Optical Detection of the Quantization of Collective Atomic Motion

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We directly measure the quantized collective motion of a gas of thousands of ultracold atoms, coupled to light in a high-finesse optical cavity. We detect strong asymmetries, as high as 3:1, in the intensity of light scattered into low- and high-energy motional sidebands. Owing to high cavity-atom cooperativity, the optical output of the cavity contains a spectroscopic record of the energy exchanged between light and motion, directly quantifying the heat deposited by a quantum position measurement's backaction. Such backaction selectively causes the phonon occupation of the observed collective modes to increase with the measurement rate. These results, in addition to providing a method for calibrating the motion of low-occupation mechanical systems, offer new possibilities for investigating collective modes of degenerate gases and for diagnosing optomechanical measurement backaction.

Quantized motion leads to a large asymmetry in the spectrum of light scattered by a ground-state oscillator. Such asymmetry is most commonly observed from microscopic oscillators, such as electrons bound within atoms and molecules, single neutral atoms [1], or few-ion ensembles [2–4]. In contrast, quantum aspects of the motion of massive or many-atom oscillators are typically obscured by thermal noise and high phonon occupation. These objects, therefore, usually modulate the spectrum of light in a classical, symmetric manner.

Controlling and measuring the motion of macroscopic objects at levels sensitive to quantum effects will be critical for operating gravitational-wave detectors [5], verifying the correspondence principle at macroscopic scales [6,7], and realizing protocols that mechanically store and exchange quantum information [8,9]. Such goals are being pursued actively using cavity optomechanical systems, wherein the motion of an object with mass ranging from attograms to kilograms is observed via its coupling to an electromagnetic cavity [10]. Implementations of cavity optomechanics using the collective motion of atomic gases [11–13] have demonstrated sensitivity to quantum-optical force fluctuations, leading to the observation of optical squeezing from ponderomotive interactions [14] and backaction from a quantum-limited position measurement [15].

In this Letter, we use a high-finesse optical cavity to detect the coherent, asymmetric scattering of light by collective modes of motion of a trapped atomic gas, occupied with as few as 0.5 phonons. Observations have previously been made of optical emission asymmetries from individual atoms [1] and ensembles of up to 14 ions [2–4] and of asymmetric absorption by a nanomechanical solid-state resonator [16]. Here, we measure the coherent scattering of light from the collective motion of many hundreds of ultracold atoms. The scattering asymmetry acts as a self-calibrating thermometer for the atoms' collective phonon occupation. Moreover, owing to high cavity-atom cooperativity and thermal isolation in our system, the cavity mode acts as the dominant channel for energy flux to our mechanical system. The spectrum of light emitted from the cavity therefore serves as a record of the energy exchanged between motion and the light field. We demonstrate that this energy transfer represents the necessary minimum diffusive heating of a continuous backaction-limited quantum position measurement.

Our experiment (Fig. 1) begins with an ensemble of 4000 ultracold ^{87}Rb atoms. The atoms are trapped in a few adjacent minima of a one-dimensional optical standing-wave potential, formed by 850-nm-wavelength light resonating within a high-finesse Fabry-Pérot optical cavity and detuned far from atomic resonance. The curvature at each potential minimum corresponds to an oscillation frequency, along the cavity axis, of $\omega_m = 2\pi \times 110$ kHz.

We create an optomechanical coupling that is linearly sensitive to atomic position by trapping the atoms at locations with strong intensity gradients of 780-nm-wavelength probe light, which also resonates within the cavity [17]. The probe light is detuned from the atomic $D2$ transition by many gigahertz. At such large detunings, the atomic gas acts as a position-dependent refractive medium, leading to an interaction Hamiltonian $\hat{H}_{\text{int}} = \sum_i g_i \hat{n} \hat{z}_i / z_{\text{HO}}$, where $\hat{n}$ is the probe's photon number operator and $\hat{z}_i$ is the position operator of atom i [15]. Here, g_i represents the change in the cavity's resonance frequency as atom i is displaced by one single-atom harmonic oscillator length z_{HO}, equivalent to $\sqrt{\hbar/2m\omega_m}$ for rubidium mass m.

The above may be rewritten [11,12] as a collective interaction described by $\hat{H}_{\text{int}} = g_{\text{om}} \hat{n} \hat{Z}/Z_{\text{HO}}$, where $\hat{Z} \equiv \sum_i g_i \hat{z}_i / \sum_i g_i$ is a collective position operator, $Z_{\text{HO}} = z_{\text{HO}}/\sqrt{N_a}$ is the collective oscillator length for N_a atoms, and $g_{\text{om}} = \sum_i g_i / \sqrt{N_a}$ is the collective optomechanical coupling rate. In our system, the g_i are all approximately
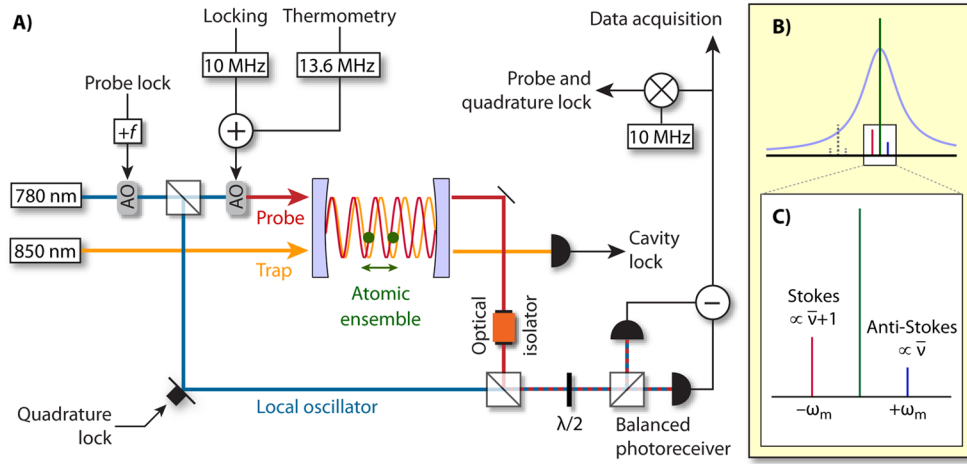
FIG. 1 (color online).   Schematic of the experiment. (a) An ensemble of 4000 ultracold ^{87}Rb atoms is trapped in three to five adjacent potential minima of an optical standing wave, within a high-finesse optical cavity. The ensemble's center-of-mass motion coherently scatters light from cavity probe light, detected using a balanced heterodyne receiver. Acousto-optic modulators (AOs) are used to lock the probe with respect to cavity resonance and to shift the probe's frequency with respect to the heterodyne detector's local oscillator. (b) Probe spectrum in the cavity. A strong resonant tone (solid green peak) applies backaction to and acquires sidebands from the collective atomic motion. A weak detuned tone (dashed gray peak) is used for locking the probe frequency with respect to the cavity and does not significantly affect the oscillator, neither via incoherent backaction nor via dynamical cooling. (c) A harmonic oscillator in its ground state can only extract energy from the optical field, leading to an asymmetry in the resonant probe's Raman sidebands, indicative of the mean phonon occupation number $\bar{\nu}$.

equal, and $\hat{Z}$ is nearly equivalent to the center-of-mass position $\hat{Z}_{\text{com}} = \sum_i \hat{z}_i / N_a$. In this case, $g_{\text{om}} = \sqrt{N_a} g_0^2 k z_{\text{HO}} / \Delta_{\text{ca}}$, where Δ_{ca} is the probe detuning from the ground $F = 2$ hyperfine manifold's $D2$ resonance, g_0 is the single-atom cavity-QED coupling rate ($2\pi \times 12.4$ MHz, averaged over the intracavity atomic distribution), and k is the probe wave number.

Unlike solid-state resonators, which are coupled to their material environments, the atoms here are isolated *in vacuo*, and the thermal "bath" for the measured collective oscillation consists of the remaining collective modes of atomic motion. After raising the trap potential to its final strength, the bath has an initial temperature of 2.9 μK, corresponding to a mode occupation of 0.54 phonons.

The atomic motion scatters cavity photons via collective virtual electronic excitations, causing transitions between collective motional energy levels (i.e., collective Raman scattering). In this process, the collective atomic oscillator can absorb a mechanical energy quantum, thereby down-converting the emitted photon by ω_m (Stokes scattering). If the oscillator is not in its ground state, it can alternatively up-convert the photon by ω_m, thereby losing an energy quantum (anti-Stokes scattering). By Fermi's golden rule, the rates for up-conversion and down-conversion, from an oscillator in its νth excited state, are proportional to $|\langle \nu | \hat{Z} | \nu - 1 \rangle|^2 \propto \nu$ and $|\langle \nu | \hat{Z} | \nu + 1 \rangle|^2 \propto \nu + 1$, respectively. A ground-state oscillator ($\nu = 0$) will thus scatter only to the Stokes sideband, with more symmetric scattering indicative of higher phonon occupation.

The Fourier spectrum $n(\omega)$ of photons emitted from the cavity, at a frequency ω relative to the probe and normalized by the discrete Fourier time window, can be calculated by solving a set of quantum Langevin equations [16,18]. In an experiment where the probe's amplitude fluctuations are dominated by shot noise [19], the solution is

$$n(\omega) = \frac{C_{\text{om}}}{2} \frac{\kappa^2}{\kappa^2 + \omega^2} \left[\frac{\Gamma_m^2 \bar{\nu}}{(\omega - \omega_m)^2 + \Gamma_m^2/4} \right. $$
$$\left. + \frac{\Gamma_m^2 (\bar{\nu} + 1)}{(\omega + \omega_m)^2 + \Gamma_m^2/4} \right], \qquad (1)$$

where $\bar{\nu}$ is the oscillator's mean phonon occupation. Here, we have parametrized the photon scattering rate by the dimensionless optomechanical cooperativity [20] $C_{\text{om}} \equiv 4\bar{n} g_{\text{om}}^2 / \Gamma_m \kappa$, which combines g_{om}, the mechanical damping rate Γ_m, the mean intracavity probe photon number $\bar{n}$, and the cavity half-linewidth κ ($2\pi \times 1.82$ MHz). In our atoms-based optomechanical system, $C_{\text{om}} \propto \bar{n}/\Delta_{\text{ca}}^2$, and we vary the cooperativity over orders of magnitude by tuning $\bar{n}$ between 0.1 and 2 and Δ_{ca} between -70 and -12 GHz.

We measure the Stokes asymmetry by integrating the optical power $P_{\pm}$ scattered to frequencies near $\pm \omega_m$, detected using a balanced heterodyne receiver. The detector measures a power spectral density $S_{\text{het}}(\omega) = S_{\text{SN}}\{1 + \varepsilon[n_0(\omega) + n(\omega)]/2\}$, where S_{SN} is the mean shot-noise spectrum as measured by the detector and $n_0(\omega)$ is the spectrum of technical noise, which contributes less than 1% to the observed spectrum at $\pm \omega_m$. The

quantum efficiency for measuring intracavity photons is $\varepsilon = 16\% \pm 2\%$. For each run of the experiment, we measure the spectral density S_A with atoms and S_0 without atoms (which measures n_0) and then measure S_{SN} by extinguishing the probe beam. The unitless photon spectrum is then $n(\omega) = 2(S_A - S_0)/\varepsilon S_{SN}$. In order to avoid excessive accumulated heating of the atomic gas, we acquire data for only 5 ms during each run of the experiment, after which time we observe the mechanical resonance to broaden significantly.

Measured spectra are shown in Fig. 2. At the lowest optomechanical cooperativites (viz. a weak probe detuned far from atomic resonance), the probe contributes only minimally to the phonon occupation. Rather, the collective mode occupation should approach the average thermal occupation (0.54 ± 0.02 phonons, measured via time-of-flight thermometry). We measure a large Stokes asymmetry, $P_-/P_+ = 3.0 \pm 0.8$, corresponding to a phonon occupation $\bar{\nu} = P_+/(P_- - P_+) = 0.49 \pm 0.10$. We note that such a "Stokes calorimeter" is self-calibrating, in that no experimental parameters are involved in extracting the phonon occupation number. Additionally, the optical power scattered into each sideband forms an independent, calibration-dependent measure of the phonon occupation [using $\bar{\nu} = n(\omega_m)/2C_{om} = n(-\omega_m)/2C_{om} - 1$]. The agreement between the measured and theoretically predicted sideband powers in Fig. 2(a) provides an independent verification of our measurement of phonon occupation via sideband asymmetry.

The mechanical resonance widths of the observed spectra are generally larger than the native mechanical damping rate. We measure the native damping rate by probing to the blue of cavity resonance and finding the phonon lasing threshold [19,21], at which point the mechanical damping rate is equal to the optomechanical amplification rate. In our system, we find a damping rate of $2\pi \times 0.46$ kHz. Comparing to the observed inhomogenous linewidth of $2\pi \times 3$ to 4 kHz indicates that $\hat{Z}$ dephases into approximately 7 to 9 collective normal modes, each containing 400 to 500 atoms. We suspect that this dephasing is due mostly to slight differences in trap curvature in adjacent potential minima arising from the probe beam [17,19] and partly to the anharmonicity of the standing-wave optical trap, as well as the variation of axial oscillation frequency with radial displacement. Note that this calculation represents an upper bound on the number of detected collective modes, as sources of broadening beyond dephasing could contribute to the observed widths (e.g., shot-to-shot fluctuations in the mechanical resonance frequency).

We next apply the sideband calorimeter to probe the effects of measurement backaction. A quantum measurement of position must be accompanied by a corresponding motional disturbance [22]. In this experiment, where the motion is driven by the light's radiation-pressure shot noise [14] and where $\omega_m \ll \kappa$, we expect an increase in the
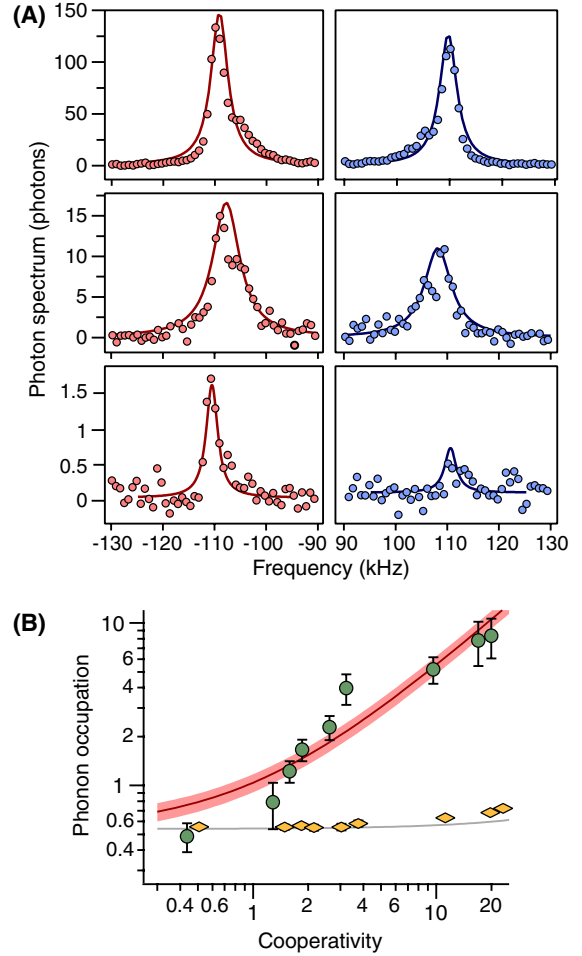

FIG. 2 (color online). Asymmetric optical scattering from quantum collective motion. (a) Photon spectra of light exiting the cavity, scattered by collective atomic motion. Shown are the measured Stokes sidebands [left panels, (red) circles] and anti-Stokes sidebands [right panels, (blue) circles] at various cooperativities (from top to bottom, $C_{om} = 9.6, 1.9, 0.4$), together with the prediction (solid lines) of Eq. (1), plotted vs cyclic frequency $f = \omega/2\pi$. The mechanical linewidths and resonance frequencies are fit to the data, but the peak heights are theoretically determined. The predicted peak areas agree with the measured areas of the photon spectra, which form a calibration-dependent measure of the phonon occupation. (b) Measured phonon occupation vs cooperativity. The collective mode occupation, determined by the green circles, increases by $C_{om}/2$, according to the zero-free-parameter measurement-backaction theory [solid (red) line; the shaded region indicates a 68% systematic confidence interval]. In contrast, the rms single-atom axial occupation, measured using time-of-flight thermometry of the gas (yellow diamonds, measured at 1 ms of probing), remains near its initial value during the measurement (the gray line indicates theoretical prediction).

phonon occupation by an amount $C_{om}/2$. As we increase the cooperativity, the sideband asymmetry decreases, as theoretically predicted. Finally, we can compare the collective mode's mean phonon occupation $\bar{\nu} = \langle (\sum_i g_i z_i/g_{om})^2 \rangle /2z_{HO}^2 - 1/2$ to an upper bound on the
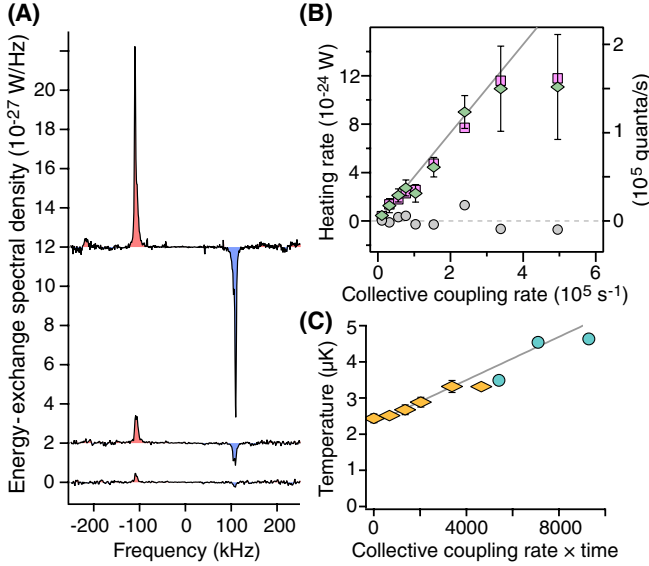
(A)

FIG. 3 (color online). Energy exchange between light and motion. (a) Energy-exchange spectral densities for various probe cooperativities (from top to bottom, $C_{om} = 9.6, 3.2, 1.6$). Traces are vertically offset for clarity (by $12, 2, 0 \times 10^{-27}$ W/Hz). (b) Heating rate due to the resonant probe (magenta squares), the weak locking tone (gray circles), and the summed heating from both tones (green diamonds), together with zero-free-parameter measurement-backaction theory (solid gray line), plotted as a function of the collective coupling rate $4\bar{n}g_{om}^2/\kappa = \Gamma_m C_{om}$. (c) Temperature of the atomic gas vs the product of the collective coupling rate and the total interrogation time (yellow diamonds at 5 ms, cyan circles at 9 ms). Also shown is the theoretical increase in the bath temperature (solid gray line).

mean single-atom phonon occupation $\bar{\nu}_a = \sum_i \langle z_i^2 \rangle / 2N_a z_{HO}^2 - 1/2$, extracted by measuring the gas temperature using time-of-flight thermometry. The upper bound is derived by assigning the gas's entire temperature increase after 1 ms of probing (equivalent to many motional equilibration times) to the mean single-atom motion along the cavity axis. The large discrepancy observed between the optically detected and single-atom occupations [Fig. 2(b)] highlights the fact that our detector senses the collective motion of the gas, rather than the motion of individual atoms.

The optical spectrum, moreover, serves as a record of the energy exchanged between light and motion. A photon recorded at frequency ω indicates the emission of an energy $\hbar\omega$ from the atoms into the cavity field. The spectral density of energy passing from the light field into motion is therefore $-\hbar\omega n(\omega)$ (units of W/Hz), and the total power passing from the light to motion is [15,18]

$$P_{om} = -\frac{1}{2\pi} \int \hbar\omega n(\omega) d\omega \approx \frac{\Gamma_m \hbar \omega_m C_{om}}{2} \frac{\kappa^2}{\kappa^2 + \omega_m^2}.$$

Heat-exchange spectra for several values of the cooperativity, as well as heating rates taken from integrating over

the sidebands, are shown in Fig. 3. The heating rates agree well with the prediction of measurement backaction. We additionally correlate the total energy exchanged with the atoms to the increase in the (finite) bath's temperature. The bath temperature should increase by an amount given by equating the backaction heat with the temperature integral of the gas's heat capacity C_N. We calculate C_N using Bose-Einstein statistics for axial motion and the ideal gas law for radial motion, yielding $C_N = \alpha(T)N_a k_B$, where $\alpha(T)$ is between 2.8 and 3.0 over the experimental temperature range. We find quantitative agreement to this theory as we vary both the cooperativity and the probe duration.

In this Letter, we have demonstrated the quantization of the collective motion of many hundreds of atoms, observing Stokes asymmetry and zero-point motion. The Stokes asymmetry provides a self-calibrating thermometer for low-occupation collective modes. We have in addition observed the spectrum of energy exchanged between light and collective atomic motion, spectroscopically identifying backaction heating from a quantum position measurement. While our system measures center-of-mass motion, other modes [12,16,17] of physical interest could be addressed by tailoring the light-motion interaction. For example, quadrupole [23–25] or scissors [26] modes could be sensed using quadratic optomechanical coupling [17], allowing for precise measurements of the effects of interactions, superfluidity, and viscosity in degenerate Bose and Fermi gases. Sideband spectroscopy of phonon modes in spatially extended gases [12] provides the means to study thermodynamics in static and driven systems [27].

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