# Measuring a Cosmological Distance-Redshift Relationship Using Only Gravitational Wave Observations of Binary Neutron Star Coalescences

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Detection of gravitational waves from the inspiral phase of binary neutron star coalescence will allow us to measure the effects of the tidal coupling in such systems. Tidal effects provide additional contributions to the phase evolution of the gravitational wave signal that break a degeneracy between the system's mass parameters and redshift and thereby allow the simultaneous measurement of both the effective distance and the redshift for individual sources. Using the population of $\mathcal{O}(10^3–10^7)$ detectable binary neutron star systems predicted for 3rd generation gravitational wave detectors, the luminosity distance-redshift relation can be probed independently of the cosmological distance ladder and independently of electromagnetic observations. We conclude that for a range of representative neutron star equations of state the redshift of such systems can be determined to an accuracy of 8%–40% for $z < 1$ and 9%–65% for $1 < z < 4$.

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*Introduction.*—Making use of gravitational wave (GW) sources as standard sirens [the GW analog of electromagnetic (EM) standard candles] was first proposed in [1]. It was noted that the amplitude of a GW signal from the coalescence of a compact binary such as a binary neutron star (BNS) is a function of the redshifted component masses and the luminosity distance. Since the former can be estimated separately from the signal phase evolution, the luminosity distance can be extracted and such systems can be treated as self-calibrating standard sirens. This indicated that GW observations do not require the cosmological distance ladder to measure distances but concluded that EM observations would be needed to measure the redshift of GW sources. Upon detection of a GW signal from a compact binary coalescence, one could localize the source on the sky using a network of GW detectors. The host galaxy of the source could then be identified and used to obtain accurate redshift information while inferring the luminosity distance from the GW amplitude. This idea that GW and EM observations could complement each other in this way was subsequently extended to include the fact that BNS events are now thought to be the progenitors of most "short-hard" gamma-ray bursts (GRBs) [2]. The expected temporal coincidence of these events would allow the more accurately measured sky position of the GRB to be used to identify the host galaxy. Recent work [3–6] has explored the technical details regarding the data analysis of BNS standard sirens with respect to the advanced, 3rd generation ground-based GW detectors with the aim of investigating the potential of GW observations as tools for performing precision cosmology. The possibility of cosmological measurements with space-based detectors events is also promising [7,8]. In addition we note that statistical arguments based on the assumed neutron star (NS) mass distribution can also be used to infer redshift information from BNS events [9]. This novel approach is similar to the work we present here in that it is independent of EM counterparts.

The operation of the initial generation of interferometric GW detectors has been successfully completed. This comprised a network of four widely separated Michelson interferometers: the Laser Interferometer Gravitational-wave Observatory (LIGO) detectors [10] in Washington and Louisiana, U.S., GEO600 [11] in Hannover, Germany, and Virgo [12] in Cascina, Italy. We now await the construction of the advanced detectors [13] which will recommence operations around 2015 and promise to provide the first direct detection of GWs. It is expected that in this advanced detector era the most likely first detections will be from compact binary coalescences of BNS systems for which detector configurations are being tuned [14]. Astrophysical estimates suggest a rate of detection of at least a few, and possibly a few dozen, per year [15] with typical signal-to-noise-ratio (SNR) $\sim$10. Already much effort has been spent on the design of a 3rd generation GW detector, the Einstein Telescope (ET) [16], which is anticipated to be operational by approximately 2025. It is designed to be $\sim$10 times more sensitive in GW strain than the advanced detectors and as such we would expect to detect $\mathcal{O}(10^3–10^7)$ BNS events per year [4,15] with SNRs ranging up to $\sim$100.

In this Letter we highlight an important feature associated with the information that we will be able to extract from BNS waveforms using 3rd generation GW interferometers, in particular, ET [16]. We show that the addition of the tidal coupling contribution to the GW waveform breaks the degeneracy present in post-Newtonian (PN) waveforms between the mass parameters and the redshift. This will then allow the measurement of the binary rest-frame masses, the luminosity distance, and redshift simultaneously for individual BNS events. We base our work on the assumption that the detections of BNS and black-hole–neutron star coalescences made using both the advanced detectors and ET (specifically the nearby high-SNR signals) would tightly constrain the universal NS core equation of state (EOS) [17–20]. Once the EOS is known, the tidal effects are completely determined by the component rest-frame masses of the system. Exploitation of these effects would then remove the requirement for coincident EM observations (so-called "multimessenger" astronomy) to obtain redshift information. In using GRB counterparts, for example, host galaxy identification [21] can sometimes be unreliable, and we also require that the emission cone from the GRB is coincident with our line of sight. Current estimates of the half-opening angles of GRBs lie in the range 8°–30° [22,23], which coupled with the fact that only some short-hard GRBs have measured redshifts imply that only a small fraction ($\sim 10^{-3}$) of BNS events will be useful as standard sirens. Removing the necessity for coincident EM observations will allow all of the $\mathcal{O}(10^3–10^7)$ BNS events seen with ET to be assigned a redshift measure independent of sky position. Each of these detected events provides a measure of the luminosity distance-redshift relation ranging out to redshift $z \approx 4$. With so many potential sources the observed distribution of effective distance (the actual luminosity distance multiplied by a geometric factor accounting for the orientation of the binary relative to the detector) within given redshift intervals will allow the accurate determination of actual luminosity distance and consequently of cosmological parameters including those governing the dark energy equation of state. Such a scenario significantly increases the potential for 3rd generation GW detectors to perform precision cosmology with GW observations alone.

In our analysis we use a Fisher matrix approach applied to a PN frequency domain waveform to estimate the accuracy to which the redshift can be measured. We also assume nonspinning component masses and treat the waveform as valid up to the innermost-stable-circular orbit (ISCO) frequency, the implications of which are discussed later in the text.

*Signal model.*—We follow the approach of [24,25] in our determination of the uncertainties in our inspiral waveform parameters. We use as our signal model the frequency domain stationary phase approximation [26] to the waveform of a nonspinning BNS inspiral,

$$\tilde{h}(f) = \sqrt{\frac{5}{24}}\pi^{-2/3} Q(\boldsymbol{\varphi}) \frac{\mathcal{M}^{5/6}}{r} f^{-7/6} e^{-i\Psi(f)}, \quad (1)$$

where we are using the convention $c = G = 1$. We define the total rest mass $M = m_1 + m_2$ and the symmetric mass ratio $\eta = m_1 m_2/M^2$, where $m_1$ and $m_2$ are the component rest masses. The chirp mass $\mathcal{M}$ is defined as $\mathcal{M} = M\eta^{3/5}$, $r$ is the proper distance to the GW source and $\Psi(f)$ is the GW phase. The quantity $Q(\boldsymbol{\varphi})$ is a factor that is determined by the amplitude response of the GW detector and is a function of the nuisance parameters $\boldsymbol{\varphi} = (\theta, \phi, \iota, \psi)$, where $\theta$ and $\phi$ are the sky position coordinates and $\iota$ and $\psi$ are the orbital inclination and GW polarization angles, respectively. The standard post-Newtonian point-particle frequency domain phase can be written as [25,27]

$$\Psi_{\rm PP}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta x^{5/2}} \sum_{k=0}^{N} \alpha_k x^{k/2}, \quad (2)$$

where we use the post-Newtonian dimensionless parameter $x = (\pi M f)^{2/3}$ and the corresponding coefficients $\alpha_k$ given in [25]. Throughout this work we use $N = 7$ corresponding to a 3.5 PN phase expansion (the highest known at the time of publication). The parameters $t_c$ and $\phi_c$ are the time of coalescence and phase at coalescence and we use $f$ to represent the GW frequency in the rest frame of the source. Note that if the signal is modeled using the point-particle phase such that $\Psi(f) = \Psi_{\rm PP}(f)$ then the detected signal $\tilde{h}(f)$ is invariant under the transformation $(f, \mathcal{M}, r, t) \to (f/\xi, \mathcal{M}\xi, r\xi, t\xi)$, where $\xi$ is a Doppler-shift parameter. For BNS systems at cosmological distances the frequency is redshifted such that $f \to f/(1+z)$, where $z$ is the source's cosmological redshift. Therefore, using the point-particle approximation to the waveform one is only able to determine the "redshifted" chirp mass $\mathcal{M}_z = (1+z)\mathcal{M}$ and the so-called luminosity distance $d_L = (1+z)r$. This implies that it is not possible to disentangle the mass parameters and the redshift from the waveform alone if the proper distance is unknown.

The leading-order effects of the quadrupole tidal response of a neutron star on post-Newtonian binary dynamics have been determined [17,28] using Newtonian and 1PN approximations to the tidal field. The additional phase contribution to a GW signal from a BNS system is given by

$$\begin{aligned}\Psi^{\rm tidal}(f) = \sum_{a=1,2} \frac{3\lambda_a}{128\eta}\bigg[&-\frac{24}{\chi_a}\bigg(1 + \frac{11\eta}{\chi_a}\bigg)\frac{x^{5/2}}{M^5} \\ &- \frac{5}{28\chi_a}(3179 - 919\chi_a \\ &- 2286\chi_a^2 + 260\chi_a^3)\frac{x^{7/2}}{M^5}\bigg], \end{aligned} \quad (3)$$

where we sum over the contributions from each NS (indexed by $a$). The parameter $\lambda = (2/3)R_{\rm NS}^5 k_2$ characterizes the strength of the induced quadrupole given an external tidal field and is a function of the $l = 2$ tidal Love number (apsidal constant) $k_2$ for each NS [19,29]. We have also defined $\chi_a = m_a/M$. Note that the tidal contributions to the GW phase in Eq. (3) have the frequency dependences of $x^5$ and $x^6$ and are 5PN and 6PN when viewed in the context of the point-particle post-Newtonian phase expansion [Eq. (2)]. However, for NSs, their coefficients are $\mathcal{O}(R_{\rm NS}/M)^5 \sim 10^5$, making them comparable in magnitude with the 3PN and 3.5PN phasing terms.

For a chosen universal NS EOS, the perturbation of a spherically symmetric NS solution for a given NS mass determines the NS radius $R_{\rm NS}$, Love number $k_2$, and therefore also the tidal deformability parameter $\lambda_a$. For the purposes of this work we use the relationship between the NS mass and the tidal deformability parameter expressed graphically in Fig. 2 of [17]. Our approach models this relationship as a first-order Taylor expansion around the canonical NS mass value such that $\lambda(m) = \lambda_{1.4} + (d\lambda/dm)_{1.4}(m - 1.4M_\odot)$, where $\lambda_{1.4}$ and $(d\lambda/dm)_{1.4}$ are the values of the tidal deformability parameter and its derivative with respect to mass, both evaluated at $m = 1.4M_\odot$.

We now highlight the fundamental feature of this work. With the addition of the tidal phase components to the total GW phase such that $\Psi(f) = \Psi_{\rm PP}(f) + \Psi_{\rm tidal}(f)$, the waveform is no longer invariant under the type of transformation discussed above. The point-particle PN phase as measured at the detector is a function of the redshifted chirp mass $\mathcal{M}_z$ and luminosity distance $d_L$ in contrast to the tidal phase component which contains terms dependent upon the unredshifted rest-frame mass components $m_1$ and $m_2$. The degeneracy between the mass parameters and the redshift is therefore broken and one can now theoretically measure both sets of quantities independently of one another. Essentially, the NS size provides a fixed scale length that is imprinted on the GW waveform. The ability to perform this measurement is based on the assumption that one knows or has a very well constrained NS EOS. As shown in [20], in the advanced detector era, departures from the point-particle limit of the GW waveform as the stars approach their final plunge and merger will place strong constraints on the EOS-dependent tidal response of neutron stars. In the 3rd generation GW detector era, specifically the ET [16], the subset of high-SNR BNS signals from local galaxies will provide even tighter constraints. The addition of future EM observational constraints on the EOS (as can be seen currently in [30,31]) will also contribute to a well-understood NS EOS by the ET era.

The choice of upper cutoff frequency for our model is an important issue. The standard approach to tidal effects on GW waveforms has been to use only the Newtonian tidal correction term and to truncate the signal corresponding to a rest-frame GW frequency of 450 Hz. The primary reason that such a choice has been made is to limit the contributions to the phase evolution from various higher order effects to $<10\%$. With the addition of the 1PN tidal phase correction, and neglecting the small known higher-multipole contributions [32], the tidal description is limited by nonlinear and resonant tidal effects [33] at the end of inspiral. Concurrently, the PN formalism also breaks down at the ISCO frequency $f_{\rm ISCO} = (6^{3/2}\pi M)^{-1}$ ($\sim 1500$ Hz for $1.4M_\odot$ BNS systems) where the secular approximation, that the mode frequency is large compared to the orbit frequency, also becomes invalid. However, recent numerical relativity (NR) simulations [34–36] show that EOS effects can be accurately modeled in the late inspiral, and that the waveform contains an EOS signature that is amenable to analytic modeling. In anticipation of future analytical models valid up to the merger phase, we choose to use the ISCO as our cutoff frequency, noting that this is applied using unredshifted mass in the source's local frame.

The standard Fisher matrix formalism [37,38] allows us to compute the uncertainties associated with the measurement of a set of signal parameters. In the large SNR regime under the assumption of Gaussian noise the signal parameters $\boldsymbol{\theta}$ have probability distribution $p(\boldsymbol{\delta\theta}) \propto \exp[-(1/2)\Gamma_{ij}\delta\theta^i\delta\theta^j]$, where $\delta\theta^i = \theta^i - \hat{\theta}^i$ and $\hat{\theta}^i$ are the best fit parameter values. The Fisher matrix $\Gamma_{ij}$ is computed via $\Gamma_{ij} = (\partial h/\partial\theta^i, \partial h/\partial\theta^j)$, where the brackets in this case indicate the noise weighted inner product. The expected errors in the measurement of the parameter set $\boldsymbol{\delta\theta}$ are then defined by the square root of the diagonal elements of the inverse Fisher matrix. We follow [25] in our treatment of the parameter estimation analysis for BNS GW signals with the addition of the redshift $z$ as a parameter. We therefore use $\boldsymbol{\theta} = (\ln\mathcal{A}, t_c, \phi_c, \ln\mathcal{M}_z, \eta, z)$ as our independent parameters where we have absorbed all amplitude information into a single parameter $\mathcal{A}$ via $\tilde{h}(f) = \mathcal{A}f^{-7/6}e^{-i\Psi(f)}$. The expected SNR of a given BNS signal is dependent upon the nuisance parameters $\boldsymbol{\varphi}$. For simplicity we have computed our results using the ET-D detector design configuration [16,39] assuming a SNR that has been appropriately averaged over each of the 4 constituent angles of $\boldsymbol{\varphi}$. The detection range of ET for BNS systems is $z \approx 1$ for such an angle averaged signal and a SNR threshold of 8. For an optimally oriented system at the same SNR threshold the horizon distance is $z \approx 4$.

*Results.*—The results of the analysis with respect to the uncertainties in the redshift measurement as a function of redshift are shown in Fig. 1 for a subset of EOSs. We have chosen the following 3 (given in order of increasing $\lambda$) labeled APR [40], SLy [41], and MS1 [42] as representative samples from the set of 18 EOSs considered in Hinderer *et al.* [17]. The tidal deformability parameter $\lambda$ for each EOS has been parametrized as a function of the NS mass as described above and is directly proportional to the level of the tidal phase contribution. Note that for our most pessimistic choice of EOS, the redshift can be measured to better than 40% accuracy for sources at $z < 1$ and then worsens to only 65% at the maximum redshift range, $z \approx 4$, of ET. For the most optimistic of our representative EOSs we see corresponding values of 8% and 15%. The general trend of the results (for all EOSs) is that the fractional redshift uncertainty remains approximately constant up to $z \approx 1$ increasing only by a factor of $\sim 1.5$ for the most distant sources at $z \approx 4$. The relationship between the accuracy of redshift determination and the EOS is, as

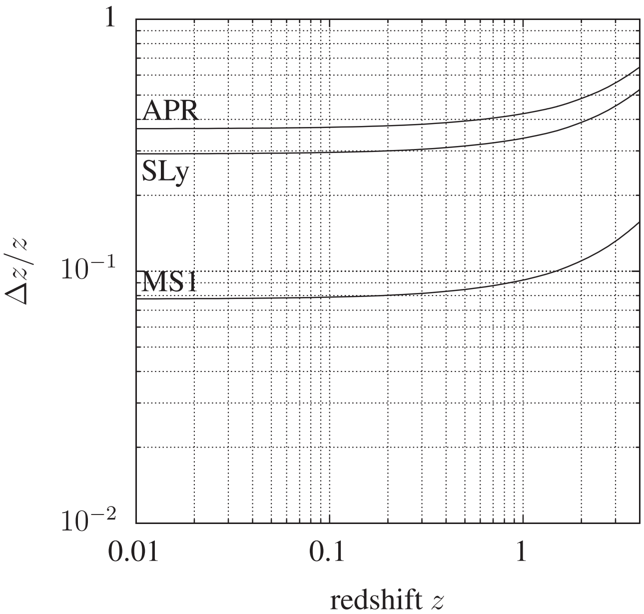


FIG. 1. The fractional uncertainties in the redshift as a function of redshift obtained from the Fisher matrix analysis for BNS systems using 3 representative EOSs: APR [40], SLy [41], and MS1 [42]. In all cases the component NSs have rest masses of $1.4M_{\odot}$ and waveforms have a cutoff frequency equal to the ISCO frequency (as defined in the BNS rest frame). We have used a cosmological parameter set $H_0 = 70.5\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_m = 0.2736$, $\Omega_k = 0$, $w_0 = -1$ to compute the luminosity distance for given redshifts and have assumed detector noise corresponding to the ET-D [16,39] design (a frequency domain analytic fit to the noise floor can be found in [45]).

expected, proportional to the NS deformability parameter $\lambda$. The consistency of the redshift determination as a function of redshift can be explained by the combination of 2 competing effects. Naively one would expect that the drop in SNR at higher redshifts would cause any parameter estimation to degrade. This is true and is the cause of the final rise in the fractional redshift error at high redshifts. In parallel, as the more distant sources have their waveforms redshifted to lower frequencies, the tidal effects which formally begin at 5PN order and have greatest effect close to the cutoff frequency are moved towards the most sensitive band of the detector ($\sim 150$ Hz). From this argument one would conclude that this ‘‘sweet spot’’ would coincide with $z \sim 10$, but this effect is diluted at higher redshifts due to a reduction in SNR as the lower frequency part of the signal moves out of band.

*Discussion.*—The analysis presented here is a proof of principle and is based on a number of assumptions and simplifications which we would like to briefly discuss and in some cases reiterate. It is likely that by the 3rd generation GW detector era our knowledge of the tidal response in BNS systems will have significantly advanced through improved NR simulations [43]. Current NR simulations have already shown that modeling these tidal phase corrections using a PN formalism, while qualitatively accurate, significantly *underestimates* the tidal phase contribution [34–36]. In addition, these same studies suggest that it is possible to accurately model tidal effects up to the merger phase. Therefore, we feel that our use of the ISCO as the upper cutoff frequency of the PN waveforms is a well justified choice for this first estimate. We have also neglected the effects of spin in our investigation, which we expect to contribute to the PN phase approximation at the level of $\sim 0.3\%$ [17]. This does not preclude the possibility that marginalizing over uncertainties in spin parameters may weaken our ability to determine the redshift. This seems unlikely given the small expected spins in these systems, as well as the difference in scalings between the spin terms and the tidal terms, $x^{-1/2}$ and $x^{5/2}$, respectively, causing the tidal effects to dominate over spin in the final stage of the inspiral. We also note that the Fisher information estimate of parameter uncertainty is valid in the limit of SNR $\gtrsim 10$ [38] and under the assumption of Gaussian noise. As such, the results at low SNR, and therefore those at high $z$, should be treated as lower limits via the Cramer-Rao bound, on the redshift uncertainty. We also mention here that since the tidal phase corrections are, at leading order, formally of 5th PN order, we have uncertainty in the effect of the missing PN expansion terms in the BNS waveform between the 3.5PN and 5PN terms. It is comforting to note that, as the PN order is increased, our results on the redshift uncertainty do converge to the point of $<1\%$ difference in accuracy between the 3PN and 3.5PN terms implying (through extrapolation) that the missing PN terms (as yet not calculated) would not affect our results. Future detailed analysis following this work will complement Fisher based estimates with Monte Carlo simulations and/or Bayesian posterior based parameter estimation techniques. Similarly, the signal parameter space should be more extensively explored beyond the canonical $1.4M_{\odot}$, equal mass case. In addition, future work will also include black-hole–neutron star systems which will also contain, encoded within their waveforms, extractable redshift information. Such systems are observable out to potentially higher redshift although tidal effects will become less important as the mass ratio increases [18]. Finally, we briefly mention that GW detector calibration uncertainties in strain amplitude (which for 1st generation detectors were typically $<10\%$) will only affect the determination of the luminosity distance. Calibration uncertainties in timing typically amount to phase errors of $<1°$ and would be negligible in the determination of the redshift. Similarly, the effects of weak lensing that would only affect the luminosity distance measurement have been shown to be negligible for ET sources [4].

*Conclusions.*—Current estimates on the formation rate of BNS systems imply that in the 3rd generation GW detector era there is the potential for up to $\sim 10^7$ observed events per year out to redshift $z \approx 4$ [16]. The results presented here suggest that redshift measurements at the level of $\sim 10\%$ accuracy can be achieved for *each* BNS event solely from the GW observation. Such systems have long been known as GW standard sirens [1], meaning that the luminosity distance can be extracted from the waveform with accuracy determined by the SNR coupled with the ability with which one is able to infer the geometric orientation of the source. Using a large number of sources all sharing the same redshift, the luminosity distance (free of the orientation parameters) can be determined statistically from the distribution of observed amplitudes. With the ability to extract both the luminosity distance and the redshift out to such cosmological distances and from so many sources, the precision with which one could then determine the luminosity distance-redshift relation is significantly enhanced. Current proposed methods for making cosmological inferences using GW standard sirens [3,5,44] rely on coincident EM counterpart signals from their progenitors in order to obtain the redshift. Our method would allow measurements to be made independently of the cosmological distance ladder.

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