. **51**, 1664 (1983).

[20] D. F. DuBois, V. Gilinsky, and M. G. Kivelson , Phys. Rev. **129**, 2376 (1963); W. Rozmus, J. Plasma Phys. **22**, 41 (1979); J. R. Jasperse and B. Basu, Phys. Fluids **29**, 110 (1986).

[21] V. Yu. Bychenkov, W. Rozmus, V. T. Tikhonchuk, A. V. Brantov, Phys. Rev. Lett. **75**, 4405 (1995); V. Yu. Bychenkov, J. Myatt, W. Rozmus, V. T. Tikhonchuk, Phys. Rev. E **52**, 6759 (1995).

[22] Z. Zheng, W. Rozmus, V. Yu. Bychenkov, A. V. Brantov, and C. E. Capjack, Phys. Plasmas **16**, 102301 (2009).