Cooling by Heating: Refrigeration Powered by Photons

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We propose a new mechanism for refrigeration powered by photons. We identify the strong coupling regime for which maximum efficiency is achieved. In this case, the cooling flux is proportional to T in the low temperature limit $T \rightarrow 0$.

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In this age of alternative energy, solar power is one of the main contenders for providing us with clean energy. In photosynthesis, the energy of solar photons is transformed into chemical energy. In solar cells, they generate electrical energy. Solar energy can also be used directly to produce heat and drive a generator, such as in solar furnaces. The thus acquired energy can be used for a myriad of purposes, including refrigeration [1]. In this Letter, we propose a simple solid state device that uses photons to perform the task of refrigeration directly, bypassing the need to first generate another form of energy. The hope is that such a cooling mechanism will be more efficient, by avoiding some of the losses that always accompany the transformation between different forms of energy. The fact that solar radiation, which has a temperature of about 5800 K, can be directly used to refrigerate may at first be surprising. The phenomenon itself is not novel, the most notable example being probably evaporative cooling (including "sweating"); see, also, the optomechanical device recently proposed in [2]. Our device however is of an entirely different nature. It has the advantage of being a nanosized solid state device, with no moving parts, and no net electric currents, even though the cooling is the result of replacing "hot" electrons by "cold" electrons. This feature suggests that the construction could also be used in nano low temperature physics, where a cold black body radiation (for example from liquid helium) could be used as photonic input to reach much lower temperatures in an electronic nanosetting.

Before turning to a quantitative analysis of the refrigerator, we explain its mode of operation. The device, which is schematically represented in Fig. 1, consists of two leads (or electron reservoirs), separated by two adjoining quantum dots, each with a lower and upper energy level. The basic idea is that under influence of the high temperature photons, cold electrons are pumped from the left to the right lead transversing through the junction of the lower energy levels of the quantum dots, while an equal amount of hot electrons are pumped the other way at the junction of the higher energy levels. The result is a net replacement of hot electrons by cold electrons in the right lead, implying its refrigeration. A qualitative explanation goes as follows. Consider first the distributions of electrons in the lead. They obey the Fermi-Dirac distribution: energy levels below the Fermi level (or chemical potential) $\epsilon_F = \mu$, corresponding to cold electrons, are almost full, while levels above ϵ_F, corresponding to hot electrons, are almost empty. The transition between "full" and "empty" levels occurs in a small region of width $k_B T$ around the Fermi level, which itself is "half-full". Consider next the exchange of electrons between leads and quantum dots. At equilibrium, there will be no flux between them so that the occupation probabilities in each level of each quantum dot is equal to the occupation probability for the corresponding energy level in the corresponding lead. We conclude that for equilibrium with the leads, the occupation probability for an electron is larger in the lower level of the left quantum dot than in the right dot, since the energy level is further below the Fermi level, and hence more heavily occupied, in the left than in the right lead. The converse is true for the upper levels. In our device, electrons can however also be exchanged between the quantum dots, and this exchange is supposed to be modulated mainly by photons (e.g., those coming from the Sun). Because these photons have a high temperature, they tend to induce an equal occupation probability. As a result, electrons have the tendency to leave the "over-occupied lower left level", and jump to the right energy level, while the opposite electron motion will take place between the two higher levels. The result is a net circulation of electrons, as

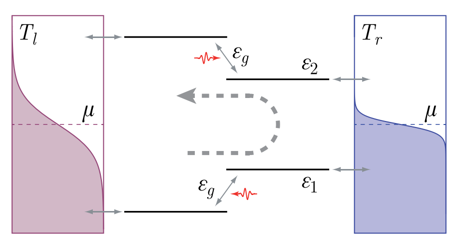

FIG. 1 (color online). Schematic representation of the solar refrigerator. Two metallic leads, at different temperatures, are connected by two quantum dots, each having two discrete energy levels. All possible electron transitions are shown by the gray arrows. Transitions induced by (solar) photons are indicated by a curly red arrow. The overall electron current through the device is shown by the dashed gray arrow.

indicated by the dashed gray arrow in Fig. 1, with cold electrons being pumped from left to right and hot electrons from right to left.

Switching to a more quantitative analysis, we have to describe in more detail the transition rules for the electrons, between lead and quantum dot and between quantum dots. Since our intention is to illustrate the principle of operation with an explicit analytic analysis, we consider a simple, quasiclassical model. We suppose that the coupling between leads and dots is weak, and that line broadening of the energy levels can be neglected. The transition rate $k^{\varepsilon}_{l\to d}$ for an electron to jump from the lead into the *empty* quantum dot is proportional to the occupation probability $f(\varepsilon)$ of the same energy level in the lead, which is the aforementioned Fermi distribution. For the inverse jump to take place, the energy level has to be empty in the lead, so that the corresponding rate $k^{\varepsilon}_{d\to l}$ is proportional to $1 - f(\varepsilon)$ (with same proportionality constant Γ):

$$\begin{aligned} k^{\varepsilon}_{l\to d} &= \Gamma f(\varepsilon); \\ k^{\varepsilon}_{d\to l} &= \Gamma(1 - f(\varepsilon)); \\ f(\varepsilon) &= [\exp((\varepsilon - \mu)/T) + 1]^{-1}. \end{aligned} \tag{1}$$

The material constant Γ sets the overall time scale of the process. Note that in order to investigate the performance as a refrigerator, we consider the case where the temperatures T_r and T_l in right and left lead, respectively, need not be the same, and it is understood that the appropriate temperature needs to be used in the above expressions for transitions from left and right lead.

Electron transitions between the quantum dots occur either between the two lower or upper energy levels. The energy difference (gap) between the levels is equal to ε_g in both cases. These transitions are mediated by the photons. An electron can jump to a higher energy level if it absorbs a photon whose energy is equal to the energy difference of the two levels involved. The corresponding transition rate $k_\uparrow$ is then proportional to the average number of photons with this energy, namely, the Bose-Einstein distribution $n(\varepsilon_g)$. The transition rate $k_\downarrow$ for the inverse process, in which the electron emits a photon, is proportional to $1 + n(\varepsilon_g)$ which takes into account both spontaneous emission and stimulated emission:

$$\begin{aligned} k_\uparrow &= \Gamma_s n(\varepsilon_g); \\ k_\downarrow &= \Gamma_s(1 + n(\varepsilon_g)); \\ n(\varepsilon_g) &= [\exp(\varepsilon_g/T_s) - 1]^{-1}. \end{aligned} \tag{2}$$

Again, the material constant Γ_s sets the overall time scale of the process. We make two additional assumptions. First, transitions between lower and higher energy levels within the same quantum dot are neglected. As a result the flow of electrons between the leads through lower and upper levels, respectively, are physically decoupled from each other, and described by an independent master equation. Second, due to Coulomb repulsion, there can be only a single electron in the quantum dots. The resulting master equation for the probabilities p_0, p_l and p_r, to find no electron, or a single electron in the left or right energy level, respectively, reads $\dot{\vec{p}}(t) = M \cdot \vec{p}(t)$ with $\vec{p}(t) = \{p_0, p_l, p_r\}^T$. The stochastic matrix M describing electron dynamics through the lower energy levels reads:

$$M = \begin{bmatrix} -k^{\varepsilon_1-\varepsilon_g}_{l\to d} - k^{\varepsilon_1}_{l\to d} & k^{(\varepsilon_1-\varepsilon_g)}_{d\to l} & k^{\varepsilon_1}_{d\to l} \\ k^{\varepsilon_1-\varepsilon_g}_{l\to d} & -k^{\varepsilon_1-\varepsilon_g}_{d\to l} - k_\uparrow & k_\downarrow \\ k^{\varepsilon_1}_{l\to d} & k_\uparrow & -k^{\varepsilon_1}_{d\to l} - k_\downarrow \end{bmatrix}. \tag{3}$$

A similar expression holds for electron dynamics through the upper levels. The solution of this equation at the steady state is a question of straightforward algebra. Of prime interest is the resulting average electron (particle) current through the dots. We define J_1 (J_2) as the particle current between the energy level ε_1 (ε_2) and the right lead. In the steady state this becomes (see also [3]):

$$J_1 = \frac{\Gamma\Gamma_s(e^{x_r} - e^{x_l+x_s})}{2\Gamma_s + (\Gamma + 2\Gamma_s)e^{x_l+x_s} + (\Gamma_s - \Gamma)e^{x_l+x_r} + (\Gamma + \Gamma_s)e^{x_l+x_r+x_s} + (\Gamma + \Gamma_s)e^{x_r+x_s} - e^{x_r}(\Gamma - 2\Gamma_s) + e^{x_l}(\Gamma_s - \Gamma) + 2\Gamma_s e^{x_s}}, \tag{4}$$

where we introduce the dimensionless variables:

$$x_l = \frac{\varepsilon_1 - \varepsilon_g - \mu}{T_l}; \qquad x_s = \frac{\varepsilon_g}{T_s}; \qquad x_r = \frac{\varepsilon_1 - \mu}{T_r}. \tag{5}$$

A similar expression holds for J_2.

The motion of the electrons between the leads and the device gives rise to an associated heat exchange. For example, an electron leaving the right lead by moving to the upper energy level ε_2 withdraws an amount of heat equal to $dQ = \varepsilon_2 - \mu > 0$ and effectively cools down that lead. The amount of heat taken by the electron follows from the more general expression $TdS(= dQ) = dU - \mu dN$ describing the change of entropy of a reservoir when an amount of energy dU and particles dN is added [4]. Similarly, an electron moving towards the right lead from the lower energy level ε_1 adds an amount of heat $\varepsilon_1 - \mu < 0$, also cooling the lead. With the aforementioned expressions for the steady state particle currents J_1 and J_2, we thus obtain the following explicit results for stationary heat fluxes $\dot{Q}_r$ (from the right lead to the device), $\dot{Q}_l$ (from the left lead to the device), and the heat flux from the photon reservoir (e.g., the Sun) $\dot{Q}_s$:

$$\dot{Q}_r = (\varepsilon_1 - \mu)(-J_1) + (\varepsilon_2 - \mu)(-J_2); \quad (6a)$$
$$\dot{Q}_l = (\varepsilon_1 - \varepsilon_g - \mu)J_1 + (\varepsilon_2 + \varepsilon_g - \mu)J_2; \quad (6b)$$
$$\dot{Q}_s = \varepsilon_g J_1 + \varepsilon_g(-J_2). \quad (6c)$$

The operation principle of the refrigerator can be formulated as follow: the photon source (Sun) is the power source which provides the energy ($\dot{Q}_s$) to draw heat from the right lead ($\dot{Q}_r$) which is then dumped in the left lead ($\dot{Q}_l$). The performance of such a refrigerator is characterized by the *coefficient of refrigerator performance* (COP) [4]:

$$\eta_{\rm cop} = \frac{\dot{Q}_r}{\dot{Q}_s}. \quad (7)$$

Before turning to a detailed analysis, we first analyze this performance using general thermodynamical arguments. During a steady state operation, the entropy of the device remains constant. As a consequence, the (negative) entropy input due to the heat exchange with the three reservoirs is compensated in the device by a positive irreversible entropy production $\dot{S}_i$:

$$\dot{S}_i = -\frac{\dot{Q}_s}{T_s} - \frac{\dot{Q}_r}{T_r} - \frac{\dot{Q}_l}{T_l} \geq 0. \quad (8)$$

Positivity of this quantity is guaranteed for any choice of model parameters [5]. Conservation of energy $\dot{Q}_l + \dot{Q}_r + \dot{Q}_s = 0$, which follows directly from Eq. (6), leads to the bilinear form:

$$\dot{S}_i = \dot{Q}_s F_s + \dot{Q}_r F_r, \quad (9)$$

where we introduce the thermodynamic forces (affinities) conjugated to the heat fluxes:

$$F_s = \frac{1}{T_l} - \frac{1}{T_s}; \qquad F_r = \frac{1}{T_l} - \frac{1}{T_r}. \quad (10)$$

The range of these forces is $F_s > 0$ and $F_r \leq 0$. Combining Eqs. (7)–(10) leads to

$$\eta_{\rm cop} = \left(1 - \frac{T_l}{T_s} - \frac{T_l \dot{S}_i}{\dot{Q}_s}\right)\left(\frac{T_r}{T_l - T_r}\right). \quad (11)$$

This expression has a simple interpretation: the first part is the Carnot efficiency of a heat engine operating between reservoirs at temperature T_s and T_l reduced by a factor related to the entropy production. This factor is a measure for the irreversibility of the processes involved. The second part gives the maximal COP for a refrigerator driven by a reversible work source [4].

The overall maximal COP is obtained when $\dot{S}_i = 0$, i.e., for reversible operation. This typically entails that both thermodynamic forces vanish, implying full equilibrium $T_r = T_l = T_s$. The latter is clearly not the case of technological interest. It is however possible to find a less stringent regime by an appropriate fine-tuning of the device parameters. The crucial step is to make the heat currents proportional to each other. By inspection of Eq. (6), one finds that this can be achieved by setting μ either equal to $(\varepsilon_1 + \varepsilon_2)/2$, or equal to the chemical potential μ^* for which the total particle current between the reservoirs $J_1 + J_2$ vanishes. Note that the latter case is of particular interest, since this implies that there is no net electric charging of the leads. In both cases we find

$$\dot{Q}_r = \frac{\varepsilon_2 - \varepsilon_1}{2}(J_1 - J_2); \qquad \dot{Q}_s = \varepsilon_g(J_1 - J_2), \quad (12)$$

which shows that $\dot{Q}_r$ and $\dot{Q}_s$ are proportional to each other upon variation of the model parameters, provided $\mu = (\varepsilon_1 + \varepsilon_2)/2$ or $\mu = \mu^*$. Such a proportionality condition between the power output ($\dot{Q}_r$) and the input energy ($\dot{Q}_s$) of an heat engine is known as tight coupling [6]. Our model underscores once more the technological importance of this condition [7,8]: the reversible regime can be reached with both $\dot{Q}_r$ and $\dot{Q}_s$ vanishing simultaneously while maintaining a nonzero COP:

$$\eta_{\rm cop} = \frac{\varepsilon_2 - \varepsilon_1}{2\varepsilon_g}. \quad (13)$$

Writing the entropy production as $\dot{S}_i = \dot{Q}_s(F_s + \eta_{\rm cop}F_r)$ shows that maximal efficiency ($\dot{S}_i = 0$) is reached when $F_s + \eta_{\rm cop}F_r = 0$, or more explicitly,

$$\varepsilon_g\left(1 - \frac{T_l}{T_s}\right) = \frac{\varepsilon_2 - \varepsilon_1}{2}\left(\frac{T_l - T_r}{T_r}\right). \quad (14)$$

Moreover, for any choice of parameters obeying Eq. (14), the output power $\dot{Q}_r$ vanishes, analogously to the maximal Carnot efficiency of a heat engine. Note that at the reversibility point, both $\dot{Q}_s$ and $(F_s + \eta_{\rm cop}F_r)$ change sign yielding $\dot{S}_i \geq 0$ as required.

We now investigate the region of parameter values for which the device functions as a refrigerator. As explained in the introduction, the photon source (Sun) operates as the energy source (and so $\dot{Q}_s \geq 0$) which drives a heat flow from the cold lead (temperature T_r) towards the hot (ambient) lead (temperature T_l). The crucial condition is thus $\dot{Q}_r \geq 0$ or $\eta_{\rm cop} \geq 0$. Since both leads have the same Fermi level μ, thermodynamic considerations require $T_s \geq T_l \geq T_r$, in agreement with the criterion that $\eta_{\rm cop} \geq 0$ cf. Eq. (11). Also in accordance with the intuitive explanation given in the introduction, we require $\varepsilon_1 \leq \mu \leq \varepsilon_2$. In case of a strongly coupled device the refrigeration window $\dot{Q}_r \geq 0$ entails both $0 \leq \eta_{\rm cop}$ and $F_s + \eta_{\rm cop}F_r \geq 0$, which summarizes to

$$0 \leq \frac{\varepsilon_2 - \varepsilon_1}{2\varepsilon_g} \leq \left(1 - \frac{T_l}{T_s}\right)\left(\frac{T_r}{T_l - T_r}\right). \quad (15)$$

When the device is not strongly coupled, one must resort to a numerical calculation for the identification of the region of refrigeration. We note [cf. Eq. (2)] that μ only appears either as $\varepsilon_1 - \mu$ or $\varepsilon_2 - \mu$. So we can conveniently set $\mu = 0$ and measure ε_1 and ε_2 with respect to this origin. For strongly coupled devices the condition on μ is then

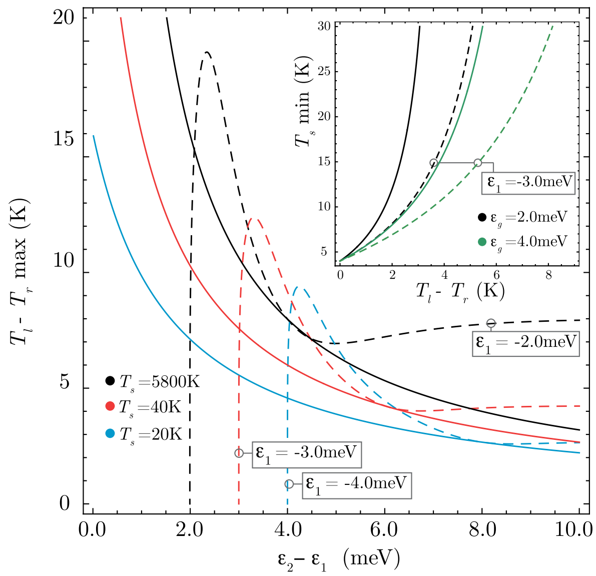

FIG. 2 (color online). Plot of the maximal temperature difference that allows cooling to take place as a function of $\varepsilon_2 - \varepsilon_1$. Parameter values: $T_r = 4$ K and $\varepsilon_g = 4.0$ meV. Inset: Minimal value of T_s required for cooling as a function of the temperature difference $T_l - T_r$. Parameter values: $T_r = 4$ K and $\varepsilon_2 - \varepsilon_1 = 4.0$ meV. The solid lines depict the results for a strongly coupled device with $\varepsilon_1 = -\varepsilon_2$.

transferred to ε_1 and ε_2. For the purpose of illustration, we choose $\Gamma_s = \Gamma$. In Fig. 2 we plot the maximum temperature difference for which cooling takes place, as a function of ε_2. In case of a strongly coupled device, this maximal temperature difference is a monotonically decreasing function of ε_2. In the inset we included the minimal temperature T_s that is required for cooling. In Fig. 3 we plot the COP in function of the cooling rate $\dot{Q}_r$. The curves are obtained by varying ε_2, with initial and final values determined by the condition $\dot{Q}_r = 0$. The other parameters are kept fixed. In the absence of strong coupling these graphs are closed loops, starting and ending in the origin. This means that $\dot{Q}_s$ does not vanish at stalling conditions: the device continues to consume energy without any cooling. In contrast, for a strongly coupled device, both $\dot{Q}_r$ and $\dot{Q}_s$ vanish at stalling conditions, yielding a nonzero COP.

We finally discuss the connection to the third law of thermodynamics: we investigate the possibility to use the device for cooling the right reservoir to absolute zero $T_r \rightarrow 0$. We focus on the case of a strongly coupled device with $\mu = 0$ and $\varepsilon_2 = -\varepsilon_1 = \varepsilon$, with the parameters T_l, T_s and ε_g fixed. Furthermore, we set

$$\varepsilon = \alpha\left(1 - \frac{T_l}{T_s}\right)\left(\frac{T_r}{T_l - T_r}\right)\varepsilon_g, \tag{16}$$

with $0 \leq \alpha < 1$, to ensure that the device operates inside the refrigeration window [cf. Eq. (15)]. We now need to extract, from the expression Eq. (12) for $\dot{Q}_r$, its behavior in the limit $T_r \rightarrow 0$. Obviously $\dot{Q}_r$ is directly proportional to

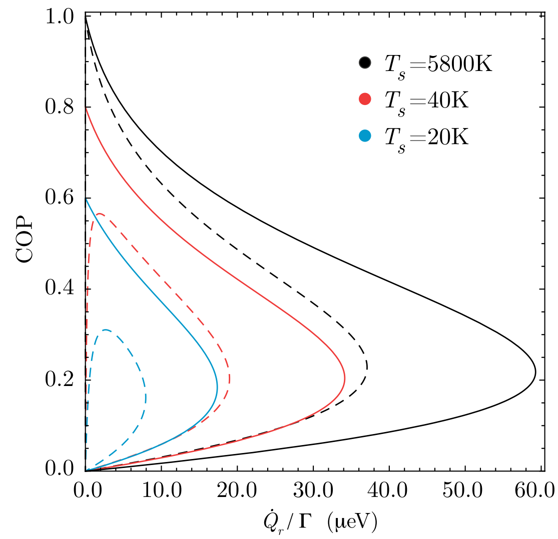

FIG. 3 (color online). Plot of the COP against cooling rate $\dot{Q}_r$. Parameter values: $T_l = 8$ K, $T_r = 4$ K, and $\varepsilon_g = 2.0$ meV. The solid lines depict the results for a strongly coupled device with $\varepsilon_1 = -\varepsilon_2$, the dashed lines correspond to $\varepsilon_1 = -2.0$ meV.

ε, and hence to T_r via Eq. (16). To investigate the behavior of the other factor $J_1 - J_2$, we find, after substitution of Eq. (16), that this quantity converges to a nonzero positive value in the limit $T_r \rightarrow 0$. In conclusion, the cooling power $\dot{Q}_r$ scales linearly with T_r, in the limit $T_r \rightarrow 0$:

$$\dot{Q}_r \sim T_r. \tag{17}$$

Such a cooling rate reaches the limit imposed by the second and third law of thermodynamics [9].

In conclusion, we propose a novel electronic photon-driven nanorefrigerator, which cools an electron reservoir (lead) by replacing its hot electrons with cold ones. The process is driven by the absorption of photons. A microscopic analysis shows that maximum efficiency for cooling can be reached by a careful fine-tuning of the system's parameters, namely, under the so-called strong coupling condition, implying proportionality of the heat fluxes. The technological interest of our device is further enhanced by the observation that this condition is compatible with and in fact derives from the absence of a net electric current.

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