BCS-BEC Crossover in 2D Fermi Gases with Rashba Spin-Orbit Coupling

Lianyi He* and Xu-Guang Huang†

Frankfurt Institute for Advanced Studies and Institute for Theoretical Physics, J. W. Goethe University, 60438 Frankfurt am Main, Germany

(Received 1 October 2011; published 3 April 2012)

We present a systematic theoretical study of the BCS-BEC crossover in two-dimensional Fermi gases with Rashba spin-orbit coupling (SOC). By solving the exact two-body problem in the presence of an attractive short-range interaction we show that the SOC enhances the formation of the bound state: the binding energy E_B and effective mass m_B of the bound state grows along with the increase of the SOC. For the many-body problem, even at weak attraction, a dilute Fermi gas can evolve from a BCS superfluid state to a Bose condensation of molecules when the SOC becomes comparable to the Fermi momentum. The ground-state properties and the Berezinskii-Kosterlitz-Thouless (BKT) transition temperature are studied, and analytical results are obtained in various limits. For large SOC, the BKT transition temperature recovers that for a Bose gas with an effective mass m_B. We find that the condensate and superfluid densities have distinct behaviors in the presence of SOC: the condensate density is generally enhanced by the SOC due to the increase of the molecule binding; the superfluid density is suppressed because of the nontrivial molecule effective mass m_B.

DOI: 10.1103/PhysRevLett.108.145302 PACS numbers: 67.85.Lm, 03.75.Ss, 05.30.Fk, 74.20.Fg

It has been widely believed for a long time that a smooth crossover from Bardeen-Cooper-Schrieffer (BCS) superfluidity to Bose-Einstein condensation (BEC) of molecules could be realized in an attractive Fermi gas [1–3]. This BCS-BEC crossover phenomenon has been successfully demonstrated in ultracold fermionic atoms by means of the Feshbach resonance [4]. Some recent experimental efforts in generating synthetic non-Abelian gauge field have opened up the opportunity to study the spin-orbit coupling (SOC) effect in cold atomic gases [5]. For fermionic atoms [6], it provides an alternative way to study the BCS-BEC crossover [7] according to the theoretical observation that novel bound states in three dimensions can be induced by a non-Abelian gauge field even though the attraction is weak [8,9].

Recently, the anisotropic superfluidity in 3D Fermi gases with Rashba SOC has been intensively studied [10–12]. Two-dimensional (2D) fermionic systems with Rashba SOC is more interesting for condensed matter systems [13] and topological quantum computation [14]. By applying a large Zeeman splitting, a non-Abelian topologically superconducting phase and Majorana fermionic modes can emerge in spin-orbit coupled 2D systems [14]. In the absence of SOC, the BCS-BEC crossover and Berezinskii-Kosterlitz-Thouless (BKT) transition temperature in 2D attractive fermionic systems were investigated long ago [15,16] (see [17] for a review), which provide a possible mechanism for pseudogap formation in high-temperature superconductors [18].

In this Letter we present a systematic study of 2D attractive Fermi gases in the presence of Rashba SOC. The main results are summarized as follows: (i) The SOC enhances the difermion bound states in 2D. At large SOC, even for weak intrinsic attraction, the many-body ground state is a Bose-Einstein condensate of bound molecules. In the presence of a harmonic trap, the atom cloud shrinks with increased SOC. (ii) The BKT transition temperature is enhanced by the SOC at weak attraction, and for large SOC it tends to the critical temperature for a gas of molecules with a nontrivial effective mass. The SOC effect therefore provides a new mechanism for pseudogap formation in 2D fermionic systems. (iii) In the presence of SOC, the superfluid ground state exhibits both spin-singlet and spin-triplet pairings, and the triplet one has a nontrivial contribution to the condensate density. In general, the condensate density is enhanced by the SOC due to the increase of the molecule binding. However, the superfluid density has entirely different behavior: it is suppressed by the SOC due to the increasing molecule effective mass.

Model and effective potential.—A quasi-2D Fermi gas can be realized by arranging a one-dimensional optical lattice along the axial direction and a weak harmonic trapping potential in the radial plane, such that fermions are strongly confined along the axial direction and form a series of pancake-shaped quasi-2D clouds [19–21]. The strong anisotropy of the trapping potentials, namely $\omega_z \gg \omega_\perp$ where ω_z ($\omega_\perp$) is the axial (radial) frequency, allows us to use an effective 2D Hamiltonian to deal with the radial degrees of freedom.

The Hamiltonian of a spin-1/2 attractive Fermi gas with Rashba SOC is given by $H = \int d^2\mathbf{r}\bar{\psi}(\mathbf{r})(\mathcal{H}_0 + \mathcal{H}_{so})\psi(\mathbf{r}) - U\int d^2\mathbf{r}\bar{\psi}_\uparrow(\mathbf{r})\bar{\psi}_\downarrow(\mathbf{r})\psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r})$, where $\psi = [\psi_\uparrow, \psi_\downarrow]^T$ represents the two-component fermion fields, $\mathcal{H}_0 = -\frac{\hbar^2\nabla^2}{2m} - \mu - h\sigma_z$ is the free single-particle Hamiltonian with μ being the chemical potential and h the Zeeman splitting, and $\mathcal{H}_{so} = -i\hbar\lambda(\sigma_x\partial_y - \sigma_y\partial_x)$ is the Rashba SOC term [22]. Here $\sigma_{x,y,z}$ are the Pauli matrices which act on the two-component fermion fields. The short-range attractive interaction is modeled by a contact coupling U [23]. In the following we use the natural units $\hbar = k_B = m = 1$.

In the functional path integral formalism, the partition function of the system is $\mathcal{Z} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\exp\{-\mathcal{S}[\psi,\bar{\psi}]\}$, where $\mathcal{S}[\psi,\bar{\psi}] = \int_0^\beta d\tau[\int d^2\mathbf{r}\bar{\psi}\partial_\tau\psi + H(\psi,\bar{\psi})]$ with the inverse temperature $\beta = 1/T$. Introducing the auxiliary complex pairing field $\Phi(x) = -U\psi_\downarrow(x)\psi_\uparrow(x)$ $[x = (\tau, \mathbf{r})]$ and applying the Hubbard-Stratonovich transformation, we arrive at $\mathcal{Z} = \int \mathcal{D}\Psi\mathcal{D}\bar{\Psi}\mathcal{D}\Phi\mathcal{D}\Phi^*\exp\{\frac{1}{2}\times \int dx\int dx'\bar{\Psi}(x)\mathbf{G}^{-1}(x,x')\Psi(x') - U^{-1}\int dx|\Phi(x)|^2\}$, where $\Psi = [\psi, \bar{\psi}]^{\rm T}$ is the Nambu-Gor'kov spinor. The inverse single-particle Green function $\mathbf{G}^{-1}(x,x')$ is given by

$$\mathbf{G}^{-1} = \begin{pmatrix} -\partial_\tau - \mathcal{H}_0 - \mathcal{H}_{\rm so} & i\sigma_y\Phi(x) \\ -i\sigma_y\Phi^*(x) & -\partial_\tau + \mathcal{H}_0 - \mathcal{H}_{\rm so}^* \end{pmatrix} \times \delta(x - x'). \tag{1}$$

Integrating out the fermion fields, we obtain $\mathcal{Z} = \int \mathcal{D}\Phi\mathcal{D}\Phi^*\exp\{-\mathcal{S}_{\rm eff}[\Phi,\Phi^*]\}$, where the effective action reads $\mathcal{S}_{\rm eff}[\Phi,\Phi^*] = U^{-1}\int dx|\Phi(x)|^2 - \frac{1}{2}{\rm Tr}\ln[\mathbf{G}^{-1}(x,x')]$.

Two-body problem.—The exact two-body problem at vanishing density can be studied by considering the Green function $\Gamma(Q)$ of the fermion pairs, where $Q = (i\nu_n, \mathbf{q})$ with $\nu_n = 2n\pi T$ (n integer) being the bosonic Matsubara frequency. In the present formalism, $\Gamma^{-1}(Q)$ can be obtained from its coordinate representation defined as $\Gamma^{-1}(x,x') = (\beta V)^{-1}\delta^2\mathcal{S}_{\rm eff}[\Phi,\Phi^*]/[\delta\Phi^*(x)\delta\Phi(x')]|_{\Phi=0}$. For $\Phi = 0$, the single-particle Green function reduces to its noninteracting form $\mathcal{G}_0(K) = {\rm diag}[g_+(K), g_-(K)]$ with $g_\pm(K) = [i\omega_n \mp (\xi_\mathbf{k} - h\sigma_z) - \lambda(\sigma_x k_y \mp \sigma_y k_x)]^{-1}$, where $K = (i\omega_n, \mathbf{k})$ with $\omega_n = (2n+1)\pi T$ being the fermionic Matsubara frequency. Here $\xi_\mathbf{k} = \epsilon_\mathbf{k} - \mu$ and $\epsilon_\mathbf{k} = \mathbf{k}^2/2$. The single-particle spectrum generally has two branches: $\omega_\mathbf{k}^\pm = \xi_\mathbf{k} \pm \sqrt{\lambda^2\mathbf{k}^2 + h^2}$.

After the analytical continuation $i\nu_n \to \omega + i0^+$, the real part of $\Gamma^{-1}(Q)$ takes the form

$$\Gamma^{-1}(\omega,\mathbf{q}) = \frac{1}{U} - \sum_{\alpha,\gamma=\pm;\mathbf{k}} \frac{1 - f(\omega_\mathbf{k}^\alpha) - f(\omega_\mathbf{p}^\gamma)}{4(\omega_\mathbf{k}^\alpha + \omega_\mathbf{p}^\gamma - \omega)}(1 + \alpha\gamma\mathcal{T}_{\mathbf{kq}}), \tag{2}$$

where $f(E) = 1/(e^{\beta E} + 1)$ is the Fermi-Dirac distribution function, and $\mathcal{T}_{\mathbf{kq}} = (\lambda^2\mathbf{k}\cdot\mathbf{p} + h^2)/\sqrt{(\lambda^2\mathbf{k}^2 + h^2)(\lambda^2\mathbf{p}^2 + h^2)}$ with $\mathbf{p} = \mathbf{k} + \mathbf{q}$. Γ^{-1} takes the form similar to that of the relativistic systems [24], due to the fact that $\mathcal{H}_{\rm so}$ behaves like a Dirac Hamiltonian. Since in 2D the bound state forms for arbitrarily small attraction [25], the contact coupling U can be regularized by the two-body problem at vanishing SOC, $U^{-1} = \sum_\mathbf{k}(2\epsilon_\mathbf{k} + \epsilon_{\rm B})^{-1}$ [15,17], where $\epsilon_{\rm B}$ is the binding energy at vanishing SOC. This equation recovers the exponential behavior $\epsilon_{\rm B} = 2\Lambda\exp(-4\pi/U)$ in 2D [26], where $\Lambda \gg \epsilon_{\rm B}$ is an energy cutoff. All physical equations are finally UV convergent in terms of $\epsilon_{\rm B}$ and we set $\Lambda \to \infty$ in the dilute limit.

From now on we consider the case $h = 0$. The binding energy $E_{\rm B}$ at nonzero SOC is determined by the solution of $\omega + 2\mu = -E_{\rm B}$ for $\Gamma^{-1}(\omega, \mathbf{q} = 0) = 0$. From the imaginary part of $\Gamma^{-1}(Q)$, the bound state corresponds to the solution in the regime $-\infty < \omega + 2\mu < -\lambda^2$ and hence $E_{\rm B} > \lambda^2$. Completing the momentum integrals analytically, we obtain a simple algebraic equation for $E_{\rm B}$ [27],

$$\ln\frac{E_{\rm B}}{\epsilon_{\rm B}} = \frac{2\lambda}{\sqrt{E_{\rm B} - \lambda^2}}\arctan\frac{\lambda}{\sqrt{E_{\rm B} - \lambda^2}}. \tag{3}$$

The solution can be generally expressed as $E_{\rm B} = \epsilon_{\rm B} + 4\eta J(\eta/\epsilon_{\rm B})$ where $\eta = \lambda^2/2$. For $\eta \ll \epsilon_{\rm B}$, we have $J \simeq 1$ and $E_{\rm B}$ is well given by $E_{\rm B} \simeq \epsilon_{\rm B} + 2\lambda^2$. For $\eta/\epsilon_{\rm B} \to \infty$, the solution approaches very slowly to the asymptotic result $E_{\rm B} \simeq \lambda^2$. In general, $E_{\rm B}$ increases with increased SOC, as shown in Fig. 1. It is straightforward to show that the bound state contains both spin-singlet and triplet components [8].

For small nonzero $\mathbf{q}$, the solution for ω can be written as $\omega + 2\mu = -E_{\rm B} + \mathbf{q}^2/(2m_{\rm B})$, where $m_{\rm B}$ is the molecule effective mass. Substituting this dispersion into the equation $\Gamma^{-1}(\omega,\mathbf{q}) = 0$ we obtain [27]

$$\frac{2m}{m_{\rm B}} = 1 - \frac{1}{2\kappa}\frac{2\sqrt{\kappa-1} - (\kappa-2)(\frac{\pi}{2} - \arctan\frac{\kappa-2}{2\sqrt{\kappa-1}})}{2\sqrt{\kappa-1} + (\frac{\pi}{2} - \arctan\frac{\kappa-2}{2\sqrt{\kappa-1}})}, \tag{4}$$

where $\kappa = E_{\rm B}/\lambda^2$. For $\lambda \to 0$, we obtain the usual result $m_{\rm B} \to 2m$. For $\lambda \to \infty$, we have $E_{\rm B} \to \lambda^2$ and $m_{\rm B}$ approaches the asymptotic result $4m$. In general, $m_{\rm B}$ is larger than $2m$, as shown in Fig. 1. Together with the result for $E_{\rm B}$, we conclude that a novel bound state (referred to as rashbon [10]) forms. It would have significant impact on the many-body problem discussed in the following.

Ground state.—For the many-body problem, we consider a homogeneous Fermi gas with fixed fermion density $n = N/V$. For convenience, we define the Fermi momentum via $n = k_F^2/(2\pi)$ and Fermi energy by $\epsilon_F = k_F^2/2$. The ground state ($T = 0$) can be studied in the self-consistent mean-field theory, where we replace the pairing field Φ by

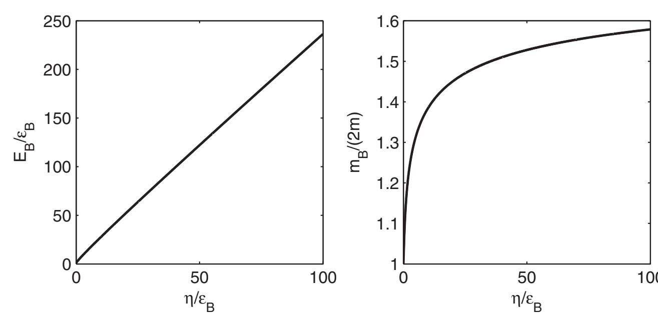

FIG. 1. The binding energy $E_{\rm B}$ (left, divided by $\epsilon_{\rm B}$) and the effective mass $m_{\rm B}$ (right, divided by $2m$) as functions of $\eta/\epsilon_{\rm B}$.

its expectation value $\langle\Phi\rangle = \Delta$. Without loss of generality, we set Δ to be real.

The mean-field ground-state energy $\Omega = \mathcal{S}_{\rm eff}[\Delta, \Delta]/(\beta V)$ can be evaluated as $\Omega = \Delta^2/U + (1/2)\sum_{\mathbf{k}}(2\xi_{\mathbf{k}} - E^+_{\mathbf{k}} - E^-_{\mathbf{k}})$, where $E^\pm_{\mathbf{k}} = [(\xi^\pm_{\mathbf{k}})^2 + \Delta^2]^{1/2}$ are the quasiparticle excitation energies with $\xi^\pm_{\mathbf{k}} = \xi_{\mathbf{k}} \pm \lambda|\mathbf{k}|$. According to the equation that $E_{\rm B}$ satisfies, Ω can be evaluated as $\Omega = \Omega_{\rm 2D}(\Delta, \mu, \epsilon_{\rm B}) + \Omega_\lambda$, where $\Omega_{\rm 2D}(\Delta, \mu, \epsilon_{\rm B}) = (\Delta^2/4\pi) \times \{\ln[(\sqrt{\mu^2+\Delta^2} - \mu)/\epsilon_{\rm B}] - 1/2 - \mu/(\sqrt{\mu^2+\Delta^2} - \mu)\}$ is formally the ground-state energy for vanishing SOC [15,17], and $\Omega_\lambda = -(\lambda/2\pi)\int_0^\lambda dk[\sqrt{(\xi_k - \eta)^2 + \Delta^2} - (\xi_k - \eta)]$ is the contribution due to the SOC effect.

From the explicit form of the ground-state energy, the gap and number equations can be expressed as

$$\begin{aligned}&[\mu^2 + \Delta^2]^{1/2} - \mu = \epsilon_{\rm B}\exp[2I_1(\mu/\eta, \Delta/\eta)],\\ &[\mu^2 + \Delta^2]^{1/2} + \mu = 2\epsilon_{\rm F} - 2\eta[1 - I_2(\mu/\eta, \Delta/\eta)],\end{aligned} \tag{5}$$

respectively. Here the functions I_1 and I_2 are defined as $I_1(a, b) = \int_0^1 dx[(x^2 - 1 - a)^2 + b^2]^{-1/2}$ and $I_2(a, b) = \int_0^1 dx(x^2 - 1 - a)[(x^2 - 1 - a)^2 + b^2]^{-1/2}$. I_1, I_2, and Ω_λ can be analytically evaluated using the elliptic functions. For vanishing SOC, we recover the well-known analytical results, $\Delta = \sqrt{2\epsilon_{\rm B}\epsilon_{\rm F}}$ and $\mu = \epsilon_{\rm F} - \epsilon_{\rm B}/2$ [15].

Now let us start from weak attraction, $\epsilon_{\rm B} \ll \epsilon_{\rm F}$. For sufficiently small SOC, we have $I_1 \to 0$ and $I_2 \to -1$, and the solution is well approximated by $\Delta \simeq \sqrt{2\epsilon_{\rm B}\epsilon_{\rm F}}$ and $\mu \simeq \epsilon_{\rm F} - \epsilon_{\rm B}/2 - 2\eta$, which indicates a BCS superfluid state. For large SOC, we expect that μ becomes negative and $|\mu| \gg \Delta$. Substituting this into the gap equation, we find $\mu \simeq -E_{\rm B}/2$, which indicates a Bose-Einstein condensate of molecules with binding energy $E_{\rm B}$. Then expanding the number equation in powers of $\Delta/|\mu|$ and keeping the leading order, we obtain $\Delta \simeq \sqrt{2E_{\rm B}\epsilon_{\rm F}\zeta(\kappa)}$, where $\zeta(\kappa) = 2\kappa^{-1}(\kappa - 1)^{3/2}(2\sqrt{\kappa - 1} + \frac{\pi}{2} - \arctan\frac{\kappa - 2}{2\sqrt{\kappa - 1}})^{-1}$. This is a transparent formula to show that the pairing gap Δ increases with increased SOC, consistent with the perturbative approach [28]. These analytical results are in good agreement with the numerical results shown in Fig. 2 even for intermediate $\lambda/k_{\rm F}$ [29].

Using the fermion Green function $\mathbf{G}(K)$, we can show that the fermion momentum distribution $n(\mathbf{k})$ is isotropic and can be expressed as $n(k) = (1/4)\sum_\alpha(1 - \xi^\alpha_k/E^\alpha_k)$ [27]. As shown in Fig. 3, with increased SOC, the distribution broadens, which indicates a BCS-BEC crossover. The new feature here is that the distribution generally displays nonmonotonic behavior. The peak in the distribution is just located at $k = \lambda$.

The pair wave functions $\phi_{\sigma\sigma'}(\mathbf{k}) \equiv \langle\psi_{\mathbf{k}\sigma}\psi_{-\mathbf{k}\sigma'}\rangle$ can be evaluated as $\phi_{\uparrow\uparrow}(\mathbf{k}) = -(i\Delta/4)e^{i\theta_{\mathbf{k}}}\sum_\alpha \alpha/E^\alpha_{\mathbf{k}}$ and $\phi_{\uparrow\downarrow}(\mathbf{k}) = -(\Delta/4)\sum_\alpha 1/E^\alpha_{\mathbf{k}}$, where $e^{i\theta_{\mathbf{k}}} = (k_x + ik_y)/|\mathbf{k}|$. Therefore, the superfluid state exhibits both singlet and triplet pairings for nonzero SOC. The numerical results for the ratio $|\phi_{\uparrow\uparrow}(k)|/|\phi_{\uparrow\downarrow}(k)|$ displayed in Fig. 3 show that the triplet pairing spreads to wider momentum regime

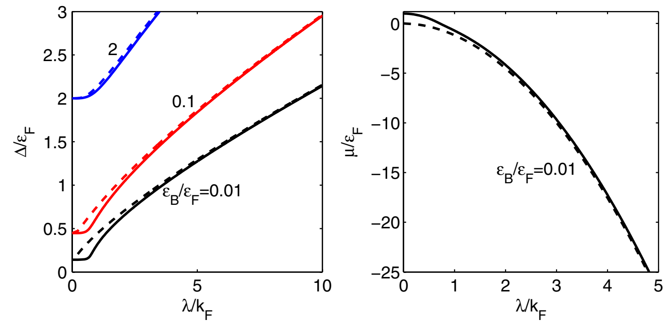

FIG. 2 (color online). The pairing gap Δ (left, divided by $\epsilon_{\rm F}$) and the chemical potential μ (right, divided by $\epsilon_{\rm F}$) as functions of $\lambda/k_{\rm F}$. The dashed lines represents the analytical results $\Delta = \sqrt{2E_{\rm B}\epsilon_{\rm F}\zeta(\kappa)}$ and $\mu = -E_{\rm B}/2$ with $E_{\rm B}$ calculated from Eq. (3).

with increased SOC. According to the general formula for the condensate number of fermion pairs [30], $N_0 = \frac{1}{2}\sum_{\sigma,\sigma'}\iint d^2\mathbf{r}d^2\mathbf{r}'|\langle\psi_\sigma(\mathbf{r})\psi_{\sigma'}(\mathbf{r}')\rangle|^2$, the condensate density reads $n_0 = \sum_{\mathbf{k}}[|\phi_{\uparrow\downarrow}(\mathbf{k})|^2 + |\phi_{\uparrow\uparrow}(\mathbf{k})|^2]$. The triplet pairing amplitude contributes, in contrast to the fermionic superfluids with only singlet pairing [31]. For large SOC, we find analytically that $2N_0/N = 1 - O(\frac{\Delta^4}{|\mu|^4}) \to 1$ (see also Fig. 3), which indicates the Bose-Einstein condensation of weakly interacting rashbons.

In the presence of a trap potential $V(r) = \frac{1}{2}\omega_\perp^2 r^2$, the chemical potential becomes $\mu(r) = \mu_0 - V(r)$ and the density distribution $n(r)$ can be solved from the constraint $N = 2\pi\int rdrn(r)$ in the local density approximation. As shown in Fig. 4, the atom cloud shrinks with increased SOC, which can be viewed as a preliminary experimental signal of the BCS-BEC crossover.

BKT transition temperature.—At finite temperature in 2D we should rewrite the complex ordering field $\Phi(x)$ in

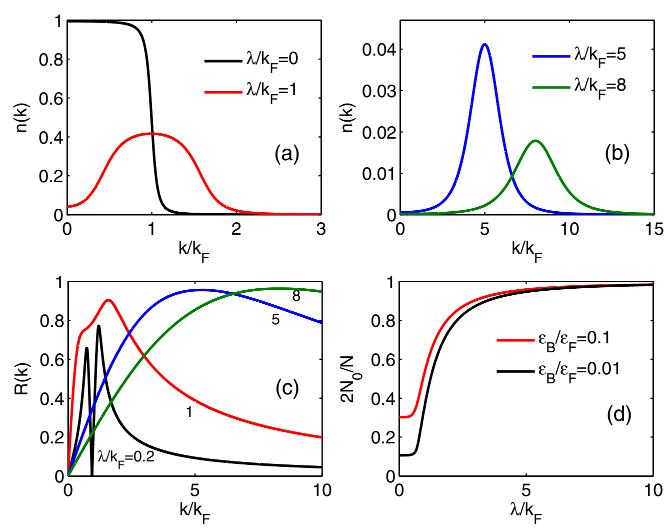

FIG. 3 (color online). (a),(b),(c) The momentum distribution $n(k)$ and the ratio $R(k) = |\phi_{\uparrow\uparrow}(k)|/|\phi_{\uparrow\downarrow}(k)|$ for various values of $\lambda/k_{\rm F}$ and $\epsilon_{\rm B}/\epsilon_{\rm F} = 0.01$. (d) The condensate fraction $2N_0/N$ as a function of $\lambda/k_{\rm F}$ for various values of $\epsilon_{\rm B}/\epsilon_{\rm F}$.

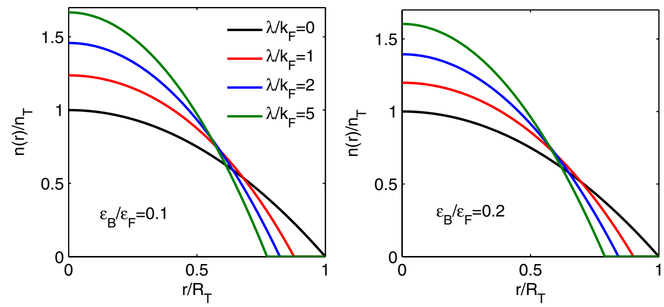

FIG. 4 (color online). The density profile $n(r)$ (divided by $n_T = \epsilon_F/\pi$) in the presence of a trap potential for various values of λ/k_F. The Fermi energy $\epsilon_F = k_F^2/2$ in trapped system is defined as $\epsilon_F = \sqrt{N}\hbar\omega_\perp$ [37], and the Thomas-Fermi radius reads $R_T = \sqrt{2\epsilon_F}/\omega_\perp$.

terms of its modulus $\Delta(x)$ and phase $\theta(x)$, i.e., $\Phi(x) = \Delta(x)\exp[i\theta(x)]$. Since the random fluctuations of the phase $\theta(x)$ forbid long-range order in 2D, we have $\langle\Phi(x)\rangle = 0$ but $\langle\Delta(x)\rangle \neq 0$ at $T \neq 0$. However, Berezinskii [32] and Kosterlitz and Thouless [33] showed that below a critical temperature T_{BKT}, there exist bound vortex-antivortex pairs and quasi-long-range order remains.

To determine the BKT transition temperature, we derive an effective action for the U(1) phase field $\theta(x)$. To this end we make a gauge transformation $\psi(x) = \exp[i\theta(x)/2]\chi(x)$ [16,17]. Then we arrive at the expression $\mathcal{Z} = \int \Delta\mathcal{D}\Delta\mathcal{D}\theta \exp\{-\beta\mathcal{U}_{\text{eff}}[\Delta(x), \partial\theta(x)]\}$, where the effective action $\beta\mathcal{U}_{\text{eff}}[\Delta(x), \partial\theta(x)] = U^{-1}\int dx\Delta^2(x) - \frac{1}{2}\,\text{Tr}\ln\mathbf{S}^{-1}[\Delta(x), \partial\theta(x)]$ now depends on the modulus-phase variables. The Green function of the initial (charged) fermions takes a new form $\mathbf{S}^{-1}[\Delta(x), \partial\theta(x)] = \mathcal{G}^{-1}[\Delta(x)] - \Sigma[\partial\theta(x)]$. Here $\mathcal{G}^{-1}[\Delta(x)] = \mathbf{G}^{-1}[\Delta(x), \Delta(x)]$ is the green function of the neutral fermion, and $\Sigma[\partial\theta] \equiv \tau_3[i\partial_\tau\theta/2 + (\nabla\theta)^2/8] - \hat{I}[i\nabla^2\theta/4 + i\nabla\theta\cdot\nabla/2] + (\lambda/2)[\tau_3\sigma_x\partial_y\theta - \hat{I}\sigma_y\partial_x\theta]$, where $\tau_i (i = 1, 2, 3)$ are the Pauli matrices in the Nambu-Gor'kov space.

Since the low-energy dynamics for $\Delta \neq 0$ is governed by long-wavelength fluctuations of $\theta(x)$, we neglect the amplitude fluctuations and treat Δ as its saddle point value [16,17]. Then the effective action can be decomposed as $\mathcal{U}_{\text{eff}}[\Delta(x), \partial\theta(x)] \simeq \mathcal{U}_{\text{kin}}[\Delta, \partial\theta(x)] + \mathcal{U}_{\text{pot}}(\Delta)$. The potential part reads $\mathcal{U}_{\text{pot}}/V = \Delta^2/U + \sum_{\mathbf{k}}[\xi_{\mathbf{k}} - \mathcal{W}(E_{\mathbf{k}}^+) - \mathcal{W}(E_{\mathbf{k}}^-)]$ where $\mathcal{W}(E) = E/2 + T\ln(1 + e^{-\beta E})$. The kinetic part can be obtained by the derivative expansion $\beta\mathcal{U}_{\text{kin}}[\Delta, \partial\theta(x)] = \sum_{n=1}^{\infty}\frac{1}{n}\,\text{Tr}(\mathcal{G}\Sigma)^n$.

Keeping only lowest-order derivatives of $\theta(x)$, we find that the kinetic term $\mathcal{U}_{\text{kin}}$ coincides with the classical spin XY model, which has the continuum Hamiltonian $H_{XY} = \frac{1}{2}\mathcal{J}\int d^2\mathbf{r}[\nabla\theta(\mathbf{r})]^2$ where the phase stiffness $\mathcal{J} = \frac{\rho_s}{4m}$ and ρ_s is the superfluid density [34]. The superfluid density in our model can be evaluated as $\rho_s = n - \rho_1 - \rho_2$, where $\rho_1 = (\lambda/8\pi)\sum_{\alpha=\pm}\int_0^\infty dk\alpha(\xi_k^\alpha + \Delta^2/\xi_k)[1 - 2f(E_k^\alpha)]/E_k^\alpha$ and $\rho_2 = -(1/4\pi)\sum_{\alpha=\pm}\int_0^\infty kdk(k + \alpha\lambda)^2 f'(E_k^\alpha)$ [27]. The BKT transition temperature is determined by $T_{BKT} = \frac{\pi}{2}\mathcal{J}$ [32–35].

For sufficiently small ϵ_B and SOC, Δ is correspondingly small and T_{BKT} recovers the mean-field result T_Δ. On the other hand, for large ϵ_B and/or SOC, ρ_s can be well approximated by its zero-temperature value for $T \sim T_{BKT}$. We are interested in the case with small ϵ_B and large SOC. For large SOC, using the fact $\Delta \ll |\mu|$, we find analytically that [27]

$$\rho_s(T \ll T_\Delta) \simeq \frac{2m}{m_B}n, \qquad \mathcal{J}(T \ll T_\Delta) \simeq \frac{n_B}{m_B}, \tag{6}$$

where $n_B = n/2$ and m_B is given by Eq. (4). Therefore, the phase stiffness $\mathcal{J}$ naturally recovers that for a Bose (rashbon) gas at large SOC. The BKT transition temperature and the phase stiffness jump $\Delta\mathcal{J}$ reaches the rashbon limit $T_{BKT} = \pi n_B/(2m_B) = (2m/m_B)\epsilon_F/8$ and $\Delta\mathcal{J} = n_B/m_B$. To verify the above analytical results, we show the numerical results for $\rho_s(T = 0)$ and T_{BKT} in Fig. 5. Even for weak attraction, a visible pseudogap phase appears in the window $T_{BKT} < T < T_\Delta$ for $\lambda \sim k_F$. The SOC therefore provides a new mechanism for pseudogap formation in 2D fermionic systems.

Finally, we point out a surprising result, $\rho_s < n$ at $T = 0$, which is in contrast to the result $\rho_s = n$ for fermionic superfluids in the absence of SOC [34,36]. Actually, at $T = 0$, the superfluid density reads $\rho_s = n - \rho_\lambda$, where the λ-dependent term $\rho_\lambda = \rho_1(T = 0)$ is always positive and is generally an increasing function of λ. Therefore, the superfluid density shown in Fig. 3 has entirely different behavior in contrast to the condensate density shown in Fig. 5: it is generally suppressed by the SOC effect. The exact two-body solution provides a very transparent explanation to this suppression. At large SOC, the effective mass $m_B > 2m$ is an increasing function of SOC and causes the suppression of the superfluid density by a factor $2m/m_B$. Our argument also applies to the suppression of the radial ($x - y$ plane) superfluid density $\rho_s^\perp$ for the 3D case [12], where the radial effective mass $m_B^\perp$ is larger than $2m$ [10].

L. He acknowledges the support from the Alexander von Humboldt Foundation, and X.-G. Huang is supported by

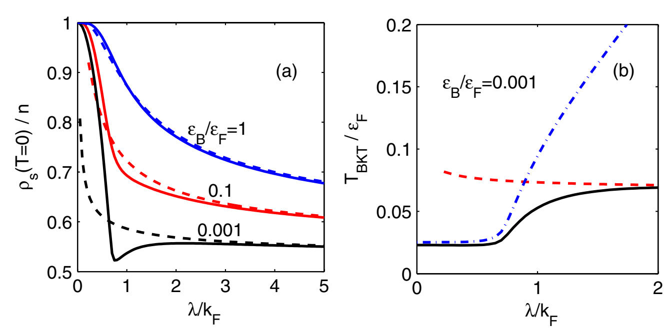

FIG. 5 (color online). (a) The superfluid density ρ_s at $T = 0$ (divided by n) as a function of λ/k_F. The dashed lines represent the results of $2m/m_B$ calculated from Eq. (4). (b) The BKT transition temperature as a function of λ/k_F. The dashed line represents the rashbon limit and the dash-dotted line is the mean-field result.

the Deutsche Forschungsgemeinschaft (Grant No. SE 1836/1-2).

Note added.—After finishing this Letter, we note that similar results of the condensate density [12,38] and the superfluid density [12] in spin-orbit coupled Fermi gases are also reported.

*lianyi@itp.uni-frankfurt.de
†xhuang@itp.uni-frankfurt.de

[1] D. M. Eagles, Phys. Rev. **186**, 456 (1969).
[2] A. J. Leggett, in *Modern Trends in the Theory of Condensed Matter* (Springer-Verlag, Berlin, 1980), pp. 13–27.
[3] P. Nozieres and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985); C. A. R. Sá de Melo, M. Randeria, and J. R. Engelbrecht, Phys. Rev. Lett. **71**, 3202 (1993).
[4] M. Greiner *et al.*, Nature (London) **426**, 537 (2003); S. Jochim *et al.*, Science **302**, 2101 (2003); M. W. Zwierlein *et al.*, Nature (London) **435**, 1047 (2005).
[5] K. Osterloh *et al.*, Phys. Rev. Lett. **95**, 010403 (2005); J. Ruseckas *et al.*, Phys. Rev. Lett. **95**, 010404 (2005); T. D. Stanescu, C. Zhang, and V. Galitski, Phys. Rev. Lett. **99**, 110403 (2007); X. J. Liu *et al.*, Phys. Rev. Lett. **102**, 046402 (2009); Y. J. Lin *et al.*, Nature (London) **462**, 628 (2009); Y. J. Lin *et al.*, Nature (London) **471**, 83 (2011).
[6] J. D. Sau *et al.*, Phys. Rev. B **83**, 140510(R) (2011).
[7] J. P. Vyasanakere, S. Zhang, and V. B. Shenoy, Phys. Rev. B **84**, 014512 (2011).
[8] J. P. Vyasanakere and V. B. Shenoy, Phys. Rev. B **83**, 094515 (2011).
[9] This phenomenon is analogous to the catalysis of the dynamical mass generation by an external non-Ablelian field in quantum field theory, see V. P. Gusynin, D. K. Hong, and I. A. Shovkovy, Phys. Rev. D **57**, 5230 (1998); I. A. Shovkovy and V. M. Turkowski, Phys. Lett. B **367**, 213 (1996).
[10] H. Hu *et al.*, Phys. Rev. Lett. **107**, 195304 (2011); Z. -Q. Yu and H. Zhai, Phys. Rev. Lett. **107**, 195305 (2011).
[11] M. Iskin and A. L. Subasi, Phys. Rev. Lett. **107**, 050402 (2011); M. Gong, S. Tewari, and C. Zhang, Phys. Rev. Lett. **107**, 195303 (2011); W. Yi and G.-C. Guo, Phys. Rev. A **84**, 031608(R) (2011); L. Han and C. A. R. Sa de Melo, Phys. Rev. A **85**, 011606(R) (2012); L. Dell'Anna, G. Mazzarella, and L. Salasnich, Phys. Rev. A **84**, 033633 (2011).
[12] K. Zhou and Z. Zhang, Phys. Rev. Lett. **108**, 025301 (2012).
[13] L. P. Gor'kov and E. I. Rashba, Phys. Rev. Lett. **87**, 037004 (2001).
[14] C. Zhang *et al.*, Phys. Rev. Lett. **101**, 160401 (2008); J. D. Sau *et al.*, Phys. Rev. B **82**, 214509 (2010); S. Tewari *et al.*, New J. Phys. **13**, 065004 (2011).
[15] M. Randeria, J.-M. Duan, and L.-Y. Shieh, Phys. Rev. Lett. **62**, 981 (1989); Phys. Rev. B **41**, 327 (1990).
[16] V. P. Gusynin, V. M. Loktev, and S. G. Sharapov, JETP **88**, 685 (1999); **90**, 993 (2000).
[17] V. M. Loktev, R. M. Quick, and S. G. Sharapov, Phys. Rep. **349**, 1 (2001).
[18] The experimental observation of pairing pseudogap in two-dimensional Fermi gases has been recently reported in M. Feld *et al.*, Nature (London) **480**, 75 (2011).
[19] S. Stock *et al.*, Phys. Rev. Lett. **95**, 190403 (2005); Z. Hadzibabic *et al.*, Nature (London) **441**, 1118 (2006).
[20] W. Zhang, G. D. Lin, and L.-M. Duan, Phys. Rev. A **77**, 063613 (2008).
[21] P. Dyke *et al.*, Phys. Rev. Lett. **106**, 105304 (2011).
[22] The sign of the coupling constant λ is not important, since all physical quantities depend only on λ^2. In this Letter we set $\lambda > 0$ without loss of generality.
[23] The validity of a contact interaction is restricted in the dilute limit, i.e., $k_F r_0 \ll 1$, where k_F is the Fermi momentum defined through the fermion density $n = k_F^2/(2\pi)$ and r_0 is the effective range of the attractive interaction. In the presence of Rashba SOC, another dilute condition $\lambda r_0 \ll 1$ should also be fulfilled, see X. Cui, Phys. Rev. A **85**, 022705 (2012).
[24] L. He and P. Zhuang, Phys. Rev. D **75**, 096003 (2007); **76**, 056003 (2007); G. Sun, L. He, and P. Zhuang, Phys. Rev. D **75**, 096004 (2007).
[25] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics-Non Relativistic Theory* (Pergamon Press, New York, 1989).
[26] For an interatomic potential described by a 2D circularly symmetric well of radius r_0 and depth v_0, the binding energy ϵ_B is given by $\epsilon_B = 1/(2r_0^2)\exp[-2/(v_0 r_0^2)]$ in the dilute limit $v_0 r_0^2 \to 0$ [25]. For quasi-2D cold atoms confined by an axial trapping frequency ω_z, the binding energy is given by $\epsilon_B = (C\hbar\omega_z/\pi)\exp[\sqrt{2\pi}l_z/a_s]$, where a_s is the 3D s-wave scattering length, $l_z = \sqrt{\hbar/\omega_z}$, and $C \simeq 0.915$. See D. S. Petrov and G. V. Shlyapnikov, Phys. Rev. A **64**, 012706 (2001).
[27] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.108.145302 for details of the derivation.
[28] G. Chen, M. Gong, and C. Zhuang, Phys. Rev. A **85**, 013601 (2012).
[29] Actually, for large ϵ_B/ϵ_F, the analytical formulas work well even for small SOC since $\Delta \ll |\mu|$ can be easily satisfied.
[30] A. J. Leggett, *Quantum Liquids: Bose Condensation and Cooper Pairing in Condensed-Matter Systems* (Oxford Universty Press, Oxford, 2006).
[31] L. Salasnich, N. Manini, and A. Parola, Phys. Rev. A **72**, 023621 (2005); L. Salasnich, Phys. Rev. A **76**, 015601 (2007).
[32] V. L. Berezinskii, Sov. Phys. JETP **32**, 493 (1971).
[33] J. M. Kosterlitz and D. Thouless, J. Phys. C **5**, L124 (1972).
[34] N. Nagaosa, *Quantum Field Theory in Condensed Matter Physics* (Springer, New York, 1999).
[35] Since $\mathcal{J}$ depends on the variables Δ, μ, and T explicitly, this equation should be accompanied with the gap equation $\partial \mathcal{U}_{pot}/\partial\Delta = 0$ and the number equation $-\partial \mathcal{U}_{pot}/\partial\mu = N$ [16,17].
[36] E. Taylor *et al.*, Phys. Rev. A **74**, 063626 (2006); N. Fukushima *et al.*, Phys. Rev. A **75**, 033609 (2007).
[37] L. He and P. Zhuang, Phys. Rev. A **78**, 033613 (2008).
[38] B. Huang and S. Wan, arXiv:1109.3970.