Only Above Barrier Energy Components Contribute to Barrier Traversal Time

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A time of arrival operator across a square potential barrier is constructed. The expectation value of the barrier time of arrival operator for a sufficiently localized incident wave packet is compared with the expectation value of the free particle time of arrival operator for the same wave packet. The comparison yields an expression for the expected traversal time across the barrier. It is shown that only the above barrier components of the momentum distribution of the incident wave packet contribute to the barrier traversal time, implying that below the barrier components are transmitted without delay. This is consistent with the recent experiment in attosecond ionization in helium indicating that there is no real tunneling delay time [P. Eckle *et al.*, Science **322**, 1525 (2008)].

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Diverse contrasting theories have been offered to compute tunneling traversal time [1,2] since the problem was raised in the early days of quantum mechanics [3]. This diversity has instigated endless controversy on the nature of quantum tunneling time [1,4]. But amidst the controversy is the consensus that tunneling does not occur instantaneously, so that there is presumably a measurable delay in the transmission of the tunneling particle. However, a recent landmark experiment by Keller's group in attosecond ionization in helium "give[s] a strong indication that there is no real tunneling delay time" [5], ruling out in the process the predicted tunneling time due to Keldysh [6]. The experiment then posits a challenge to the presumed nonzero tunneling traversal time, imploring for a new theoretical treatment consistent with zero delay time. This Letter offers such a treatment.

In this Letter, we investigate quantum traversal time across arbitrary continuous potential barriers under the hypothesis that we can meaningfully construct a time of arrival operator T corresponding to an arrival at some point x in the configuration space for a given interaction potential $V(q)$. Our theory models the situation where at time $t = 0$ a wave packet $\psi(q)$ is launched in the presence of $V(q)$ toward a detector located at x to indicate arrival of the particle there. We hypothesize that the average time elapsed between the launching of the wave packet and a successful registration of the particle at the detector is given by the expectation value $\langle\psi|\mathsf{T}|\psi\rangle$, where T is the time of arrival (TOA) operator corresponding to $V(q)$. We will show that only the above barrier components of the momentum distribution of the incident wave packet contribute to any measurable barrier traversal time and that below the barrier components are transmitted without delay.

Foremost, let us exemplify the measurements that serve to define the barrier traversal time: A detector D_T to announce the arrival of a particle is located at the origin. (The nature of the detector will generally depend on the projectile; for example, for neutrons, the detector must provide the necessary nuclear reaction to "turn" the neutron into charge particles that will be detected directly by means, say, of a scintillation detector.) A similar detector D_R is located at the far left of D_T. A potential barrier $V(q)$ with length L is placed between D_T and D_R. A localized wave packet $\psi(q)$ is prepared between D_R and $V(q)$ and launched at $t = 0$ towards the barrier. The time of arrival is recorded when D_T clicks; otherwise, no data are collected when D_R clicks. This is repeated a large number of times, with $\psi(q)$ as the initial state for every repeat, and the average time of arrival $\bar{\tau}_B$ at D_T is computed. The same experiment is performed without the barrier, and the average free time of arrival $\bar{\tau}_F$ at D_T is computed from the new time of arrival data. The expected traversal time across the barrier is deduced from the difference $\Delta\tau = \bar{\tau}_F - \bar{\tau}_B$.

It is the objective of this Letter to obtain a theoretical prediction for $\Delta\tau$ and from this quantity obtain the tunneling traversal time. The treatment starts from constructing the TOA operator in the presence of the barrier, T_B, and the time of arrival operator in the absence of the barrier, T_F. The operator T_F is the free TOA operator, which is the quantization of the classical time of arrival [7–9]; the operator T_B is to be constructed here for the first time by quantization, using the theory of quantum time of arrival in the presence of an interaction potential we have developed elsewhere [10,11]. We then make the identifications $\bar{\tau}_B = \langle\psi|\mathsf{T}_B|\psi\rangle$ and $\bar{\tau}_F = \langle\psi|\mathsf{T}_F|\psi\rangle$. These identifications are justified on the grounds that the above described experiments reduce to simple classical time of flight measurements when the wave packet is replaced by a classical particle and that the expectation values $\langle\psi|\mathsf{T}_B|\psi\rangle$ and $\langle\psi|\mathsf{T}_F|\psi\rangle$ give the correct classical values (where the classical TOA exists) in the limit as $\hbar$ approaches zero. For the free particle case, this has been established in Ref. [12], and, for the barrier case, it will be established in the development to follow.

Despite earlier claims that the classical time of arrival cannot be quantized in the presence of an interacting potential, it is shown in Ref. [10] that it can be quantized, at least, for analytic potentials. In coordinate representation, the quantized TOA operator for arrival at the origin is the integral operator

$$(\mathsf{T}_0\varphi)(q) = \int_{-\infty}^{\infty} \frac{\mu}{i\hbar} T_0(q, q')\mathrm{sgn}(q - q')\varphi(q')dq', \quad (1)$$

where $\mathrm{sgn}(x)$ is the sign function and

$$T_0(q, q') = \frac{1}{2}\int_0^{\eta} ds\, {}_0F_1\left(;1;\frac{\mu}{2\hbar^2}\zeta^2\{V(\eta) - V(s)\}\right) \quad (2)$$

in which ${}_0F_1$ is a specific hypergeometric function, $\zeta = (q - q')$, and $\eta = (q + q')/2$. Numerical simulations done in Refs. [8,11] identify T_0 as a first time of arrival operator. First time of arrival distributions can be extracted from the operator T_0 by means of successive coarse grainings [9,13]. Readers are referred to Refs. [13,14] for full accounts of the theory of time of arrival operators. (See [15] for a distinct treatment of the TOA problem in the interacting case.)

Now the quantized free particle TOA operator for arrival at the origin is obtained from Eq. (2) by substituting $V(q) = 0$. We obtain $T_F(q, q') = (q + q')/4$, substitution of which back into Eq. (1) gives the free TOA operator. This result can be reached by a direct symmetric quantization of the classical free time of arrival $t = -\mu q/p$. Quantization yields $\mathsf{T}_F = -(\mu/2)(\mathsf{q}\mathsf{p}^{-1} + \mathsf{p}^{-1}\mathsf{q})$ [7]. In coordinate representation, T_F is the integral operator $(\mathsf{T}_F\psi)(q) = \int_{-\infty}^{\infty}\langle q|\mathsf{T}_F|q'\rangle\psi(q')dq'$, where the kernel is given by $\langle q|\mathsf{T}_F|q'\rangle = -(\mu/2)(q + a')(1/2\pi\hbar)\int_{-\infty}^{\infty} p^{-1}\times e^{i(q-q')p/\hbar}dp$. Using the identity $\int_{-\infty}^{\infty} x^{-1}e^{i\sigma x}dx = i\pi\mathrm{sgn}(\sigma)$ ([16], p. 360, no. 19) leads to Eq. (1) for the free case.

Now Eq. (1) has been derived under the assumption that the interaction potential is analytic. We assume for the moment that it extends to piecewise continuous potential by appropriate subdivision of the integral and apply it to a square potential barrier to construct the corresponding time of arrival operator. Let us place the barrier to the left of the arrival point, which is the origin. With $a < b < 0$ the potential is $V(q) = V_0 > 0$ for $a < q < b$ and zero elsewhere. We change variables from (q, q') to (η, ζ), where $\eta = (q + q')/2$ and $\zeta = (q - q')$, so that $T_0(q, q') = \tilde{T}_0(\eta, \zeta)$. Then we can directly use Eq. (2) to obtain $\tilde{T}_0(\eta, \zeta)$. In the η coordinate the potential becomes $V(\eta) = V_0 > 0$ for $a < \eta < b$ and zero elsewhere.

We now obtain $\tilde{T}_0(\eta, \zeta)$. For $\eta > b$ we have $V(\eta) = 0$, and in the entire region of integration from 0 to η we have $V(\eta') = 0$. Then Eq. (2) yields $\tilde{T}_{0,1}(\eta, \zeta) = \frac{\eta}{2}$, for $\eta > b$. For $a \leq \eta \leq b$ we have $V(\eta) = V_0$. We have to divide the integration in two parts: $b < \eta' < 0$, where $V(\eta') = 0$, and $\eta < \eta' < b$, where $V(\eta') = V_0$. Then $\tilde{T}_{0,2}(\eta, \zeta) = \frac{\eta}{2} + \frac{b}{2}[I_0(\kappa_0|\zeta|) - 1]$ for $a \leq \eta \leq b$, where $I_0(x)$ is a modified Bessel function and $\kappa_0 = \sqrt{2\mu V_0}/\hbar$. For $\eta < a$ we have $V(q) = 0$. We have to divide the integration in three parts: $b < \eta' < 0$, where $V(\eta') = 0$, $a \leq \eta \leq b$, where $V(\eta') = V_0$, and $\eta < 0$, where $V(\eta') = 0$. Then $\tilde{T}_{0,3}(\eta, \zeta) = \frac{\eta}{2} + \frac{(a-b)}{2}[J_0(\kappa_0|\zeta|) - 1]$ for $\eta < a$, where $J_0(x)$ is a Bessel function of the first kind.

To prove that Eq. (1) gives a quantization of the classical time of arrival across the barrier, we now show that the constructed TOA operator in the presence of the barrier gives the correct classical limit. The limit is obtained by taking the inverse Weyl-Wigner transform of the kernel $\langle q|\mathsf{T}_0|q'\rangle$, and it is given by $t_0 = \int_{-\infty}^{\infty}\langle q_0 + \frac{v}{2}|\mathsf{T}_0|q_0 - \frac{v}{2}\rangle\times e^{-ip_0v/\hbar}dv$, where q_0 and p_0 are now the respective classical position and momentum at $t = 0$. With the kernel $\langle q|\mathsf{T}_0|q'\rangle = \frac{\mu}{i\hbar}T_0(q, q')\mathrm{sgn}(q - q')$, we have $t_0 = \int_{-\infty}^{\infty}\tilde{T}(q_0, v)\mathrm{sgn}(v)e^{-ip_0v/\hbar}dv$, where the integral is understood in the distributional sense. We will need the identity $\int_{-\infty}^{\infty} v^{m-1}\mathrm{sgn}(v)e^{-ixv}dv = 2(m-1)!i^{-m}x^{-m}$ (the inverse Fourier transform of Ref. [16], p. 360, no. 18) to obtain the classical limit.

For $q_0 > b$ the limit is obtained by substituting $\tilde{T}_{0,1}(q_0, v)$ for $\tilde{T}_0(q_0, v)$ in t_0; the result is $t_0 = -\mu q_0/p_0$. For $a < q_0 < b$ we use $\tilde{T}_{0,2}(q_0, v)$ and expand $I_0(\kappa v)$ in power series. Integrating the resulting expression term by term and simplifying yields $t_0 = -\mu(q_0 - b)/p_0 - \mu b/p_0\sqrt{1 + 2\mu V_0/p_0^2}$ provided $2\mu V_0/p_0^2 < 1$; the first term is the traversal time from the b edge of the barrier to the origin, and the second term is the traversal time on top of the barrier. For $q_0 < a$ we use $\tilde{T}_{0,3}(q_0, v)$ and expand $J_0(\kappa v)$ in power series. The same procedure leads to the limit $t_0 = -\mu(q_o + L)/p_0 + \mu L/p_0\sqrt{1 - 2\mu V_0/p_0^2}$ provided $2\mu V_0/p_0^2 < 1$; the first term is the traversal time across the interaction free region, and the second term is the traversal time across the barrier. Then T_B reduces to the correct classical TOA expression in the classical limit, hence, a quantization of the classical TOA.

Another important property of T_B is its conjugacy with the free Hamiltonian H_F inside and outside of the barrier. That is, $\langle\phi|[\mathsf{T}_F, \mathsf{H}_F]|\varphi\rangle = i\hbar\langle\phi|\varphi\rangle$ for all $\phi(q)$ and $\varphi(q)$ with compact supports to the left of the barrier, similarly to the right and inside of the barrier. This follows from the fact that the $T_{0,k}(q, q')$'s satisfy the required partial differential equation for a TOA operator to be conjugate with its Hamiltonian [10]; that is, $-\partial_q^2 T_{0,k}(q, q') + \partial_{q'}^2 T_{0,k}(q, q') = 0$ for all $k = 1, 2, 3$. Then $T_{0,3}(q, q')$ gives the TOA operator for a wave packet incident from the left of the barrier.

Now the expected time of arrival for an initial incident wave packet $\psi(q)$ for a TOA operator T is given by

$$\langle\psi|\mathsf{T}|\psi\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{\psi}(q)\psi(q')\frac{\mu}{i\hbar}T(q, q')\mathrm{sgn}(q - q')dq'dq. \quad (3)$$

Let the initial wave packet $\psi(q)$ be incident with the momentum expectation value $\hbar k_0$ or with the group velocity

$\hbar k_0/m$. Such a wave packet can be written in the form $\psi(q) = \varphi(q)e^{ik_0 q}$, where $\varphi(q)$ satisfies $\int_{-\infty}^{\infty}\bar{\varphi}(q)\varphi'(q)\times dq = 0$. By changing variables to $\zeta = (q - q')$ and $\eta = (q + q')/2$, the expectation value assumes the form $\langle\psi|\mathsf{T}|\psi\rangle = \mathrm{Im}\tau^*$, where

$$\tau^* = -\frac{2\mu}{\hbar}\int_0^\infty\int_{-\infty}^\infty \tilde{T}(\eta,\zeta)\bar{\varphi}\left(\eta - \frac{\zeta}{2}\right)\varphi\left(\eta + \frac{\zeta}{2}\right)e^{i\zeta k_0}d\eta d\zeta \tag{4}$$

It will be convenient for us to work with the complex-expected TOA τ^* and related complex valued quantities introduced below (indicated by an asterisk *), their imaginary parts yielding their corresponding physical quantities.

We now assume that our incident wave packet is infinitely differentiable and with support to the left of the barrier. In the absence of the barrier, the complex-expected TOA is given by

$$\tau_F^* = -\frac{\mu}{\hbar}\int_0^\infty\int_{-\infty}^\infty e^{ik_0\zeta}\bar{\varphi}\left(\eta - \frac{\zeta}{2}\right)\varphi\left(\eta + \frac{\zeta}{2}\right)\eta d\eta d\zeta, \tag{5}$$

where we substituted the free kernel $\tilde{T}_F(\eta,\zeta) = \eta/2$ back into Eq. (4) to obtain this expression. In the presence of the barrier, the complex-expected TOA is given by

$$\tau_B^* = -\frac{\mu}{\hbar}\int_0^\infty\int_{-\infty}^\infty e^{ik_0\zeta}\bar{\varphi}\left(\eta - \frac{\zeta}{2}\right)\varphi\left(\eta + \frac{\zeta}{2}\right)(\eta + L)d\eta d\zeta + \frac{\mu L}{\hbar}\int_0^\infty\int_{-\infty}^\infty e^{ik_0\zeta}\bar{\varphi}\left(\eta - \frac{\zeta}{2}\right)\bar{\varphi}\left(\eta + \frac{\zeta}{2}\right)J_0(\kappa\zeta)d\eta d\zeta, \tag{6}$$

where the contribution comes from $\tilde{T}_{0,3}(\eta,\zeta)$ only because the support of $\varphi(q)$ does not extend inside and to the right of the barrier.

The directly measurable quantity of the theory is the time of arrival difference

$$\Delta\tau = \langle\psi|\mathsf{T}_F|\psi\rangle - \langle\psi|\mathsf{T}_B|\psi\rangle. \tag{7}$$

In terms of the complex quantities τ_B^* and τ_F^*, this difference reduces to $\Delta\tau = \mathrm{Im}(\Delta\tau^*) = \mathrm{Im}(\tau_F^* - \tau_B^*)$. Let $\Phi(\zeta) = \int_{-\infty}^{\infty}\bar{\varphi}(\eta - \frac{\zeta}{2})\varphi(\eta + \frac{\zeta}{2})d\eta$. Then we have $\Delta\tau^* = (L/v_0)Q^* - (L/v_0)R^*$, where $Q^* = k_0\int_0^\infty e^{ik_0\zeta}\Phi(\zeta)d\zeta$ and $R^* = k_0\int_0^\infty e^{ik_0\zeta}\Phi(\zeta)J_0(\kappa\zeta)d\zeta$, or $\Delta\tau^* = (L/v_0)\times(Q^* - R^*)$. To understand the underlying physical contents of the quantities $(L/v_0)Q$ and $(L/v_0)R$, where $Q = \mathrm{Im}(Q^*)$ and $R = \mathrm{Im}(R^*)$, we investigate their respective classical limits by taking the high energy $k_0 \to \infty$ limit for fixed κ_0, followed by the substitutions $k_0 = \sqrt{2\mu E_0}/\hbar$ and $\kappa_0 = \sqrt{2\mu V_0}/\hbar$.

Since Q^* is a Fourier integral with respect to the asymptotic parameter k_0, it is straightforward to establish by integration by parts the asymptotic relation

$$Q^* \sim i\sum_{n=0}^\infty \frac{1}{k_0^{2n}}\Phi_1^{(n)} - \sum_{n=0}^\infty\frac{1}{k_0^{2n+1}}\Phi_2^{(n)}, \qquad k_0 \to \infty, \tag{8}$$

where $\Phi_1^{(n)} = (-1)^n\Phi^{(2n)}(0) = \int_{-\infty}^{\infty}|\varphi^{(n)}(\eta)|^2 d\eta$ and $\Phi_2^{(n)} = (-1)^n\Phi^{(2n+1)}(0) = \int_{-\infty}^{\infty}\bar{\varphi}^{(n)}(\eta)\varphi^{(n+1)}(\eta)d\eta$. Substituting $k_0 = \sqrt{2\mu E_0}/\hbar$ and taking the imaginary part yield the asymptotic value

$$Q \sim \sum_{n=0}^\infty\frac{\hbar^{2n}}{(2\mu E_0)^n}\Phi_1^{(n)}, \qquad \hbar \to 0. \tag{9}$$

The normalization condition $\int_{-\infty}^{\infty}\bar{\varphi}(\eta)\varphi(\eta)d\eta = 1$ gives $Q \sim 1$ or $(L/v_0)Q \sim (L/v_0)$ in the classical limit, which is the classical traversal time of a free particle with velocity v_0 across the barrier length L. Hence the quantity $\tau_F = (L/v_0)Q$ lends to the interpretation as the expected quantum traversal time for the free particle across the barrier length.

Now R^* is likewise a Fourier integral with respect to k_0, so that its asymptotic expansion can also be found by repeated integration by parts. The result is

$$R^* \sim i\sum_{j=0}^\infty\frac{1}{k_0^{2j}}\sum_{m=0}^j\binom{2j}{2m}\frac{1}{2^{2m}}\binom{2m}{m}\kappa_0^{2m}\Phi_1^{(j-m)} - \sum_{j=0}^\infty\frac{1}{k_0^{2j+1}}\sum_{m=0}^j\frac{1}{2^{2m}}\binom{2j+1}{2m}\binom{2m}{m}\kappa_0^{2m}\Phi_2^{(j-m)}, \tag{10}$$

as $k_0 \to \infty$. Substituting $\kappa_0 = \sqrt{2\mu V_0}/\hbar$ and $k_0 = \sqrt{2\mu E_0}/\hbar$ back into (10), collecting equal powers of $\hbar$, and taking the imaginary part give

$$R \sim \sum_{m,j=0}^\infty\frac{\hbar^{2j}\Phi_1^{(j)}}{(2\mu E_0)^j 2^{2m}}\binom{2j+2m}{2m}\binom{2m}{m}\left(\frac{V_0}{E_0}\right)^m, \tag{11}$$

as $\hbar \to 0$.

Only the $j = 0$ term in the first term survives in the classical limit and is given by

$$R \sim \sum_{m=0}^\infty\frac{1}{2^{2m}}\binom{2m}{m}\left(\frac{V_0}{E_0}\right)^m = \sqrt{\frac{E_0}{E_0 - V_0}}, \tag{12}$$

which is just the ratio of the free velocity v_0 and the velocity on top of the barrier v_0', i.e., $R \sim v_0/v_0'$ as $\hbar \to 0$. Then $(L/v_0)R \sim L/v_0'$ in the classical limit, which is the classical traversal time across the barrier. Hence the quantity $\tau_B = (L/v_0)R$ lends to the interpretation as the quantum traversal time across the barrier. In analogy with optics, the classical limit $R \sim v_0/v_0'$ identifies the quantity R as the effective index of refraction of the barrier with respect to the incident wave packet, R being the ratio of the "reference speed" v_0 to the "phase speed" v_0' in the "medium."

We now establish that the expected quantum traversal time across the barrier comes from the above barrier components of the momentum distribution. We rewrite the index of refraction by introducing the Fourier transform $\varphi(q) = (2\pi)^{-1/2}\int_{-\infty}^{\infty}\phi(\tilde{k})e^{i\tilde{k}q}d\tilde{k}$ back into the definition of $\Phi(\zeta)$. Direct substitution yields $\Phi(\zeta) = \int_{-\infty}^{\infty} |\phi(\tilde{k})|^2 e^{i\zeta\tilde{k}} d\tilde{k}$. We substitute this expression for $\Phi(\zeta)$ back into R^*, interchange the order of integrations, and change variable to $\tilde{k} = k - k_0$ to arrive at the expression $R^* = k_0 \int_0^{\infty} dk |\phi(k-k_0)|^2 \int_0^{\infty} d\zeta e^{ik\zeta} J_0(\kappa_0 \zeta)$. One readily identifies $\phi(k-k_0)$ as the Fourier transform of the full incident wave function $\psi(q) = e^{ik_0 q}\varphi(q)$; that is, $\tilde{\psi}(k) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} e^{-ikq} \psi(q) dq = \phi(k-k_0)$. Only the imaginary part of the integral $\int_0^{\infty} d\zeta e^{ik\zeta} J_0(\kappa_0\zeta)$ is relevant and is evaluated by using the integral identity $\int_0^{\infty} J_0(ax)\sin(bx)dx = H(b-a)/\sqrt{b^2-a^2}$ ([17], p. 718, no. 7), where $H(x)$ is the Heaviside step function. Then on taking the imaginary part of R^* we have

$$R = k_0 \int_{\kappa_0}^{\infty} \frac{|\tilde{\psi}(k)|^2}{\sqrt{k^2-\kappa_0^2}} dk - k_0 \int_{\kappa_0}^{\infty} \frac{|\tilde{\psi}(-k)|^2}{\sqrt{k^2-\kappa_0^2}} dk. \quad (13)$$

Clearly, Eq. (13) shows that only those components of $\tilde{\psi}(k)$ with $|k| > \kappa_0$ contribute to any measurable traversal time across the barrier. When the support of the momentum distribution of the incident wave packet has a corresponding energy distribution that lies below the potential height, the index of refraction is zero and the traversal time under the barrier vanishes. This inevitably leads to the conclusion that below the barrier energy components are transmitted without delay across the barrier—that is, quantum tunneling, whenever it occurs, happens instantaneously. (This is reminiscent of Ref. [18], where it was shown that it does not make sense to ascribe a velocity to a tunneling electron.)

Equation (13) delineates the contributions of the positive and negative momentum components. For arrivals at the transmission channel, only the positive components are relevant. For this case the measurable traversal time is given by

$$\tau_{\text{trav}} = \frac{L\mu}{\hbar} \int_{\kappa_0}^{\infty} \frac{|\tilde{\psi}(k)|^2}{\sqrt{k^2-\kappa_0^2}} dk, \qquad \kappa_0 > 0. \quad (14)$$

Now $\hbar\sqrt{k^2-\kappa_0^2}/\mu$ is the velocity $v(k)$ on top of the barrier, so that $L/v(k)$ is the traversal time $\tau(k)$ across the barrier for a given k. Then Eq. (14) reduces to $\int_{\kappa_0}^{\infty} \tau(k)|\tilde{\psi}(k)|^2 dk$, which means that τ_{trav} is the weighted sum of all classical above barrier traversal times with weights $|\tilde{\psi}(k)|^2$.

Equation (14) can be written as an expectation value of a traversal time operator by substituting the Fourier transform $\tilde{\psi}(k) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} e^{-ikq} \psi(q) dq$. Reversing the order of integrations yields the expression

$$\tau_{\text{trav}} = -\frac{L\mu}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi^*(q) Y_0(\kappa_0|q-q'|)\psi(q') dq' dq, \quad (15)$$

where $Y_0(x)$ is a Bessel function of the second kind; Eq. (15) is obtained by using the known Fourier transform $\int_{-\infty}^{\infty} e^{ix\sigma}(x^2-1)_+^{\lambda} dx = -\Gamma(\lambda+1)\sqrt{\pi}|\sigma/2|^{-\lambda-1/2} \times Y_{-\lambda-1/2}(|\sigma|)$ for $\lambda \neq -1, -2, \ldots$ ([16], p. 363, no. 41). By inspection of Eq. (15), we find that τ_{trav} is the expectation value of the operator T_{trav} with kernel $\langle q|\mathsf{T}_{\text{trav}}|q'\rangle = -(L\mu/2)Y_0(\kappa_0|q-q'|)$ in coordinate representation. Since Eq. (14) follows from the assumption that the incident wave packet has support to the left of the barrier, the kernel $\langle q|\mathsf{T}_{\text{trav}}|q'\rangle$ corresponds to $\tilde{T}_{0,3}$.

The above conclusions hold in general. Let the initial wave packet be incident upon an arbitrary potential barrier $V_0(q)$ in the interval $[a, b]$, where $V_0(q)$ is positive definite and continuous in the entire barrier length. Again only the piece of the kernel in the region $\eta < a$ contributes in the expected traversal time. Similar calculation yields $\tilde{T}(\eta, \zeta) = \frac{1}{2}(\eta + L) - \frac{L}{2}J(\zeta)$ for $\eta < a$, where $J(\zeta) = \frac{1}{L}\int_a^b J_0(|\zeta|\kappa(s))ds$ and $\kappa(s) = \sqrt{2\mu V_0(s)}/\hbar$. The continuity of $V_0(q)$ in the interval $[a, b]$ implies the existence of a unique point s_0 in $[a, b]$ such that $J(\zeta) = J_0(|\zeta|\kappa(s_0))$, in accordance with the mean value theorem. Then the barrier is equivalent to the square barrier with height $\kappa_0 = \kappa(s_0) > 0$.

Finally, while Keller's experiment involves a time-dependent potential barrier, the character of their experiment reduces to the simple model of the present Letter: A quantum particle is prepared in one side of a potential barrier and is detected at the other side, and the time it takes for the particle to tunnel through the barrier is measured. Since the experiment has been controlled to ensure that the energy of the tunneling electron is well below the potential barrier, their result of a zero delay time within experimental accuracy is consistent with the main result of this Letter that below the barrier energy components are transmitted instantaneously.

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[1] E. H. Hauge and J. A. Stovneng, Rev. Mod. Phys. **61**, 917 (1989); R. Landauer and Th. Martin, Rev. Mod. Phys. **66**, 217 (1994); A. M. Steinberg, Superlattices Microstruct. **23**, 823 (1998); V. S. Olkhovsky, E. Recami, and J. Jakiel, Phys. Rep. **398**, 133 (2004).
[2] E. Recami, J. Mod. Opt. **51**, 913 (2004); P. Krekora, Q. Su, and R. Grobe, Phys. Rev. A **64**, 022105 (2001); A. Peres, Am. J. Phys. **48**, 552 (1980); M. Buttiker, Phys. Rev. B **27**, 6178 (1983); P. C. w. Davies, Am. J. Phys. **73**, 23 (2005).
[3] L. A. MacColl, Phys. Rev. **40**, 621 (1932); T. E. Hartman, J. Appl. Phys. **33**, 3427 (1962).
[4] G. Nimtz, Prog. Quantum Electron. **27**, 417 (2003); H. G. Winful, Phys. Rep. **436**, 1 (2006).
[5] P. Eckle, A. N. Pfeiffer, C. Cirelli, A. Staudte, R. Dörner, H. G. Muller, M. Büttiker, and U. Keller, Science **322**, 1525 (2008).
[6] L. V. Keldysh, Sov. Phys. JETP **20**, 1307 (1965).

[7] J. G. Muga and C. R. Leavens, Phys. Rep. **338**, 353 (2000); I. L. Egusquiza and J. G. Muga, Phys. Rev. A **61**, 012104 (1999).
[8] E. A. Galapon, R. Caballar, and R. T. Bahague, Phys. Rev. Lett. **93**, 180406 (2004); Phys. Rev. A **72**, 062107 (2005).
[9] E. A. Galapon, F. Delgado, J. G. Muga, and I. Egusquiza, Phys. Rev. A **72**, 042107 (2005).
[10] E. A. Galapon, J. Math. Phys. (N.Y.) **45**, 3180 (2004).
[11] E. A. Galapon, Int. J. Mod. Phys. A **21**, 6351 (2006).
[12] E. A. Galapon, Phys. Rev. A **80**, 030102(R) (2009).
[13] E. A. Galapon, in *Time in Quantum Mechanics II*, edited by J. G. Muga, A. Ruschhaupt, and A. Del Campo, p. 25–63 (2009); Proc. R. Soc. A **465**, 71 (2009).
[14] E. A. Galapon and A. Villanueva, J. Phys. A **41**, 455302 (2008).
[15] G. C. Hegerfeldt, J. G. Muga, and J. Munoz, Phys. Rev. A **82**, 012113 (2010); G. C. Hegerfeldt and J. G. Muga, J. Phys. A **43**, 505303 (2010).
[16] I. M. Gelfand and G. E. Shilov, *Generalized Functions* (Academic, New York, 1964), Vol. I.
[17] I. S. Gradshteyn and I. M. Ryzhik, in *Table of Integrals, Series and Products*, edited by A. Jeffrey and D. Zwillinger (2007), 7th ed.
[18] J. A. Stovneng and E. H. Hauge, Phys. Rev. B **44**, 13 582 (1991).