## Dynamics and Dissipation Induced by Single-Electron Tunneling in Carbon Nanotube Nanoelectromechanical Systems

Marc Ganzhorn and Wolfgang Wernsdorfer

Institut Néel, CNRS & Université Joseph Fourier, BP 166, 25 Avenue des Martyrs, 38042 Grenoble Cedex 9, France (Received 13 November 2011; revised manuscript received 28 January 2012; published 24 April 2012)

We demonstrate the effect of single-electron tunneling (SET) through a carbon nanotube quantum dot on its nanomechanical motion. We find that the frequency response and the dissipation of the nanoelectromechanical system to SET strongly depends on the electronic environment of the quantum dot, in particular, on the total dot capacitance and the tunnel coupling to the metal contacts. Our findings suggest that one could achieve quality factors of  $10<sup>6</sup>$  or higher by choosing appropriate gate dielectrics and/or by improving the tunnel coupling to the leads.

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Carbon nanotubes (CNT) have become an essential building block for nanoelectromechanical systems (NEMS). Their low mass and high Young's modulus allow for instance ultrasensitive mass [\[1](#page-3-0)[–3\]](#page-3-1) or force detection [\[4,](#page-3-2)[5](#page-3-3)] (electric and magnetic) over a wide range of frequencies and its small diameter enables even single-molecule detection [[5](#page-3-3)[–7\]](#page-3-4). Moreover, CNT devices exhibit remarkable electronic transport properties, ranging from Kondo physics [[8](#page-3-5)] to Coulomb blockade at high temperature [\[9](#page-3-6)]. It was recently demonstrated that a CNT NEMS' nanomechanical motion at very low temperature (i.e., in Coulomb blockade regime) is strongly affected by the electronic transport through the CNT quantum dot (QD), and vice versa: For instance, single-electron tunneling (SET) caused a frequency softening and increased dissipation when tuning the CNT dot's potential through a Coulomb peak [\[10–](#page-3-7)[12](#page-3-8)].

Here we demonstrate that the response and dissipation of a CNT NEMS at very low temperature induced by zero bias SET through the CNT NEMS QD critically depends on the dot capacitance, the tunnel coupling to the metal leads, and temperature. We studied the frequency and dissipation response of nanomechanical bending modes to zero bias SET in suspended CNT devices with tunable tunnel couplings and different gate dielectrics, i.e., different dot capacitance. We observe that SET causes a frequency softening for small dot capacitance and/or tunnel coupling, whereas a frequency hardening or no frequency modulation is observed for large dot capacitance and/or tunnel coupling. We show that the dissipation of the CNT NEMS is mainly dominated by the capacitance, when electron tunneling through the dot is suppressed (i.e.. in the Coulomb valley), whereas it is limited by the mean tunneling rate  $\Gamma$  and the conductance, when electron tunneling through the dot is allowed (i.e., on a Coulomb peak). Finally we demonstrate that the tunnel current is the dominant dissipation mechanism in CNT NEMS at low temperature. Our findings are in fair agreement with a theoretical model provided previously [\[10](#page-3-7)[,12\]](#page-3-8).

Low capacitance CNT quantum dots ( $C_{dot} \approx 20{\text -}40 \text{ aF}$ ) were obtained by using silicon dioxide as gate dielectric [Fig. [1\(a\)](#page-0-0)]. First, source-drain electrodes are patterned by optical deep ultraviolet (DUV) lithography and  $e$ -beam evaporation of Mo (20 nm) and Pt (160 nm) on 500 nm of thermal  $SiO<sub>2</sub>$ . To ensure the suspension of the CNT, 150 nm of  $SiO<sub>2</sub>$  are dry etched in CHF<sub>3</sub> plasma. High capacitance CNT quantum dots ( $C_{dot} \approx 160-260$  aF) are obtained with high- $\kappa$  gate dielectrics Al<sub>2</sub>O<sub>3</sub> [Fig. [1\(b\)\]](#page-0-0). First, a 1  $\mu$ m-wide metallic local gate is patterned by optical DUV lithography and e-beam evaporation of Mo (20 nm) on 300 nm of thermal  $SiO<sub>2</sub>$ . A layer of 100 nm of  $Al_2O_3$  is then deposited by atomic layer deposition. Using optical DUV lithography and e-beam evaporation of Mo (20 nm) and Pt (160 nm), source-drain electrodes are aligned above the local gate. Suspended CNT are finally grown by chemical vapor deposition at 800 °C from a  $CH_4$ feedstock and Fe/Mo catalyst spots patterned on the



<span id="page-0-0"></span>FIG. 1 (color online). False color SEM image of (a) a low capacitance device based on a  $SiO<sub>2</sub>$  covered Si backgate (grey) and (b) a high capacitance device based on an  $Al_2O_3$  covered local metallic gate (blue). The rf actuation signal is injected into the local metallic gate and Si backgate, respectively, through a home-built bias-T. As the induced mechanical motion changes the charge flow through the CNT quantum dot and vice versa, we can detect the CNT resonance through a change in zero bias conductance. (c) Mechanical resonance of a typical CNT NEMS at low driving power  $P_{\text{rf}} = -100$  dBm. The resonance width  $\delta f = 2.5$  kHz leads to a quality factor of  $Q \approx 140000$ .

source-drain electrodes next to the junction. The CNT device length is approximately 800 nm  $(\pm 50 \text{ nm})$  and the dot capacitances are deduced from the dot's charging energy at low temperature. The spread in capacitance values of nominally identically fabricated devices is due to variations in the nanotubes' length and slack (see [\[13](#page-3-9)]).

The measurements are carried out in a  ${}^{3}$ He $/{}^{4}$ He dilution refrigerator with a base temperature of 30 mK. The NEMS actuation and detection scheme used in our experiment is similar to the one used by Steele and co-workers [\[11](#page-3-10)[,12\]](#page-3-8). The rf actuation signal is injected into the gate electrode via a home-built bias T. As the induced mechanical motion changes the charge flow through the CNT quantum dot and vice versa, we can detect the CNT resonance through a change in zero bias conductance. The actuation power is kept to a minimum ( $P_{\text{rf}} \approx -100$  dBm) in order to ensure a quasilinear regime of the oscillator and a high Q Lorentzian resonance shape [Fig. [1\(c\)\]](#page-0-0). All measurements were done under zero bias with a standard lock-in technique.

SET in a CNT NEMS QD can be considered an external perturbation to the CNT mechanical motion, and vice versa. This perturbation can be described as an electrodynamic force acting on the CNT. The contribution of this force, which is in phase with the mechanical motion, is responsible for a frequency modulation  $\Delta f$ , whereas a <span id="page-1-1"></span>contribution of the force being out of phase with mechanical motion induces a modification of the dissipation and the quality factor  $Q$ .  $\Delta f$  and  $Q$  can be expressed as follows [[10](#page-3-7)]:

$$
\Delta f = -\frac{f_0}{2} \frac{C_g^2}{k} \frac{V_g^2}{C_{dot}} \left(\frac{2G}{C_{dot}\Gamma} - 1\right),\tag{1}
$$

$$
\frac{1}{Q} = 2\pi f \frac{C_g^2}{k} V_g^2 \left(\frac{2}{\Gamma C_{\text{dot}}}\right)^2 G,\tag{2}
$$

<span id="page-1-0"></span>where  $\Gamma$  is the mean tunnel coupling of the dot to the leads, yielding  $\Gamma = (\Gamma_d + \Gamma_s)/2$ , and  $C_{dot}$  is the total dot capacitance. We can estimate the tunnel coupling  $\Gamma$  for each device from the line shape of the Coulomb blockade peaks (see [\[13\]](#page-3-9)). Equations [\(1](#page-1-0)) and [\(2](#page-1-1)) are valid in the regime of the Coulomb blockade and if  $\Gamma \gg f_0$ .

First, we compare the frequency response  $\Delta f$  (scaled with  $fV_g^2$ ) of devices with different dot capacitances  $C_{dot}$ . We studied the frequency response for comparable Coulomb peaks height, i.e., for comparable conductance  $G(V_g) \approx 20 \mu$ S. The tunnel coupling are on the order of 10 GHz. As depicted in Fig. [2\(a\),](#page-1-2) we observe a strong frequency softening for small dot capacitances, whereas for high dot capacitances a frequency hardening or no modulation is observed. As shown in Fig. [2\(b\),](#page-1-2) the



<span id="page-1-2"></span>FIG. 2 (color online). Modulation of the resonance frequency due to SET: (a) Frequency shift  $\Delta f$  (scaled with  $fV_g^2$ ) for different dot capacitance  $C_{dot}$ , with comparable tunneling rates  $\Gamma$ . Inset shows a typical Coulomb peak centered at  $\Delta V_g = 0$ . (b) Maximum scaled frequency shift at maximum tunnel current  $G_{\text{max}}(\Delta V_g = 0)$  vs the dot capacitance for all measured devices. The solid red line corresponds to the fit using Eq. [\(1\)](#page-1-0) with  $C_g^2/k = 8 \times 10^{-22} \text{ F}^2/\text{N m}$  and  $\Gamma_d \approx 10 \text{ GHz}$ . The data are in fair agreement with the model. (c) Scaled frequency shift for different Coulomb peaks, i.e., tunneling rates  $\Gamma$ , with a fixed dot capacitance  $C_{dot} = 260$  aF (device 1). The tunneling rate  $\Gamma$  is estimated from the line shape of the respective Coulomb peaks (see [\[13\]](#page-3-9)). (d) Maximum frequency shift  $\Delta f_{\text{max}}$ at maximum tunnel current  $G_{\text{max}}(\Delta V_g = 0)$  vs temperature for a device yielding  $C_{\text{dot}} = 160$  aF (device 2). The inset shows the evolution of  $G_{\text{max}}$  as a function of temperature.

experimental data are in fair agreement with the model. A fit with Eq. ([1\)](#page-1-0) yields fitting parameters  $C_g^2/k =$  $8 \times 10^{-22}$  F<sup>2</sup>/N m and  $\Gamma \approx 10$  GHz, which are in rather good agreement with previous experiments [\[10](#page-3-7)[,12](#page-3-8)[,14\]](#page-3-11). Figure [2\(c\)](#page-1-2) shows the (scaled) frequency modulation for different Coulomb peaks and tunneling rates  $\Gamma$  on a given device (device 1). It was shown previously that one can tune a CNT quantum dot from a Coulomb blockade regime to a regime of strong tunnel coupling by simply changing the CNT's gate potential [\[15](#page-3-12)[,16\]](#page-3-13). Hence, we can change the tunnel coupling  $\Gamma$  by tuning the gate voltage of our CNT device and estimate the values of  $\Gamma$  from the Coulomb peak line shape (see [[13](#page-3-9)]). The dot capacitance  $C_{dot}$  = 260 aF remains unchanged by tuning the gate voltage. As  $\Gamma$  increases, the CNT becomes stiffer, resulting in a sign change of  $\Delta f$  and the transition from a frequency softening to frequency hardening, in qualitative agreement with Eq.  $(1)$  $(1)$  [Fig.  $2(c)$ ].

Figure [3\(a\)](#page-2-0) depicts the effect of SET on the dissipation of the CNT NEMS, i.e., the Q factor (scaled with  $f(\hat{V}_g^2)$ , for devices with different dot capacitances. From Eq. [\(2](#page-1-1)), we expect an increase of the  $Q$  factor with increasing capacitance in the limit of suppressed SET through the dot. Indeed, in a region of suppressed SET ( $|\Delta V_g| \gg 0$ ), the Q factor is larger for devices with higher dot capacitance [Fig. [3\(a\)\]](#page-2-0). The tunnel coupling  $\Gamma \approx 10$  GHz and the conductance  $G(V_g) \approx 18 \mu \text{S}$  are comparable for both traces. Figure [3\(b\)](#page-2-0) shows the Q factor (scaled with  $fV_g^2$ ) in the limit of suppressed SET ( $|\Delta V_g| \gg 0$ ) as a function of the dot capacitance for all measured devices. The fit with Eq. ([2](#page-1-1)) yields  $C_g^2/k = 1 \times 10^{-21} \text{ F}^2/\text{Nm}$  and  $\Gamma \approx 10 \text{ GHz}$ , which is consistent with the fitting parameters of Fig. [2\(b\)](#page-1-2) and previous experiments [\[10,](#page-3-7)[12,](#page-3-8)[14\]](#page-3-11). Despite the good agreement between theory and experiment, we observe deviations from the model for some devices in Fig. [2\(b\)](#page-1-2) and [3\(b\)](#page-2-0) that we attribute to variations in  $C_g^2/k$  (see [\[13](#page-3-9)]). Moreover, we expect from Eq. ([2\)](#page-1-1) and previous experiments [\[10,](#page-3-7)[12\]](#page-3-8) an enhanced electromechanical dissipation in a given device if the conductance  $G$  (i.e., the tunnel current) and the tunnel resistance at the nanotube-electrode interface  $R \sim 1/\Gamma_d$  are increased, in analogy to the current dissipation in a simple resistance. Indeed we observe increased dissipation (decreased  $Q$  factor) when tuning the gate voltage through a Coulomb peak [Figs. [3\(a\)](#page-2-0) and [3\(c\)\]](#page-2-0). In Fig. [3\(c\)](#page-2-0) (device 2), this effect becomes more pronounced as we move to gate voltage regions with a smaller tunneling rate  $\Gamma$ , i.e., larger tunnel resistance at the nanotube-electrode interface. The dot capacitance  $C_{dot}$ yields 160 aF for this device and the height of the Coulomb peak is comparable for all traces ( $\tilde{G} \approx 7 \mu$ S).



<span id="page-2-0"></span>FIG. 3 (color online). Modulation of the Q factor (scaled with  $fV_g^2$ ) due to SET: (a) For different dot capacitances, with comparable tunneling rates  $\Gamma$ . The solid lines are to guide the eye and the inset shows a representative Coulomb peak centered at  $\Delta V_g = 0$ . (b) Maximum quality factor  $Q_{\text{max}}$  at minimal tunnel current  $G(|\Delta V_g| \gg 0)$  vs the dot's capacitance for all measured devices. The solid red line represents the fit with Eq. [\(2\)](#page-1-1) with  $C_g^2/k = 1 \times 10^{-21} \text{ F}^2/\text{N m}$  and  $\Gamma \approx 10 \text{ GHz}$ . Our findings are in good agreement with the model. (c) Modulation of the Q factor (scaled with  $fV_g^2$ ) for different tunneling rates  $\Gamma$ , with a fixed dot capacitance  $C_{dot} = 160$  aF (device 2). The solid lines are to guide the eye and the tunnel coupling  $\Gamma$  is estimated from the line shape of the respective Coulomb peaks (see [[13](#page-3-9)]). (d) Maximum quality factor  $Q_{\text{max}}$  at minimal tunnel current  $G(\Delta V_g | \gg 0)$  vs temperature for a device yielding  $C_{dot} = 160$  aF (device 2). The inset shows the evolution of  $G_{min}$  as a function of temperature.

Therefore, the CNT capacitance is the limiting factor for the dissipation in regions of suppressed SET, whereas the mean tunnel coupling and the tunneling current itself define the dissipation in regions of strong SET. We conclude that the electron transport through the CNT is the main dissipation mechanism in our nanoelectromechanical system.

Finally, we study the temperature dependence of the frequency response and the dissipation. Figure [2\(d\)](#page-1-2) depicts the frequency shift  $\Delta f_{\text{max}}$  for  $G_{\text{max}} = G(\Delta V_g = 0)$ whereas Fig. [3\(d\)](#page-2-0) shows the quality factor  $Q_{\text{max}}$  for  $G_{\text{min}} =$  $G(|\Delta V_g| \gg 0)$  as a function of temperature. A fit of the data yields  $\Delta f_{\text{max}} \sim T$  and  $Q_{\text{max}} \sim 1/T$ . As temperature increases the Coulomb blockade peak broadens and be-comes smaller (see[\[13\]](#page-3-9), Fig. S3). As a result  $G_{\text{max}} =$  $G(\Delta V_g = 0)$  decreases whereas  $G_{\text{min}} = G(|\Delta V_g| \gg 0)$  increases with temperature, as depicted in the insets of Figs.  $2(d)$  and  $3(d)$ . From the data fit we obtain  $G_{\text{max}} \sim T$  and  $G_{\text{min}} \sim T$ . It was previously demonstrated that carbon nanotubes show Luttinger liquid behavior [\[17–](#page-3-14)[19\]](#page-3-15), where the conductance follows a characteristic power law dependence as a function of temperature  $G(T) \sim T^{\alpha}$ ,  $\alpha$  being related to the Luttinger parameter g by  $\alpha = (g - 1 + g - 2)/2$  [[17](#page-3-14),[18](#page-3-16)]. We find  $\alpha \approx 1$  and a Luttinger parameter of  $g = 0,1$ , which is close to previous measurements [\[17\]](#page-3-14) and theoretical predictions [\[18](#page-3-16)[,19\]](#page-3-15). Comparing the temperature dependence of the conductance, the quality factor, and the frequency shift, we finally obtain  $\Delta f_{\text{max}} \sim -G_{\text{max}}$  and  $Q_{\text{max}} \sim 1/G_{\text{min}}$ , which is in full agreement with Eqs. ([1](#page-1-0)) and [\(2\)](#page-1-1). We can conclude that the current is the dominant dissipation mechanism for carbon nanotube-based NEMS in a Coulomb blockade regime at cryogenic temperatures.

We have demonstrated that the response and the dissipation to single-electron tunneling of carbon nanotube NEMS at low temperature depends on the dot capacitance and the tunnel coupling to the leads, the tunnel current itself being the main dissipation mechanism in the system. By choosing high- $\kappa$  dielectrics (HfO<sub>2</sub> or ZrO<sub>2</sub>) and improving the tunnel contact to the metal leads, one could significantly enhance the quality factor  $Q$  to values exceeding  $10<sup>6</sup>$ . It was proposed in theoretical calculation that one can use such high -Q CNT NEMS as magnetic torque or force detectors for nanoparticles [[5\]](#page-3-3) or single-molecule magnets [[20](#page-3-17),[21](#page-3-18)] grafted to the CNT NEMS. In principle, one can achieve a sensitivity of one  $\mu_B$  at low temperature, whereas the best magnetometers, for instance the micro-SQUID, only have a sensitivity of  $10^3$   $\mu_B$  [[22](#page-3-19)]. Finally, such molecular quantum spintronic device would allow single spin manipulation on a molecular level.

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