Quantum Phase Slips in Superconducting Wires with Weak Inhomogeneities

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(Received 17 August 2011; published 3 May 2012)

Quantum phase slips are traditionally considered in diffusive superconducting wires which are assumed homogeneous. We present a definite estimate for the amplitude of phase slips that occur at a weak inhomogeneity in the wire where local resistivity is slightly increased. We model such a weak link as a general coherent conductor and show that the amplitude is dominated by the topological part of the action. We argue that such weak links occur naturally in apparently homogeneous wires and adjust the estimate to that case. The fabrication of an artificial weak link would localize phase slips and facilitate a better control of the phase-slip amplitude.

DOI: 10.1103/PhysRevLett.108.187002 PACS numbers: 74.81.Fa, 74.25.F−, 74.78.Na

The phase-slip processes in superconducting wires and long Josephson-junction arrays remain an active research subject both experimentally and theoretically [1–4]. In the course of a phase slip, the superconducting order parameter fluctuates to zero at a point in the wire while the superconducting phase difference along the wire changes by $\pm 2\pi$. The *incoherent* phase slips provide a mechanism for superconducting wires to retain a finite resistance at temperatures below the superconducting transition. Phase-slip events are thermally activated at temperatures close to critical [5] and triggered by quantum fluctuations at low temperatures [6]. Progress in microfabrication has enabled production of superconducting wires with diameters of a few tens of nanometers in which incoherent quantum phase slips have been studied experimentally [7–10]. Recently, much attention has been paid to *coherent* phase slips [11–14]. It has been argued that a wire where coherent phase slips take place may be regarded as a new circuit element—the phase-slip junction [12]—which is a dual counterpart of the Josephson junction with superconducting phase difference replaced by charge. The phase-slip qubit [11] [see Fig. 1(b)] and other coherent devices [13] have been proposed. The novel functionality may be useful in realization of the fundamental current standard dual to the Josephson voltage standard [12].

The coherent phase slips in a wire are characterized by a quantum amplitude E_S rather than a rate of an event [1,15]. The amplitude depends exponentially on the instanton action which is usually dominated by the phase-slip "core" $\mathcal{S}_{\rm core} = \alpha(G_Q R' \xi)^{-1}$ where R' is a wire normal-state resistance per unit length, ξ is the coherence length, and $G_Q \equiv e^2/\pi\hbar$ (hereafter $\hbar = 1$). The numerical factor α depends on the details of the core profile which are unknown. Therefore, the amplitude $E_S \propto e^{-\mathcal{S}_{\rm core}}$ is exponentially small for not very resistive wires and is difficult to predict for the specific experimental settings since even a small arbitrariness in α would amount to orders of magnitude ambiguity in E_S [11].

In this Letter, we report on a definite estimate of E_S [Eq. (1)] for a weak link in diffusive wire where resistivity is slightly and locally enhanced. We argue that such weak links occur naturally in apparently homogeneous wires and adjust the estimate to that case as well.

To justify the model, let us first note that much attention is paid experimentally to making the wires as homogeneous as possible [10]. Indeed, if the resistance of the wire is dominated by a single weak link, the device would be a Josephson junction which is the opposite of the phase-slip junction intended. However, a *weak* inhomogeneity, where the local resistivity of the wire is only *slightly* larger, will not spoil the phase-slip character of the junction. The condition for this is just that the resistance of the weak link is much smaller than the overall normal-state resistance of the wire. Such weak links occur naturally in apparently homogeneous wires. Owing to exponential dependence on resistivity, the phase slips will be localized at the weak links. Thus, making such a weak link artificially would provide a better control for E_S, since one knows where the phase slips occur.

This motivates us to consider a simple yet general model of a weak link where the link is described as a short (length much smaller than ξ) coherent conductor characterized by a set of spin-degenerate transmission eigenvalues $\{T_p\}$. We

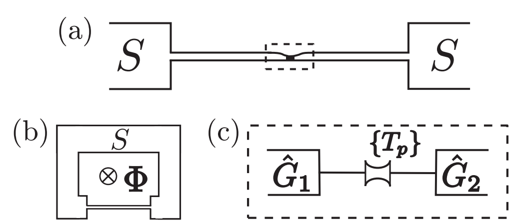

FIG. 1. (a) Superconducting diffusive wire with a weak link (dashed rectangle) connecting bulk superconducting electrodes. (b) Embedding the wire into a superconducting loop makes a phase-slip flux qubit [11]. (c) We model the weak link as a general coherent conductor characterized by a set of transmission eigenvalues $\{T_p\}$.

solve this model and obtain the accurate estimate for the amplitude

$$E_S \approx 2\Delta\sqrt{\sum_p T_p}\prod_p\sqrt{1-T_p} \quad (1)$$

under approximations specified in the text, where Δ is the superconducting order parameter in the wire. We show that the amplitude is dominated by the topological part of the action emerging from $\pm 2\pi$ phase winding in the phase-slip process. Finally, we use the known transmission distribution of a diffusive conductor and obtain an estimation of E_S valid for homogeneous wires as well.

The system under consideration is depicted in Fig. 1. The weak link (or, "contact") is modeled as a general coherent conductor with conductance $G_c = G_Q \sum_p T_p$. The wire is much thinner than ξ and is characterized by the length L ($L \gg \xi$), normal-state resistance R', and capacitance C', where $'$ signifies that these quantities are defined per unit length. For a wire thickness in tens of nanometers range, the geometric inductance $\mathcal{L}'_g$ is negligible with respect to the kinetic inductance $\mathcal{L}'_k \equiv R'/\pi\Delta$. For concreteness, we consider the wire in a phase-slip qubit configuration [Fig. 1(b)]. This does not affect the evaluation of E_S.

Generally, the quantum dynamics of such systems is described by an imaginary-time action that is path integrated over fluctuating superconducting order parameter $\Delta(\tau, x)$, where x is the coordinate along the wire. Our model brings about drastic simplifications. The modulus of order parameter can be regarded as constant, its phase $\phi(\tau, x)$ being the only dynamical variable. The action comprises two terms, $\mathcal{S}[\phi] = \mathcal{S}_c[\phi] + \mathcal{S}_w[\phi]$, which describe the weak link and the wire, respectively. The action $\mathcal{S}_c$ for tunnel coupling was obtained in [16]. We generalize the result to generic coherent contact along the lines of Ref. [17]. The action reads

$$\mathcal{S}_c = -\frac{1}{2}\sum_p \mathrm{Tr}\ln\left(1 + \frac{T_p}{4}(\{\hat{G}_1, \hat{G}_2\} - 2)\right) \quad (2)$$

with $\hat{G}_j(\tau, \tau') = e^{i\phi_j(\tau)\hat{\tau}_3/2}\hat{G}_0(\tau - \tau')e^{-i\phi_j(\tau')\hat{\tau}_3/2}$. Here, $\hat{G}_{1,2}$ are imaginary-time Green's functions in a wire on the left and right side of the weak link [cf. Fig. 1(c)], $\phi_{1,2}$ are the corresponding phases, $\hat{G}_0(\omega) = (\omega\hat{\tau}_3 + |\Delta|\hat{\tau}_1)/\sqrt{\omega^2 + |\Delta|^2}$ is the Green's function of a homogeneous superconductor, and $\hat{\tau}_i$ are the Pauli matrices in Nambu space. We see that this action depends on the phase difference $\phi(\tau) \equiv \phi_2(\tau) - \phi_1(\tau)$ only.

The resistance of the weak inhomogeneity in the wire is naturally much smaller than the total resistance of the wire, $R_c \ll LR'$ ($R_c \equiv G_c^{-1}$). The same pertains to inductance. Under these conditions, the minima of the action correspond to a well-defined fluxon states where the winding of the phase along the wire takes values $2\pi n$, n being integer. The energies of the states are given by $E_n = (\Phi - n\Phi_0)^2/2L\mathcal{L}'_k$, where Φ is the flux penetrating the loop and $\Phi_0 = \pi/e$ is the flux quantum. Technically, it is convenient to ascribe the phase difference to the weak link and concentrate on $\Phi = \Phi_0/2$, where minima $n = 0$, 1 are degenerate. The phase-slip amplitude E_S is then computed from analysis of instantons in $\phi(\tau)$ connecting these two energy-degenerate minima and equals to the energy splitting of the resulting qubit states [11].

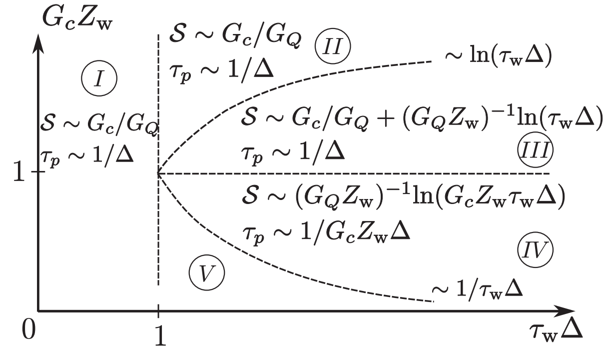

FIG. 2. The phase-slip regimes (see text) in parameter space ($G_c Z_w$, $\tau_w\Delta$), where Z_w is the wave impedance of the wire and τ_w is the characteristic time of charge propagation through the wire. We concentrate on the regions *I* and *II*, where E_S does not depend on the wire parameters.

The wire provides an electromagnetic environment for the phase propagation. In our situation, $\xi\partial_x\phi \ll 1$ and the effective environment is linear. Owing to this, the quadratic action $\mathcal{S}_w$ can be expressed in terms of $\phi(\tau)$ [18], $\mathcal{S}_w[\phi] = (8\pi^2 G_Q)^{-1}\int_0^\infty d\omega\,\omega Y(\omega)|\phi(\omega)|^2$, where $Y(\omega) = [\mathcal{L}'(\omega)/C']^{-1/2}\,[\tanh(\omega L_1/v_p) + \tanh(\omega L_2/v_p)]^{-1} - (L\mathcal{L}'_k\omega)^{-1}$, L_1 (L_2) is the length of the wire left (right) from the contact and $v_p(\omega) = 1/\sqrt{\mathcal{L}'(\omega)C'}$. Here $\mathcal{L}'(\omega)$ is the inductance in imaginary frequency obtained by analytic continuation of impedance. It accounts for the fact that the wire is inductive with $\mathcal{L}' = \mathcal{L}'_k$ at subgap energies $\omega \ll 2\Delta$, and resistive with $\mathcal{L}'(\omega) = R'/\omega$ at large energies $\omega \gg 2\Delta$. This completes the theoretical description of the model. The instanton solution $\phi(\tau)$ minimizes $\mathcal{S}[\phi]$ satisfying $\phi(-\infty) = 0$ and $\phi(\infty) = 2\pi$.

We want to concentrate on the case when the estimation of E_S does not depend on wire parameters. This is not always so and we need to discuss various regimes that may be realized in the system (Fig. 2). The relevant wire parameters are the wave impedance $Z_w = \sqrt{\mathcal{L}'_k/C'}$ and the characteristic charge propagation time τ_w, which is estimated as either the plasmon propagation time $L\sqrt{\mathcal{L}'_k C'}$ ($\tau_w\Delta \gg 1$, superconducting response) or RC time $L^2R'C'$ ($\tau_w\Delta \ll 1$, dissipative response). Let τ_p be the optimal instanton duration. The weak-link action can be then estimated as $\mathcal{S}_c \simeq (G_c/G_Q)\max(1, \tau_p\Delta)$. As to the wire action, it corresponds to the dissipative response $\mathcal{S}_w \simeq (G_Q Z_w)^{-1}\ln(\tau_w/\tau_p)$ if charge propagation does not reach wire ends for the time τ_p and to the capacitive response $\mathcal{S}_w \simeq LC'/G_Q\tau_p$ otherwise. The τ_p is found from minimizing $\mathcal{S} = \mathcal{S}_c + \mathcal{S}_w$ which gives rise to five regimes depicted in Fig. 2.

For "short" wires ($\tau_w\Delta \ll 1$) the action is dominated by the weak link and $\tau_p \simeq 1/\Delta$ (region *I*). For "long" wires ($\tau_w\Delta \gg 1$), we encounter the variety of regimes. At sufficiently large G_c, the above estimations still hold (region *II*). Upon decreasing G_c, the dissipative wire response starts to dominate while $\tau_p \simeq 1/\Delta$ (region *III*). At $G_c \simeq Z_w$, the instanton duration τ_p increases. It is determined from the competition of inductive response of the weak link and dissipative response of the wire (region *IV*). Upon further decrease of G_c, the τ_p matches τ_w. Below this, the wire response is capacitive and τ_p is determined from the competition of inductive response of the weak link and the capacitive response of the wire (region *V*), very much like in traditional theory of macroscopic phase tunnelling [16]. We conclude that there is a large part of the parameter space (regions *I*, *II*) where instanton action is dominated by $\mathcal{S}_c$ and concentrate on the minimization of this part of the action.

For an arbitrary transmission set $\{T_p\}$ the analytical solution cannot be obtained, and we have treated the problem numerically [19]. However, the analysis of the numerical results permitted us to formulate a good analytical approximation. To outline this, let us note that the action in Eq. (2) can be expressed in terms of the eigenvalues Λ_n of a Hermitian operator $\hat{\Lambda} \equiv (\hat{G}_1 - \hat{G}_2)/2$,

$$\mathcal{S}_c[\phi] = -\frac{1}{2}\sum_{p,n} \ln(1 - T_p\Lambda_n^2). \qquad (3)$$

One can deduce some properties of the eigenvalues that do not depend on details of the instanton profile $\phi_{\rm in}(\tau)$. First of all, $|\Lambda_n| \leq 1$. Importantly, there is a single eigenvalue precisely at $\Lambda = 1$. This is guaranteed by topological properties of $\hat{\Lambda}$ with respect to variations of $\phi_{\rm in}(\tau)$; a similar discussion is provided in [20]. Generally, the number of these special eigenvalues is set by the winding number of $\phi(\tau)$, which is 1 in the case under consideration. All other eigenvalues come in pairs $\pm\Lambda$.

The special eigenvalue gives a topological contribution to the action

$$\mathcal{S}_{c1} = -\frac{1}{2}\sum_{p} \ln(1 - T_p), \qquad (4)$$

which presents a lower bound for $\mathcal{S}_c$. This lower bound could have been realized if there was an instanton profile for which all nonspecial Λ_n are zero. In the normal-metal case such instantons indeed exist and can even be found analytically [21]. This is not the case for superconducting action. However, the numerics prove that for the optimal instanton all nonspecial Λ_n are small and the topological contribution gives an accurate estimation of the overall

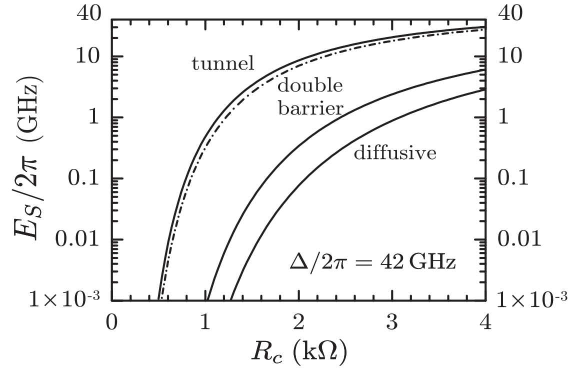

FIG. 3. Phase-slip amplitude E_S for tunnel contact, double-barrier junction, and short diffusive bridge estimated using topological action $\mathcal{S}_{c1}$ (solid curves). The true E_S with a non-topological contribution taken into account is shown in the tunnel limit (dash-dotted curve).

action. For instance, in the tunnel limit ($T_p \ll 1$) $\mathcal{S}_c = 0.528G_c/G_Q$ while the topological bound is $\mathcal{S}_{c1} = 0.5G_c/G_Q$. In all cases investigated, the relative accuracy of the topological approximation was better than 6%. Formally, the exponential dependence of E_S could amplify even this small error by orders of magnitude; yet this does not happen for any E_S of interest (see Fig. 3).

This gives us the value of the action. We also need to compute the prefactor. The prefactor is evaluated by the standard instanton techniques yielding $E_S = 2(\int d\tau\dot{\phi}_{\rm in}^2/2\pi)^{1/2}(D')^{-1/2}e^{-\mathcal{S}_{\rm in}}$. The ratio of determinants $D' = \det'(\delta^2\mathcal{S}/\delta\phi^2|_{\rm in})/\det(\delta^2\mathcal{S}/\delta\phi^2|_0)$ takes into account fluctuations with respect to the instanton and trivial trajectories; the prime $'$ denotes that the zero eigenvalue intrinsic to the instanton is omitted in the numerator.

It is important to note that the high eigenvalues h_n at $n \gg 1$ of $\delta^2\mathcal{S}/\delta\phi^2$ are linear in n. This is related to the frequency dependence of the integral kernels in the action: for rapidly varying $\phi(\tau)$, the action reads $\mathcal{S}_c = (G_c/16\pi^2G_Q)\int d\omega|\omega||\phi(\omega)|^2$ (assuming $\omega \gg \Delta$). This implies logarithmic divergence of $\ln(D')$ at large energies. In principle, account of the wire capacitance might provide an upper cutoff needed. However, we find it more consistent to cancel the divergence by taking into account the renormalization of transmission eigenvalues.

Indeed, it is known that Coulomb interaction leads to energy-dependent renormalization of T_p [22]. Under current-bias conditions, which is the case under consideration, the renormalization reads $dT_p/d\ln E = T_p(1 - T_p)/\sum_p T_p$. Correcting the transmissions in $\mathcal{S}_{c1}$ with the above equation indeed cancels the divergence of $(D')^{-1/2}$. It implies that the T_p in all formulas must be taken at $E \simeq \Delta$ rather than at unphysical high energy. The procedure is similar to the common treatment of ultraviolet divergencies in the instanton determinant [23]. This brings us to Eq. (1). We stress that by virtue of

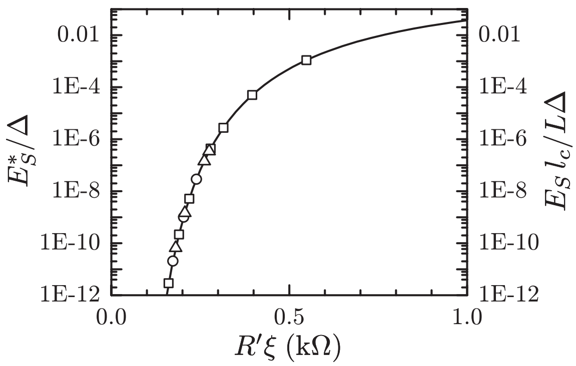

FIG. 4. Estimation of phase-slip amplitude for a long diffusive superconducting wire. Squares (□), circles (○), and triangles (△) give the estimated values of $R'\xi$ for superconducting nanowire samples as reported in [8,6,10], respectively.

instanton approximation, this relation is only valid for $E_S \ll \Delta$.

To make concrete predictions (Fig. 3), we need to specify the type of weak link. Using known transmission distributions [24] we find $\mathcal{S}_{c1} = \alpha G_c/G_Q$ with $\alpha = 1/2$, 1, $\pi^2/8$ for a tunnel junction, double tunnel junction, and diffusive weak link, respectively. The phase-slip amplitude E_S for these types of weak links is shown in Fig. 3 for $T_c = 1.2$ K. For qubit applications, E_S should be in the gigahertz range. In this range, E_S at a given R_c varies by 2 orders of magnitude depending on the type of the weak link. The dash-dotted curve for the tunnel junction illustrates the accuracy of topological approximation.

Let us use the results for the weak link to suggest a better estimation of E_S in a homogeneous wire. There, the spatial extent of the phase-slip core is of the order of ξ [1]. Let us separate the wire into pieces of the length l_c and treat each piece as a diffusive weak link of corresponding resistance, $R_c = R' l_c$. We can find l_c by comparing the critical current of a single weak link, $I_c = 1.32\pi\Delta/2eR_c$, and that of a homogeneous wire, $I_{cw} = \pi\Delta/3\sqrt{3}eR'\xi$ [25]. This gives $l_c \approx 3.43\xi$ and $E_S^* = 1.08\Delta(G_Q R'\xi)^{-1/2} e^{-0.360/G_Q R'\xi}$ per link. The amplitudes of the pieces add to $E_S = E_S^* L/l_c$.

The amplitude E_S^* versus $R'\xi$ is plotted in Fig. 4 along with several values of $R'\xi$ for fabricated nanowires. Owing to exponential dependence on $R'\xi$, the phase-slip amplitude varies by 9 orders of magnitude. We conclude that for most wires the expected E_S^* is smaller than $10^{-6}\Delta$, with an exception of Ref. [8] where the wires have been fabricated by metal coating of a nanotube.

Let us use the above formula to estimate the expected homogeneity of E_S in realistic wires. We assume that fabrication imperfections induce normally distributed fluctuations of G_c in each weak link with standard deviation δG_c. For $\delta G_c = 0$, the total E_S scales with the length. However, if the fluctuations of E_S^* are sufficiently large, the total E_S can be just dominated by a single weak link of the lowest conductance. The criterion of crossover between these two regimes is derived to be $\ln(L/l_c) = (4.64\ \mathrm{k}\Omega/R'\xi)^2(\delta G_c/G_c)^2$. It sharply depends on $R'\xi$. Let us assume $\delta G_c/G_c = 20\%$, a typical width variation of ultranarrow wires. For the smallest experimental $R'\xi$ in Fig. 4, the homogeneity is only realized if $L > 10^{17}\xi$! For the largest $R'\xi$, $L > 60\xi$ would suffice. The smallest possible δG_c is determined by mesoscopic fluctuations. For the quantity given by Eq. (1), these fluctuations have been computed in [21]. Substitution leads to the homogeneity criterion $\ln(L/l_c) = (1/8)\ln(G_c/G_Q)$ [19]. This criterion is not restrictive for the values of $R'\xi$ in Fig. 4.

We see that even for apparently homogeneous wires E_S may be strongly inhomogeneous. In addition, high values of E_S are hard to achieve for the wires under experimental consideration. We suggest that fabrication of an *artificial* weak link may solve the problem. To do so, one can try to reduce selectively the wire width in a given point by, say, a factor of 2, either by laser or ion beam.

In conclusion, we have studied the quantum phase slips generated at a weak inhomogeneity in a superconducting wire. We have shown that the phase-slip action can be approximated by its topological part with accuracy better than 6%, thereby establishing a correspondingly accurate analytic estimate for the phase-slip amplitude. We have analyzed the consequences of that estimation when applied to realistic, imperfectly homogeneous wires. We suggest the fabrication of an artificial weak link would provide a better control needed for practical realization of the phase-slip devices.

The authors are indebted to J. E. Mooij for valuable discussions. This research was supported by the Dutch Science Foundation NWO/FOM. M. V. acknowledges support by the Serbian Ministry of Science, Project No. 171027.

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