Nondivergent Cherenkov Radiation in a Wire Metamaterial

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The electromagnetic radiation of a charge moving in an infinite 3D structure made of parallel wires is considered. The periods of the structure are assumed to be small; therefore, it can be described by an effective permittivity tensor. The charge velocity is perpendicular to the wires. Analytical and numerical investigations are performed, and some unusual properties of the radiation are noted. It is shown that the radiation propagates along the wires and concentrates near certain rays behind the charge. The wave field does not vary with distance from the charge along these rays (if energy loss in the medium is negligible). The prospects for the use of the structure under consideration for diagnostics of bunches are noted.

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Cherenkov radiation (CR) of charged particles is used very widely, both for the generation of large electromagnetic fields and for the detection of charged particles [1]. One of the defects of traditional techniques used for particle detection is that the radiation intensity essentially decreases with distance from the charge. Overcoming this defect seems impossible if we deal only with some homogenous medium (unless we use special devices such as lenses or mirrors to concentrate the radiation or waveguides to channel it). However, the properties of radiation in the presence of dispersive and anisotropic media can be very unusual [1–8]. Further, we will demonstrate that the divergence of CR can be negligible if certain specific types of material are used.

At the present time, special attention is being given to so-called "metamaterials," which are artificial periodic structures with relatively small spacing. A metamaterial can be considered as a medium that is characterized by an effective permittivity and permeability. Metamaterials can possess various properties that are not observed in traditional media. For example, they can behave similarly to a left-handed medium (LHM). The radiation of moving charges in the presence of an LHM has been investigated intensively [3–6]. Other metamaterials are known as well; however, the radiation of moving charges in them has rarely been analyzed [4,7,8].

Here we consider the case in which the charge moves through a volume boundless system of parallel wires (Fig. 1). It is assumed that $d \ll \lambda$, where d is a structural period and λ is a typical wavelength or a typical distance of a field variation. (For simplicity, we assume that the periods for two perpendicular directions are identical, but this is not necessarily [9,10].) Different approaches to the derivation of the effective ("macroscopic") characteristics of this structure can be found in the literature [9–13]. If the x axis is parallel to the wires, the expression of the permittivity tensor has the form

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{\parallel} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \varepsilon_{\parallel} = 1 - \frac{\omega_p^2}{\omega^2 - k_x^2 c^2 + 2i\omega_d \omega}, \tag{1}$$

where $\omega_p^2 = 2\pi c^2 d^{-2}[\ln(d/r_0) - C]^{-1}$. Here c is the velocity of light in vacuum, r_0 is the wire radius, ω_d is a small constant that is responsible for energy losses (we consider that $\omega_d \ll \omega_p$), and C is some constant. Note that the value of this constant depends on the ratio d/r_0. In the literature, different estimations of C can be found; however, all of them yield values on the order of 1 [9–13]. It should be stressed that the medium under consideration possesses both a frequency dispersion and a spatial one, as can be seen from (1).

We will now analyze the case in which the charge velocity is perpendicular to the wires. In the opposite case, in which the charge moves parallel to the wires, radiation can be generated only if the wires are covered with nonconducting coating and the properties of the radiation are fundamentally different (this type of problem was considered in [14] for the case in which the structure is situated in a waveguide).

It is assumed that the point charge moves with a constant velocity $\vec{V} = V\vec{e}_z = c\beta\vec{e}_z$. For this source, the current density is $\vec{j} = \vec{e}_z c\beta q\delta(x, y, \zeta)$, where $\zeta = z - c\beta t$, and its Fourier transform is $\vec{j}_{\omega,\vec{k}} = \vec{e}_z c\beta q\delta(\omega - k_z c\beta)/(2\pi)^3$. The electromagnetic field is determined from the Maxwell

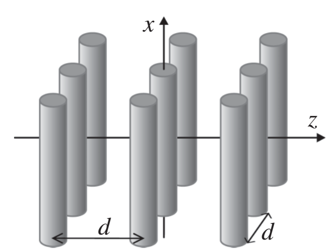

FIG. 1. Scheme of the structure.

equations which should be supplemented the condition of exponential decrease of the field with increase in distance $\rho = \sqrt{x^2 + y^2}$ (we take into account small energy losses). The solution was found with the help of the Fourier transformation technique. Note that the initial fourfold Fourier integrals are reduced to threefold ones due to the Dirac delta function $\delta(\omega - k_z c\beta)$. Omitting transformations, we write here the exact result for the electric force in the limit of negligible losses ($\omega_d \to +0$):

$$\vec{E} = -iq(2\pi^2 c^3 \beta)^{-1} \int_{-\infty}^{-\infty} d\omega \int_{-\infty}^{+\infty} dk_y \int_{-\infty}^{+\infty} dk_x \vec{u} \times \frac{\exp(ik_x x + ik_y y + i\omega\zeta/c\beta)}{(k_x^2 - k_{ox}^2)(k_x^2 - k_{eix}^2)(k_x^2 - k_{eax}^2)}, \tag{2}$$

where

$$u_x = k_x c^2 (k_x^2 - k_{eix}^2)(k_x^2 - k_{eax}^2), \tag{3a}$$
$$u_y = k_y c^2 [(k_x^2 - k_{eix}^2)(k_x^2 - k_{eax}^2) - \omega^2 \omega_p^2 c^{-4}], \tag{3b}$$
$$u_z = c\beta\omega(k_x^2 - k_{eix}^2)(k_x^2 - k_{eax}^2)(1 - \beta^2)\beta^{-2} - c\beta\omega[(k_x^2 - k_{eix}^2)\omega_p^2 c^{-2} + k_y^2 \omega_p^2 c^{-2}]; \tag{3c}$$

$$k_{ox}^2 = -\omega^2 c^{-2}(1 - \beta^2)\beta^{-2} - \omega_p^2 c^{-2} - k_y^2, \tag{4a}$$
$$k_{eix}^2 = -\omega^2 c^{-2}(1 - \beta^2)\beta^{-2} - k_y^2, \tag{4b}$$
$$k_{eax}^2 = \omega^2 c^{-2}. \tag{4c}$$

Note that we should take into account that $k_{eax} = \omega/c + i0$; i.e., the pole $k_x = k_{eax}$ is situated above the integration path, and the pole $k_x = -k_{eax}$ is situated below it (this result is obtained if we initially consider $\omega_d \neq 0$ and take the limit as this value approaches 0 in the final result only). Expressions (4) also follow from the dispersion equations for the three types of plane waves that can exist in the medium under consideration [10]:

$$k_x^2 + k_y^2 + k_z^2 = \omega^2/c^2, \tag{5a}$$
$$k_x^2 + k_y^2 + k_z^2 = (\omega^2 - \omega_p^2)/c^2, \tag{5b}$$
$$k_x^2 = \omega^2/c^2. \tag{5c}$$

Equations (5) correspond to an ordinary wave (5a), an ''isotropic'' extraordinary wave (5b), and an ''anisotropic'' extraordinary wave (5c). The wave vector module of an ordinary wave or an isotropic extraordinary wave does not depend on the wave vector direction. An anisotropic extraordinary wave has a fixed projection k_x but arbitrary projections k_y and k_z. Taking into account that $k_z = \omega/V$ in our problem, one can see that Eqs. (5) give expressions (4). From (4), we see that only an anisotropic extraordinary wave is a propagating one in the situation under consideration, while the two other types of waves are evanescent, because k_{ox} and k_{eix} are imaginary. This fact is illustrated geometrically in Fig. 2, where the perpendicular from A intersects only planes $k_x = \pm k_{eax} = \pm\omega/c$. Note that the phase velocity of the anisotropic extraordinary wave

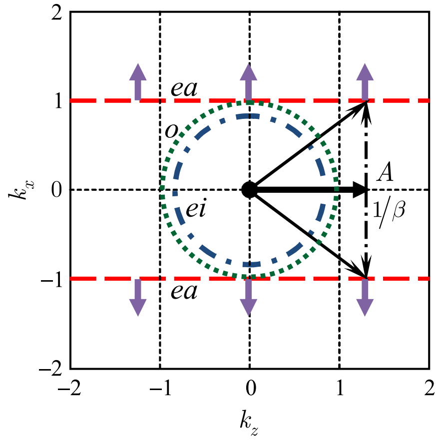

FIG. 2 (color online). Dispersive curves for three plane waves. The small arrows show the group velocity direction for the extraordinary anisotropic wave. The long arrows represent the wave vector and its projections for the problem under consideration (in units of ω/c).

$\omega/\sqrt{\omega^2 c^{-2} + k_y^2 + k_z^2}$ is not greater than c, but the group velocity is always equal to c and directed along the x axis.

Writing the integrals over k_x in (2) as the sum of residues, we obtain double integrals. It is interesting that the integral for E_x can be expressed exactly in the explicit form [15]:

$$E_x = \frac{2qx}{c^3}\sqrt{\frac{\omega_p^3\beta^5}{2\pi}}\frac{K_{3/2}[\frac{\omega_p\beta}{1-\beta^2}(\frac{\zeta^2(1-\beta^2)}{c^2\beta^2} + \frac{\rho^2\beta^2}{c^2})^{1/2}]}{(1-\beta^2)[\frac{\zeta^2(1-\beta^2)}{c^2\beta^2} + \frac{\rho^2\beta^2}{c^2}]^{3/4}}, \tag{6}$$

where $K_\nu(\xi)$ is a modified Hankel function, and $\rho = \sqrt{x^2 + y^2}$. Because of the modified Hankel function, this component exponentially decreases with an increase in ρ and ζ. Thus it is a part of a quasistatic field, and the wave field (the field of the CR) does not contain it.

The behaviors of E_y and E_z are fundamentally different. They both are cumbersome expressions that consist of the contributions of poles $\pm k_{ox}$, $\pm k_{eix}$, and $\pm k_{eax}$. Two of them are imaginary and produce contributions characterized by an exponential decrease (they pertain to the quasistatic field). Only the contributions of poles $\pm k_{eax}$ give the wave field (henceforth denoted by index W):

$$\begin{Bmatrix} E_y^W \\ E_z^W \end{Bmatrix} = \frac{-q\omega_p^2}{2\pi c^5 \beta^2}\int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} dk_y \begin{Bmatrix} k_y c\beta \\ \omega \end{Bmatrix} \times \frac{\exp(\frac{\omega}{c}|x| + i\frac{\omega}{c\beta}\zeta + ik_y y)}{(\frac{\omega^2}{c^2\beta^2} + k_y^2)(\frac{\omega^2}{c^2\beta^2} + k_y^2 + \frac{\omega_p^2}{c^2})}. \tag{7}$$

The integrals over k_y in these expressions can be determined by their residues, and then the integrals over ω are analytically calculated using tabulated integrals [15]:

$$E_y^W = -2\beta q \frac{y(\zeta + \beta|x|)}{\xi^4}\left[1 - \frac{\omega_p^2}{2c^2}\xi^2 K_2\left(\frac{\omega_p \xi}{c}\right)\right], \qquad (8)$$

$$E_z^W = -\beta q\left[\frac{y^2 - (\zeta + \beta|x|)^2}{\xi^4} - \frac{\omega_p}{\xi c}K_1\left(\frac{\omega_p\xi}{c}\right) - \frac{(\zeta + \beta|x|)^2\omega_p^2}{\xi^2 c^2}K_2\left(\frac{\omega_p\xi}{c}\right)\right], \qquad (9)$$

where $\xi^2 = y^2 + (\zeta + \beta|x|)^2$.

One can make similar transformations for the magnetic field components as well. The result is that the wave part of the magnetic force is orthogonal to the electric one and of equal modulo to it:

$$H_z^W = E_y^W \mathrm{sgn}x, \qquad H_y^W = -E_z^W \mathrm{sgn}x. \qquad (10)$$

Note that these formulas also follow from the properties of a plane wave in the medium under consideration [10].

Using expansions of the modified Hankel functions, one can find the approximate expressions for the field components under the condition $\omega_p\xi/c \ll 1$ (i.e., within some small vicinity of the lines $\zeta + \beta|x| = 0$ in the plane $y = 0$):

$$E_y^W \approx -\frac{\beta q \omega_p^2}{c^2}\frac{y(\zeta + \beta|x|)}{\xi^2}, \qquad (11)$$

$$E_z^W \approx \frac{\beta q \omega_p^2}{2c^2}\left[\frac{(\zeta + \beta|x|)^2}{\xi^2} + \ln\frac{\omega_p\xi}{c} + C_E - \frac{1}{2}\right], \qquad (12)$$

where $C_E \approx 0.577$ is the Euler constant.

One can see that components E_y^W and H_z^W tend to zero on the lines $\zeta = -\beta|x|$, $y = 0$, and components H_y^W and E_z^W each have a logarithmic singularity on these lines. When the viewpoint is shifted along these lines (i.e., conditions $y = \mathrm{const}$ and $\zeta + \beta|x| = \mathrm{const}$ are fulfilled), the wave field does not vary. The Poynting vector $\vec{S} = c(4\pi)^{-1} \times [\vec{E} \times \vec{H}]$ consists only of a component that lies along the wires, and it is proportional to the full electric wave field squared. The energy flow through some square $\int_\Sigma \vec{S} d\vec{\Sigma}$ is limited and does not depend on the distance from the charge trajectory.

Note that the behavior of the wave field far from the lines $\zeta = -\beta|x|$, $y = 0$, under the condition $\omega_p\xi/c \gg 1$, is determined by the first summands in (8) and (9) because of the exponential decrease of the modified Hankel functions. One can see that the wave field decreases in this zone proportionally to $R^{-2} = (x^2 + y^2 + \zeta^2)^{-1}$.

Thus, a charge moving perpendicularly to the wires emits CR concentrated within some small vicinity of the lines $\zeta = -\beta|x|$ in the plane $y = 0$ behind the charge. These waves propagate along the wires and exist for an arbitrary velocity of the charge. Figure 3 shows the

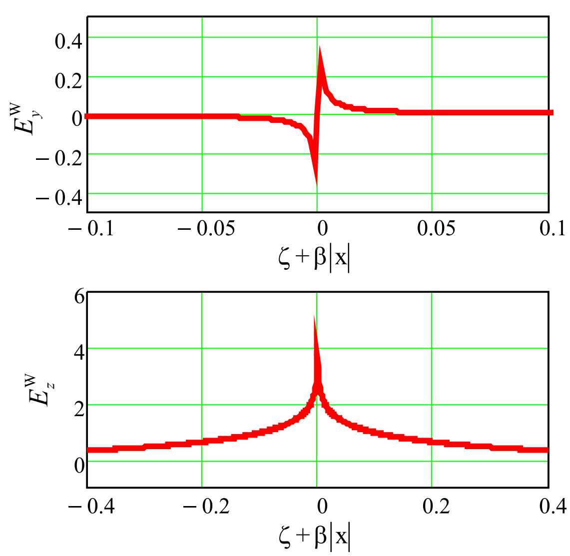

FIG. 3 (color online). Electric wave field components of a point charge as a function of $\zeta + \beta|x|$ for $y = 0.001c/\omega_p$. Distances are given in units of c/ω_p, and field intensity in units of $q\omega_p^2/c^2$.

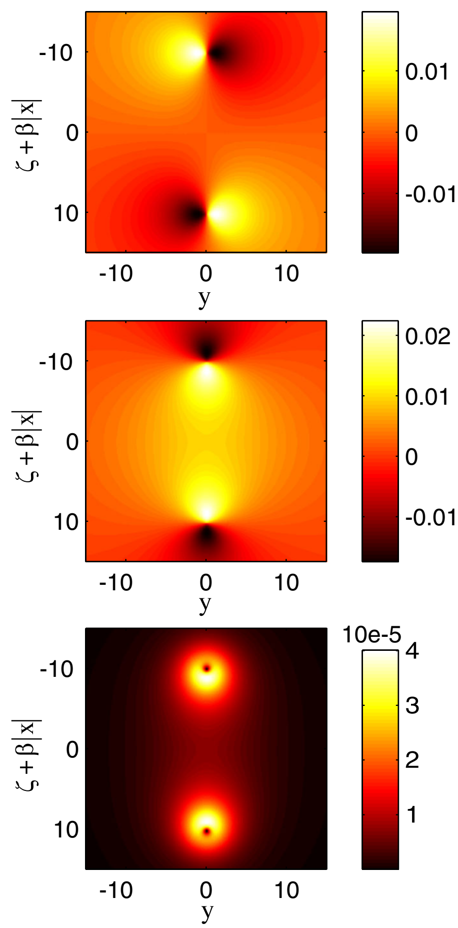

FIG. 4 (color online). "Snapshots" of E_y^W (top), E_z^W (middle), and the absolute value of the Poynting vector $\vec{S}^W$ (bottom) in the case of a bunch with $\sigma = 10c/\omega_p$ at $x = \mathrm{const}$; the energy flow density is given in units of $q^2\omega_p^4/(4\pi c^3)$, while the other units are the same as in Fig. 3.

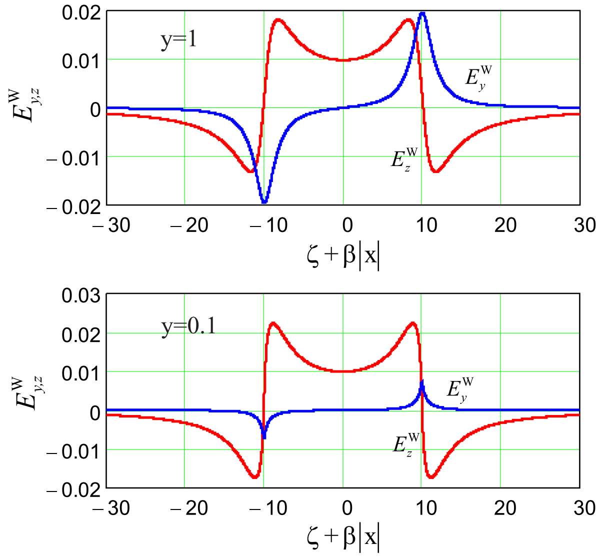

FIG. 5 (color online). Wave field components of a bunch as a function of $\zeta + \beta|x|$ for $\sigma = 10c/\omega_p$. The values of y and $\zeta + \beta|x|$ are given in units of c/ω_p, the field components in units of $q\omega_p^2/c^2$.

behavior of the electric field components as a function of ζ (note that the behavior as a function of y is similar).

We will now analyze the field of a bunch that has negligible thickness but some finite length. We consider a bunch with a homogeneous charge distribution along the z axis, where the charge density has the form $\rho = q\delta(x)\delta(y)/(2\sigma)$ for $|\zeta| < \sigma$, and $\rho = 0$ for $|\zeta| > \sigma$. The electromagnetic field of the bunch moving perpendicular to the wires has a convolution product that can be written in the following form:

$$\vec{E}(\vec{r}, t) = \frac{1}{2\sigma}\int_{-\sigma}^{+\sigma} \vec{E}_\delta(x, y, \zeta - \zeta', t)d\zeta', \qquad (13)$$

where $\vec{E}_\delta(x, y, \zeta, t)$ is the field of the point charge obtained above. We will now discuss some results of the computations of these integrals for the wave part of the field.

Figure 4 illustrates the intensities of the electric field components and the magnitude of the Poynting vector on the plane $x = \text{const}$. Components E_z^W and H_y^W each have a maximum at $\zeta = -\beta|x|$, $y = 0$. They decrease with an increase in $|y|$ more slowly than with an increase in $|\zeta + \beta|x||$. The components E_y^W and H_z^W are equal to zero at $\zeta = -\beta|x|$, $y = 0$ and have extremes in all four quadrants. The behavior of the absolute value of $\vec{S}^W$ is similar to the behavior of E_z^W because the latter is usually greater than E_y^W.

Figure 5 shows the behavior of electric wave field components as a function of $\zeta + \beta|x|$ for different values of y. One can see that the field variation distance is much more than the value of $c/\omega_p \sim d$ excluding some small vicinities of lines $\zeta + \beta|x| = \sigma$. This circumstance justifies the use of continuous medium model for the wire structure.

Figures 4 and 5 prove that the properties of the wave field can be used for measurement of the length and velocity of the particle bunch.

In conclusion, note the main properties of the Cherenkov radiation in the problem under consideration. We have shown that the radiation is nondivergent. It propagates along the wires and has two components of electric force: one is parallel to the charge velocity, and the other is perpendicular to it and to the wires. The magnetic force is orthogonal to the electric one and of equal modulo to it. The radiation field does not depend on the distance from the charge along certain rays behind the charge (if losses are negligible). Computations of the wave fields of bunches with finite length show that the structure under consideration can be used for bunch diagnostics.

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