

# Superconducting and Ferromagnetic Phases in SrTiO<sub>3</sub>/LaAlO<sub>3</sub> Oxide Interface Structures: Possibility of Finite Momentum Pairing

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We introduce a model to explain the observed ferromagnetism and superconductivity in LAO/STO oxide interface structures. Because of the polar catastrophe mechanism, 1/2 charge per unit cell is transferred to the interface layer. We argue that this charge localizes and orders ferromagnetically via exchange with the conduction electrons. Ordinarily, this ferromagnetism would destroy superconductivity, but, due to strong spin-orbit coupling near the interface, the magnetism and superconductivity can coexist by forming a Fulde-Ferrell-Larkin-Ovchinnikov-type condensate of Cooper pairs at finite momentum, which is surprisingly robust in the presence of strong disorder.

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**Introduction.**—It is known that a conducting electronic state can form at the interface between two insulating oxides [1]. A particularly well studied example is the TiO<sub>2</sub> interface between SrTiO<sub>3</sub> and LaAlO<sub>3</sub>. The carrier density can be controlled by a backgate on the SrTiO<sub>3</sub> side, and superconductivity (SC) has been discovered over a range of densities with maximum  $T_c$  of about 0.3 K [2]. Recently, signs of ferromagnetism (FM) have also been reported [3–7]. In particular, Li *et al.* [5] showed that SC and FM coexist in the same sample and that the FM moment is large,  $\approx 0.3\text{--}0.4\mu_B$  per interface unit cell. Assuming that the FM and SC arise from the interface, these observations raise the question of whether the SC has to be unconventional in order to coexist with FM. Before addressing this question, we need to first understand the nature of the electronic state at the interface, and up to now no clear picture has emerged [8–10]. Are most of the electrons localized or extended? Does the FM come from local moments or the mobile electrons, and what is its origin? In this Letter, we propose a model for the interface electrons which is consistent with existing transport data. Based on this model, we explain the existence of FM and the coexistence of SC and FM. For the latter, the key idea is that a large Rashba-type spin-orbit coupling exists at the interface [11]. Such a Rashba term is particularly favorable for the formation of a condensate at finite momentum, called a Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state which coexists with FM [12,13]. This general idea was pointed out earlier by Barzykin and Gor'kov [14]. However, they considered only the clean case, and their solution is quickly destroyed by disorder. Surprisingly, with increasing disorder, the FFLO state is revived [15]. We suggest that the SC observed at the interface is described by this disordered stabilized FFLO state. This state is sometimes referred to as a “helical” FFLO state [15], since pairing occurs at a single momentum  $\mathbf{q}$ , so that  $\Delta(\mathbf{r}) = \Delta e^{i\mathbf{q}\cdot\mathbf{r}}$ , unlike the usual FFLO state, where pairing occurs at both  $\pm\mathbf{q}$ , so that  $\Delta(\mathbf{r}) = \Delta \cos\mathbf{q}\cdot\mathbf{r}$ .

**The model.**—As shown in Fig. 1, LaAlO<sub>3</sub> consists of layers with alternating charged, while SrTiO<sub>3</sub> has charged-neutral layers. As a result of the charge discontinuity at the interface, an electric potential proportional to the number of LaAlO<sub>3</sub> layers is built up. This phenomenon is termed the polar catastrophe. Since the Ti ions allow for mixed valence charge compensation, to avoid the polar catastrophe, half an electron per unit cell is transferred from the surface AlO<sub>2</sub> layer to the TiO<sub>2</sub> across the interface. The electrons are expected to occupy the  $d_{xy}$  orbital on the Ti atoms. Because of the relatively narrow bandwidth, an on-site repulsion  $U$  and a nearest-neighbor Coulomb repulsion  $V$  will cause these electrons to be localized on every other interface site. This picture of local moment formation at the interface has been proposed before [9]. Superexchange

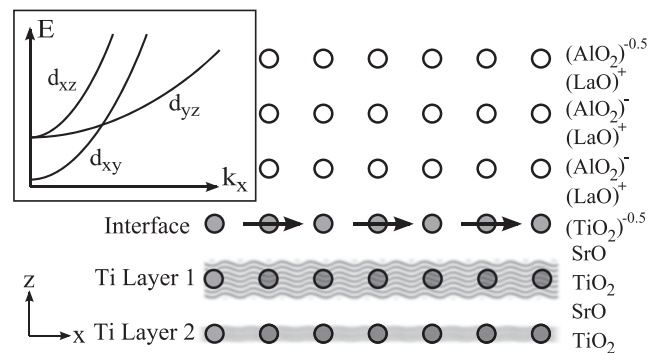


FIG. 1. Schematic depiction of the SrTiO<sub>3</sub>/LaAlO<sub>3</sub> oxide interface structure. Filled and empty circles depict Ti and Al ions. The half-electron charge per unit cell is transferred to the interface TiO<sub>2</sub> layer, which localizes and orders magnetically (shown as arrows on the interface layer) via exchange polarization of conduction electrons on the subsequent Ti layers (shown as wavy clouds in Ti Layers 1 and 2). Inset: Dispersion of electron bands arising from 3d orbitals on Ti layers near the interface.

via the oxygens is expected to provide a weak antiferromagnetic (AF) exchange.

The application of a backgate or the presence of defects forces more electrons to the interface. We assume that the effect of  $U$  and  $V$  makes it too costly to place these electrons at the interface layer. Instead, the additional electrons mainly occupy the Ti layer next to it (called layer 1), and their wave functions also spill over to adjacent layers. We model these electrons as occupying two-dimensional conducting bands. These are the electrons seen in transport measurements, with a typical areal density of  $3 \times 10^{13} \text{ cm}^{-2}$ , or about 10% of the density compared with the localized electrons. These electrons initially go into the  $d_{xy}$  band [16]. Hall effect measurements show nonlinearity in the magnetic field  $H$  with increasing gate voltage, which has been interpreted as the appearance of a second carrier with lower mobility [16–18]. We assume that these are the  $d_{xz}$  and  $d_{yz}$  bands, illustrated in the inset of Fig. 1. These bands are highly anisotropic, with a heavy mass in one direction which may be responsible for the lower mobility. The  $d_{xz}$  and  $d_{yz}$  bands are higher in energy because their lobes point towards the negative charge at the interface and because their bandwidths are narrower.

In addition, transport measurements show that the elastic scattering rate  $1/\tau$  drops rapidly with increasing carrier density [18]. Furthermore, from the analysis of magnetoresistance, it was found that a Rashba term  $H_R = \alpha \hat{z} \cdot (\boldsymbol{\sigma} \times \mathbf{k})$  grows rapidly with the gate voltage [11], reaching a spin splitting  $\Delta_{so} = 2\alpha k_F$  of 10 meV near the peak of the superconducting dome, a value comparable to the Fermi energy of  $\approx 40$  meV. Since a backgate voltage tends to pull carriers from the interface, the decrease of  $1/\tau$  is reasonable but the increase of  $\Delta_{so}$  is counterintuitive. We believe this trend is a consequence of an increasing admixture of the  $d_{xz}$  and  $d_{yz}$  bands with increasing carrier density. The Rashba energy is determined by the polarization of the electron wave function due to the asymmetric environment at the interface, and the contributions come mainly from near the atomic core, where the electron is subject to a large electric field  $-\partial V/\partial z$ :

$$\Delta_{so} \propto |k_{\parallel}| \frac{2}{c^2} \int d\mathbf{r} \frac{dV}{dz} |\psi_{k_{\parallel}}(\mathbf{r})|^2, \quad (1)$$

where the wave function  $\psi_{k_{\parallel}}(\mathbf{r}) = \sum_{\ell} a_{\ell} \phi_{k_{\parallel}}^{\ell}(\mathbf{r})$  and  $\ell$  denotes various angular momenta which are admixed due to the asymmetric environment of the interface [19]. Let us restrict ourselves to admixtures between  $d$ ,  $p$ , and  $s$  states. Since  $\nabla V \propto \hat{z}$ , nonvanishing contributions involving the  $d$  bands in Eq. (1) come only from the cross term between  $d_{xz}$  and  $p_x$  and between  $d_{yz}$  and  $p_y$ . Furthermore, the  $d_{xy}$  band can have a nonzero  $\Delta_{so}$  only via the admixture of either  $s$  and  $p$  or, more importantly, of  $d_{xz}$  with  $p_y$  and  $d_{yz}$  with  $p_x$ . We expect that the latter hybridization between  $d$  and  $p$  orbitals gives rise to  $\Delta_{so}$ , which increases with chemical potential.

The final ingredient of our model is the exchange coupling between the local moments and the conduction electrons. We write the standard phenomenological form

$$H_J = J_K \sum_i \int d\mathbf{r} \hat{S}_i \cdot \hat{s}(\mathbf{r}) \delta(\mathbf{R}_i - \mathbf{r}), \quad (2)$$

where  $\hat{s}(\mathbf{r}) = \frac{1}{2} \psi_{\alpha}^{\dagger} \boldsymbol{\sigma}_{\alpha\beta} \psi_{\beta}(\mathbf{r})$  is the electron spin density operator in the  $d_{xy}$  band and  $\hat{S}_i$  is the local spin operator on site  $i$ . We introduce a similar coupling  $J'_K$  for the  $d_{xz}$  and  $d_{yz}$  bands. It is useful to introduce  $J_0 = J_K/n_0$ , where  $n_0$  is the inverse of the interface unit cell area, and similarly  $J'_0 = J'_K/n_0$ . The Schrieffer-Wolff expression is  $J_0 = 2\tilde{t}^2(\frac{1}{U+\epsilon_d} - \frac{1}{\epsilon_d})$ , where  $\tilde{t}$  is the hybridization between the local moment and the conduction band orbital and  $\epsilon_d < 0$  is the orbital energy of the local moment relative to the chemical potential. We note that the  $d_{xz}$  and  $d_{yz}$  orbitals in layer 1 are orthogonal to the localized  $d_{xy}$  orbital in the interface layer, so that the hybridization  $\tilde{t}$  vanishes except for the admixture of other orbitals in the  $d_{xz}$  and  $d_{yz}$  bands. We therefore expect  $J'_0 \ll J_0$ .

*The origin of ferromagnetism.*—We note that the present problem is in the opposite limit to the familiar problem of dilute Kondo impurities, where the Kondo screening of the local moments competes with RKKY interactions between them. Here, the density of local moments  $n_i = \frac{1}{2}n_0$  is much greater than the conduction electron density  $n_c$ ; i.e., the separation between local moments  $1/\sqrt{n_i}$  is much smaller than  $k_F^{-1}$ . We can still view the local moments as interacting via RKKY interactions, but this interaction will be ferromagnetic and with a relatively long range of  $(2k_F)^{-1}$ . The FM ordering temperature  $T_F$  can be worked out [20], and, apart from a numerical constant, the result was shown to be equivalent to a mean field treatment of  $H_J$ , which we shall adopt below. In this picture, which was introduced by Zener [21] and is referred to as the Zener kinetic exchange mechanism, the local moments order by polarizing the conduction electrons. This mechanism has been applied successfully to explain the FM of Mn substitution in GaAs, and we borrow the results here [22]. We introduce the average localized spin order per site  $\mathcal{S} = \frac{1}{N_i} \sum_i \langle \hat{S}_i \rangle$  and the average electron density  $\mathbf{s} = \frac{1}{\text{vol}} \int d\mathbf{r} \langle \hat{s}(\mathbf{r}) \rangle$ . To quadratic order, the total free energy density takes the form

$$E_{\text{tot}} = \frac{1}{2} \frac{|\mu_0 \mathcal{S}|^2}{\chi_0} n_i + \frac{1}{2} \frac{|\mu_0 \mathbf{s}|^2}{\chi_c} + J_0 \frac{n_i}{n_0} \mathcal{S} \cdot \mathbf{s}. \quad (3)$$

The last term is the mean field decoupling of Eq. (2). In the first term,  $\xi_0 = \frac{\mu_0^2 S(S+1)}{T+\Theta}$ , where  $\mu_0 = g\mu_B$ ,  $S = 1/2$ ,  $g = 2$ , and  $\Theta > 0$  is the Weiss term due to the weak AF superexchange which we shall ignore below. In the second term,  $\chi_c = \frac{1}{4} \mu_0^2 \nu(0)$ , where  $\nu(0) = m^*/\pi\hbar^2$  is the density of states, including the spin of a free electron gas. (The presence of a Rashba term does not change the spin sus-

ceptibility of a free electron gas [14,23].) Minimizing Eq. (3) with respect to  $\mathcal{S}$  leads to a purely quadratic term in  $|\mathcal{S}|^2$ , and the sign change of its coefficient determines the FM transition temperature

$$k_B T_F = \frac{S(S+1)}{12} J_0^2 \frac{n_i}{n_0} \frac{\nu(0)}{n_0}. \quad (4)$$

We find that  $\nu(0)/n_0 = 0.64(m_{xy}^*/m_e) \text{ eV}^{-1}$ . For  $m_{xy}^*/m = 0.7$ ,  $J_0 = 1.3 \text{ eV}$  will give the estimated  $T_F \approx 300 \text{ K}$ . By comparison, for Mn/GaAs,  $J_0$  is  $\approx 1 \text{ eV}$ . Here,  $\tilde{t}$  is smaller, but  $|\varepsilon_d|$  is also smaller because the same orbital is involved in the local moment and the conduction electron, so the estimated  $J_0$  appears reasonable. Thus, we conclude that the Zener kinetic exchange mechanism can account for a robust FM state.

Next, we estimate the polarization of the conduction electron. In the mean field theory, the effect of  $\mathcal{S}$  on the conduction electrons is described by an effective Zeeman field  $H_J \approx \int dr \mu_0 \mathbf{H}_{\text{MF}} \cdot \mathbf{s}(r)$ , where  $\mathbf{H}_{\text{MF}} = J_0 \frac{n_i}{n_0} \mathcal{S}$ . Li *et al.* [5] reported an ordered moment of  $= 0.3 \mu_B$  per interface unit cell, i.e.,  $0.6 \mu_B$  per local moment in our picture, which implies  $|\mathcal{S}| = 0.3$ . Using  $J_0 = 1.3 \text{ eV}$ , we estimate a Zeeman spin splitting  $|\mu_0 \mathbf{H}_{\text{MF}}| \approx 200 \text{ meV}$ , which is comparable to or exceeds the Fermi energy. Thus, the  $d_{xy}$  band is largely spin polarized. Since  $J_0' \ll J_0$ , we expect a smaller (but still significant) polarization of the  $d_{xz}$  and  $d_{yz}$  bands.

*Nature of the superconducting state.*—We assume that the superconductivity originates from a conventional electron-phonon coupling mechanism, which is modeled by an attractive short-range interaction  $g$  with a cutoff given by the Debye frequency  $\omega_D$ . Since the superconducting transition temperature is  $T_c \approx 0.3 \text{ K}$ , the pairing gap  $\approx 0.04 \text{ meV}$  is the lowest energy scale in the problem. In the  $d_{xy}$  band, we estimate a Zeeman splitting of  $0.2 \text{ eV}$ , which exceeds the Pauli limit by more than 3 orders of magnitude and precludes the possibility of pairing in the  $d_{xy}$  band. The  $d_{xz}$  and  $d_{yz}$  bands will also be partially polarized due to the exchange interaction. However, the exchange splitting,  $\mu_0 B$ , in these bands is expected to be much smaller (although likely still  $\mu_0 B \gg \Delta$ ). Moreover, as we argue above, we expect that the Rashba spin-orbit coupling,  $\Delta_{\text{so}}$ , is even larger in the  $d_{xz}$  and  $d_{yz}$  bands than that observed in the  $d_{xy}$  bands. It is natural to look to strong spin-orbit coupling to preserve pairing in the  $d_{xz}$  and  $d_{yz}$  bands, despite large Zeeman splitting.

The enhancement of  $B_c$  due to Rashba spin-orbit coupling was first demonstrated in Ref. [14] for the case of weak Rashba coupling ( $\Delta_{\text{so}} \ll \varepsilon_F$ ) and no disorder and later in Ref. [15] for the case of weak Rashba coupling and moderate disorder. They showed that an FFLO state is favored, where the pairing occurs with a finite center-of-mass momentum [24]. Here, we extend their analysis to treat arbitrarily strong values of  $\Delta_{\text{so}}$  and disorder. We begin

by neglecting disorder and find the susceptibility to form Cooper pairs at finite pair momentum  $\mathbf{q} = q\hat{y}$ . The dispersion for the  $\pm$  Rashba branches is  $\varepsilon_{\mathbf{k}+q/2}^{\pm} = \frac{(\mathbf{k}+q/2)^2}{2m} - \mu \pm \alpha \sqrt{k_x^2 + (k_y + q/2 + \mu_0 B/\alpha)^2}$  [25]. In the physically relevant limit  $v_F q, B \ll \Delta_{\text{so}}$ , and we can expand in  $q$  and  $B$  to

$$\varepsilon_{\mathbf{k}+q/2}^{\pm} \approx \varepsilon_{\mathbf{k}}^{\pm}(B=0) + (v_F q/2 \pm \mu_0 B) \sin \phi_k + \mathcal{O}[(v_F q)^2/\Delta_{\text{so}}, (\mu_0 B)^2/\Delta_{\text{so}}], \quad (5)$$

where  $v_F = \sqrt{\alpha^2 + 2\mu/m}$  is the Fermi velocity for the Rashba bands and  $\phi_k = \tan^{-1}(\frac{k_x}{k_y})$ . The key is that choosing  $q = \frac{2\mu_0 B}{v_F}$  [26] ensures  $\varepsilon_{\mathbf{k}+q/2}^{\pm} = \varepsilon_{\mathbf{k}+q/2}^{\pm} + \mathcal{O}(\frac{B^3}{\Delta_{\text{so}}})$  for all angles  $\phi_k$ . This should be contrasted with the usual FFLO case without spin-orbit coupling, where the linear terms cannot be cancelled for all angles for any choice of  $q$ . However, we cannot prevent an energy mismatch in both bands simultaneously. By choosing  $\mathbf{q} = 2\mu_0 B \hat{y}/v_F$ , we optimize for the  $\varepsilon_-$  branch, which has a larger density of states,  $\nu_-$ , and find

$$\mu_0 B_c \approx \Delta_0 (\Delta_{\text{so}}/\Delta_0)^{(1+\alpha/v_F)/(2+\alpha/v_F)}, \quad (6)$$

where  $\Delta_0 = \omega_D \exp[\frac{-1}{(v_+ + v_-)g}]$  is the superconducting gap in the absence of the Zeeman field  $\mu_0 B$ . In the limit  $\alpha \ll v_F$ , we recover the results of Barzykin and Gor'kov:  $\mu_0 B_c \approx \Delta_0 \sqrt{\Delta_{\text{so}}/\Delta_0}$ . For the oxide interface system, we expect stronger spin-orbit coupling,  $\Delta_{\text{so}} \approx \varepsilon_F$ , and find an even larger enhancement:  $\mu_0 B_c \approx \Delta_0 (\Delta_{\text{so}}/\Delta_0)^{2/3}$ .

The above calculation is only valid in very clean systems, for which  $\Delta_0 \tau \gg 1$ . In practice, we expect to be in the dirty limit,  $\Delta_0 \tau \ll 1$ . To incorporate impurity scattering, we consider spinless, short-ranged impurities and compute the disorder-averaged Cooper-channel susceptibility in the limit  $\varepsilon_F \tau \gg 1$ , by summing the ladder diagrams for impurity scattering (called the Cooperon). As shown in Fig. 2, there are 3 regimes. First, in the weak disorder regime ( $\tau^{-1} < \Delta_0$ ), we find that the critical field drops rapidly to the Pauli limit  $B_c \approx \Delta_0 (\Delta_0 \tau)^{(1+\alpha/v_F)/(1-\alpha/v_F)}$ . This can be understood as

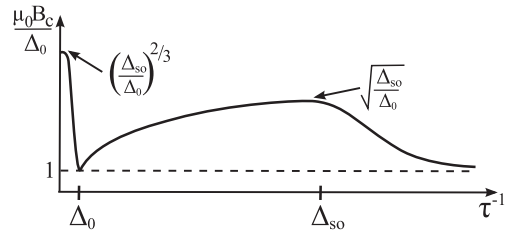


FIG. 2. Critical Zeeman splitting,  $B_c$ , measured with respect to the bare SC gap  $\Delta_0$  as a function of disorder strength  $\tau^{-1}$ . The FFLO state identified in [14] is rapidly destroyed as  $\Delta_0 \tau \rightarrow 1$ . Remarkably, for stronger disorder, the FFLO state reemerges and  $B_c$  is enhanced beyond the Pauli limit (shown as a dashed line).



follows: as in the clean case, the pair momentum minimizes the effect of the magnetic field in the  $\varepsilon_-$  branch while keeping the pair breaking in the  $\varepsilon_+$  branch. However, impurities can scatter Cooper pairs from the  $-$  band to the  $+$  band, where they rapidly decohere. Thus, disorder enhances the dephasing effects of the Zeeman field, which becomes fully pair breaking as  $\tau^{-1} \rightarrow \Delta_0$ . On the other hand, for very strong disorder,  $\tau^{-1} \gg \Delta_{\text{so}}$ , the Rashba bands  $\varepsilon^\pm$  lose their identities due to the rapid impurity scattering. Here, spin and momentum become decoupled and the problem reduces to that of conventional parabolic bands with effective spin-orbit scattering rate  $\tilde{\tau}_{\text{so}}^{-1} = \Delta_{\text{so}}^2 \tau \ll \tau^{-1}$ . This is the D'yakonov-Perel limit where the spin diffuses in small steps between rapid impurity scattering [27]. In this limit, it was demonstrated in [28] that  $\mu_0 B_c \approx \Delta_0 / \sqrt{\Delta_0 \tilde{\tau}_{\text{so}}}$ , and SC occurs at  $q = 0$ .

The interesting limit is the intermediate regime  $\Delta_0 \ll \tau^{-1} \ll \Delta_{\text{so}}$  [15]. Here, the disorder is weak enough that the Rashba bands maintain their identity, but a pair in the  $\varepsilon^+$  band can readily be scattered to a pair in the  $\varepsilon^-$  band. Unlike the weak disorder case, the pairing is strongly admixed and it is not possible to sacrifice the pair breaking of one band in favor of the other. On the other hand, due to the spin-orbit coupling, both spin (the Zeeman energy) and momentum show diffusive behavior. As a consequence, the pair breaking effect of the magnetic field is weaker. The pairing interaction is dominated by the Cooperon,  $\mathcal{C}$ , which ordinarily develops a diffusion pole  $\mathcal{C} = \frac{1}{i\omega + Dq^2}$ , with  $D = \frac{v_E^2 \tau}{2}$ , but, in the present case,  $q$  is replaced by  $q \pm \frac{2\mu_0 B}{v_F}$  for the  $\pm$  bands, respectively. Because of the strong mixing of the pairing channels, the effective pair breaking strength is given by the combination  $\sum_{\lambda=\pm} \frac{v_\lambda}{4m} (v_F q + \lambda 2\mu_0 B)^2$ . Since  $v_+ \neq v_-$ , this combination is minimized by finite momentum  $q = \frac{4\alpha B}{\alpha^2 + v_F^2}$ , and we predict that an FFLO state exists in the intermediate regime but with a different  $q$  from the clean case. The corresponding  $B_c$  is  $\mu_0 B_c \approx \frac{\Delta_0}{\sqrt{\Delta_0 \tau}}$ . Physically, on this limit,  $\tau_B \ll \tau$ , and, unlike the weak disorder, the pair dephasing time  $\tau_B^2/\tau$  grows with increasing disorder.

*Discussion.*—To summarize, we propose a model to explain the coexistence of SC and FM observed in STO/ALO oxide interface structures. In this model, a half-charge per unit cell is transferred to the interface layer and forms a lattice of local moments due to Coulomb repulsion. These local moments then order ferromagnetically via exchange with lower-density bands of mobile electrons residing in Ti layers near the interface. The large FM exchange would ordinarily kill SC in these mobile bands. However, the presence of a large spin-orbit coupling enables the formation of an FFLO state which can coexist with strong magnetism. In this FFLO state, Cooper pairs form with finite pair momentum perpendicular to the direction of magnetic ordering. Unlike the usual FFLO state

without spin-orbit coupling [12,13], spin-orbit coupling parametrically enhances  $B_c$  beyond the Pauli limit and enables FFLO pairing to survive to much stronger disorder (up to  $\Delta_{\text{so}} \tau \gtrsim 1$ ). Experimentally,  $\tau^{-1}$  decreases with backgate voltage  $V_G$  while simultaneously  $\Delta_{\text{so}}$  increases. In our picture, the observed  $T_c$  vs  $V_G$  dome is related to the behavior of  $B_c$  shown in Fig. 2 sweeping from  $\tau^{-1} > \Delta_{\text{so}}$  to  $\tau^{-1} < \Delta_{\text{so}}$ .

We wish to emphasize that our model suggests that the FM and SC occur in different bands and are uniformly distributed at the interface. Another possible explanation of the coexistence is in the spatial separation of the two phases. Indeed, a recent experiment [6] shows that the FM is arranged in domains and might not occupy the entire interface layer. However, the large total magnetic moment of the system [5] implies that the domains should occupy most of the area at the interface. The SC order parameter, in contrast, does not go to zero at any point in the plane [6].

Before concluding, we briefly discuss some experimental signatures of our model. First, if the exchange coupling for the  $d_{xz}$  and  $d_{yz}$  bands is such that the conduction electrons polarize in the opposite direction of the interface moments, then the external in-plane field  $H_{\parallel}$  would align the local moments along  $H_{\parallel}$  but would reduce the total Zeeman field seen by the  $d_{xz}$  and  $d_{yz}$  bands to  $\mu_0(B - H_{\parallel})$ . This leads to the unusual prediction that  $T_c$  should exhibit a maximum at finite  $H_{\parallel}$ . Second, one can look for  $\mathbf{q} \neq 0$  pairing by creating a Josephson tunneling junction with a conventional SC film. By applying a magnetic field parallel to the junction, Cooper pairs tunnel at finite momentum  $\Delta \mathbf{k}$  and the Josephson current would peak when  $\Delta \mathbf{k} = \mathbf{q}$ .

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- [25] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.108.117003> for a more detailed analysis, including the full anisotropic dispersions of the  $d_{xz}$  and  $d_{yz}$  bands. In the Supplement, we show that including band anisotropy does not introduce any important changes, and hence, we choose to illustrate our results for the simpler isotropic Rashba band.
- [26] Here and throughout, we have assumed  $\alpha > 0$ ; for  $\alpha < 0$ , one should replace  $\mathbf{q} \rightarrow -\mathbf{q}$ .
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