Fermionization of Two Distinguishable Fermions

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We study a system of two distinguishable fermions in a 1D harmonic potential. This system has the exceptional property that there is an analytic solution for arbitrary values of the interparticle interaction. We tune the interaction strength and compare the measured properties of the system to the theoretical prediction. For diverging interaction strength, the energy and square modulus of the wave function for two distinguishable particles are the same as for a system of two noninteracting identical fermions. This is referred to as fermionization. We have observed this phenomenon by directly comparing two distinguishable fermions with diverging interaction strength with two identical fermions in the same potential. We observe good agreement between experiment and theory. By adding more particles our system can be used as a quantum simulator for more complex systems where no theoretical solution is available.

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A powerful tool for solving complex quantum systems is to map their properties onto systems with simpler solutions. For interacting bosons in one dimension there is a one-to-one correspondence of the energy and the square modulus of the wave function $|\psi(x_1, \ldots, x_n)|^2$ to a system of identical fermions [1]. As one consequence the local pair correlation $g^{(2)}(0)$ of an interacting 1D Bose gas vanishes for diverging interaction strength just like in a gas of noninteracting identical fermions. Thus, a large decrease of $g^{(2)}(0)$ in a repulsively interacting 1D Bose gas is strong evidence for the existence of fermionization [2].

The many-body properties of such 1D bosonic systems have been studied in [3,4]. However, the essential property of a such a gas—namely the fermionization [1,5]—is already present in a system of two interacting particles, regardless of the particles being identical bosons or distinguishable fermions [6]. This two-particle problem is of significant interest because it is the main building block of all 1D quantum systems with short-range interactions. It is also one of the few quantum mechanical systems for which an analytic solution exists. In contrast to measurements of bulk properties such as compressibility and collective oscillations or measurements of local pair correlations [2], we access the energy and the square modulus of the wave function of the fundamental two-particle system. We directly observe fermionization of two distinguishable fermions by comparing two distinguishable fermions with two identical fermions in the same potential. In optical lattices the energy of similar two-particle systems has been measured for large but not diverging interaction strength [7,8].

We realize such a two-particle system with tunable interaction using two fermionic ^{6}Li atoms in the ground state of a potential created by an optical dipole trap and a magnetic field gradient [Figs. 1(a) and 1(b)]. We can prepare this state with a fidelity of $(93 \pm 2)\%$ [9]. The energy of such two particles interacting via contact interaction—which is fully described by one parameter, the 1D coupling strength g—can be analytically calculated for a harmonically trapped 1D system [10,11]. The problem can be separated into center-of-mass and relative motion because of the harmonic trapping potential and because the interaction term only depends on the relative distance between the two particles. Then the solution can be written as a product of the center-of-mass and the relative wave function. The latter is shown in Fig. 2(a) for different

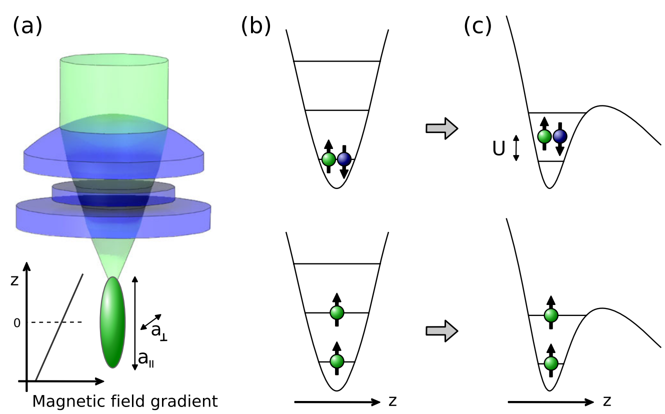

FIG. 1 (color online). Trap setup and sketch of the performed experiment. (a) Our trap consists of an optical potential created by a tight focus of a laser beam and a magnetic field gradient. (b) Deterministic preparation of two fermions in the ground state of a potential well. (c) We measure the tunneling dynamics through a potential barrier for a repulsively interacting system of two distinguishable fermions for various interaction energies. The mean interaction energy per particle is indicated by the parameter U. These results are then compared with the tunneling dynamics of two noninteracting identical fermions in the same potential.

values of the coupling strength. For diverging coupling strength the square modulus of the wave function of a system of two distinguishable fermions is the same as for two noninteracting identical fermions. This is the point where fermionization occurs.

In our setup the particles are confined in a three-dimensional cigar-shaped potential with an aspect ratio of about 1:10, which can be harmonically approximated with trap frequencies of $\omega_{\parallel} = 2\pi \times (1.234 \pm 0.012)$ kHz in longitudinal direction and $\omega_{\perp} = 2\pi \times (11.88 \pm 0.22)$ kHz in perpendicular direction. It has been shown in [12] that the energy of two interacting particles in the ground state of such a potential is well described by the 1D solution given in [10]. Hence we treat our system in this 1D framework. The combined optical and magnetic potential in one-dimensional form reads:

$$V_{r=0}(z) = pV_0\left(1 - \frac{1}{1 + (z/z_r)^2}\right) - \mu_m B' z, \qquad (1)$$

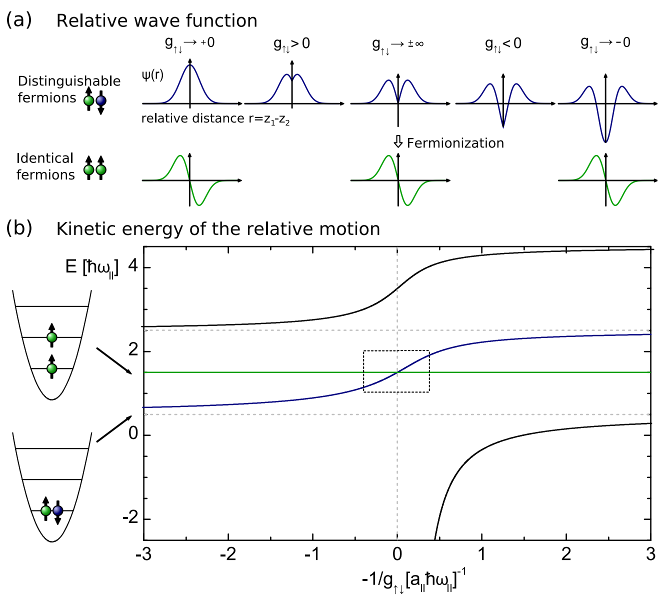

FIG. 2 (color online). Two particles in a 1D harmonic potential. (a) Relative wave function of two interacting fermions (blue [dark gray]) and two identical fermions (green [light gray]) in a 1D harmonic potential. For infinitely strong interaction ($-1/g_{|\uparrow\downarrow\rangle} \to 0$) the probability to find the two distinguishable fermions at the same position vanishes. In this case the square modulus of the total wave function of two distinguishable fermions is the same as for two identical fermions. (b) Kinetic energy of the relative motion. The blue [dark gray] and black curves show the energy of two interacting fermions in state $|\uparrow\downarrow\rangle$ depending on the coupling strength $g_{|\uparrow\downarrow\rangle}$ given in units of $a_{\parallel} = \sqrt{\hbar/\mu\omega_{\parallel}}$. The green [light gray] line shows the energy of two identical fermions in state $|\uparrow\uparrow\rangle$. The energy is plotted versus $-1/g_{|\uparrow\downarrow\rangle}$ for a better comparison with the experimental results. The experimentally studied region is indicated by the dashed rectangle.

where $V_0 = k_B 3.326\ \mu$K is the initial depth of the optical potential, p is the optical trap depth in units of the initial trap depth, $z_R = \frac{\pi w_0^2}{\lambda}$ is the Rayleigh range of the optical trapping beam with minimal waist $w_0 = 1.838\ \mu$m and wavelength $\lambda = 1064$ nm, μ_m is the magnetic moment of the atoms, and $B' = 18.92$ G/cm is the strength of the magnetic field gradient. The determination of the trap parameters is described in the Supplemental Material [13]. The 1D coupling constant g can be calculated from the 3D scattering length a_{3D} and depends strongly on the confining potential, which is characterized by the harmonic oscillator length $a_{\perp} = \sqrt{\hbar/\mu\omega_{\perp}}$ [14], where $\hbar$ is the reduced Planck constant and $\mu = \frac{m}{2}$ the reduced mass of two ^{6}Li atoms with mass m. The coupling constant is given by

$$g = \frac{2\hbar^2 a_{3D}}{\mu a_{\perp}^2} \frac{1}{1 - C a_{3D}/a_{\perp}}, \qquad (2)$$

with $C = \zeta(\frac{1}{2}) = 1.46\ldots$ and ζ the Riemann zeta function. The value of g can be changed by tuning the 3D scattering length via a magnetic Feshbach resonance [15,16]. When a_{3D} approaches the extension of the confining harmonic oscillator potential $a_{\perp}$, a confinement-induced resonance (CIR) occurs for $a_{3D} = a_{\perp}/C$ [17,18]. Figure 4(b) shows $g_{|\uparrow\downarrow\rangle}$ for two distinguishable atoms in the two lowest ^{6}Li hyperfine states, $|F = \frac{1}{2}, m_F = -\frac{1}{2}\rangle$ and $|F = \frac{1}{2}, m_F = \frac{1}{2}\rangle$—labeled $|\uparrow\rangle$ and $|\downarrow\rangle$—as a function of the magnetic offset field [19]. For two identical fermions s-wave scattering is forbidden and thus $g_{|\uparrow\uparrow\rangle} = 0$ for all values of the magnetic offset field.

To determine the energy of the two-particle system in state $|\uparrow\downarrow\rangle$ we modify the trapping potential such that there is a potential barrier of fixed height through which the particles can tunnel out of the trap (see Fig. 1(c) and Supplemental Material [13]). In the presence of repulsive interactions the energy of the system is increased according to the blue [dark gray] curve in Fig. 2(b). This decreases the effective height of the barrier and the particles tunnel faster. We allow the particles to tunnel out of the trap for different durations and record the number of particles remaining in the trap. By choosing an adequate barrier height we ensure that the time scale for tunneling is smaller than the lifetime of our samples in the ground state (about 60 s). Additionally, obtaining meaningful tunneling time constants requires the time scale of the tunneling to be much larger than the inverse longitudinal trap frequencies of 0.7 ms. By averaging over many experimental realizations we obtain the expectation value of the particle number in the potential for different hold times (Fig. 3). By performing this measurement for various values of the coupling strength we can determine the dependence of the system's energy on $g_{|\uparrow\downarrow\rangle}$.

We find that for the observed range of interaction energies—which are on the order of $\hbar\omega_{\parallel}$—only one particle leaves the potential even for long hold times. In a

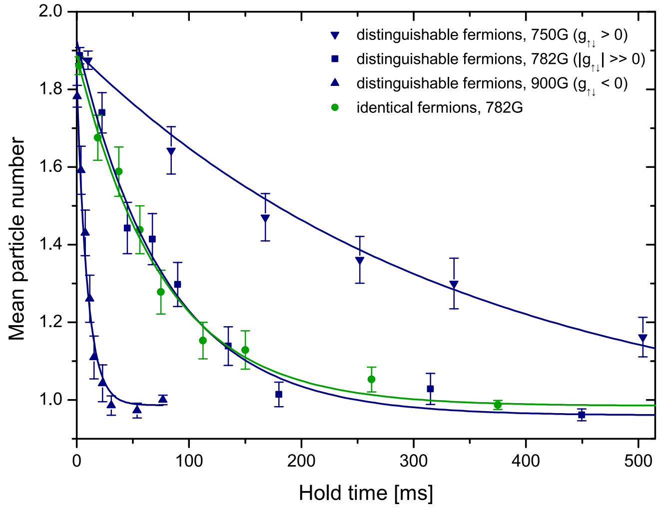

FIG. 3 (color online). Mean number of particles remaining in the potential well. After modifying the initial potential the particles can tunnel through a barrier of fixed height for a certain hold time. Subsequently, tunneling is switched off and the mean particle number left in the potential is recorded by averaging over many experimental realizations. Exponential fits to the data (solid lines) allow us to extract the tunneling time constants of two interacting distinguishable fermions for different interaction strengths (blue [dark gray]) and of two identical fermions (green [light gray]). Each data point is the average of about 70 measurements except for the first and the last data point in each series (about 230 realizations). The errors are the standard errors of the mean.

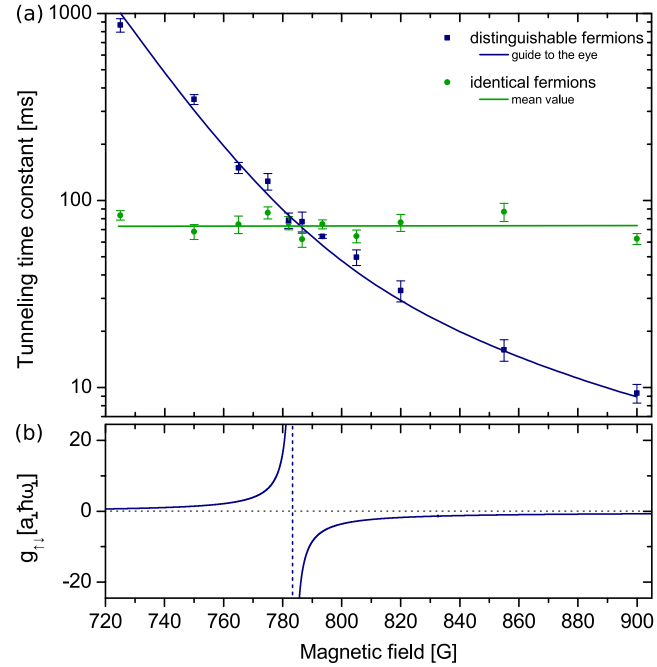

FIG. 4 (color online). (a) Tunneling time constants for different values of the 1D coupling strength. The tunneling time constant of two repulsively interacting distinguishable fermions (blue [dark gray] curve) decreases by 2 orders of magnitude with increasing magnetic field. This is attributed to the gain in interaction energy when ramping across the CIR. The tunneling time constant of two noninteracting identical fermions (green [light gray] line) remains unaffected by the magnetic field within our experimental accuracy. At the magnetic field value where both curves cross we identify the fermionization of two distinguishable fermions. The errors are the statistical errors of the fits shown in Fig. 3. The blue [dark gray] line is a guide to the eye. (b) One-dimensional coupling constant $g_{|\uparrow\downarrow\rangle}$ with a CIR at $(783.4 \pm 0.4)G$. For the calculation we used the perpendicular harmonic oscillator length $a_\perp = \sqrt{\hbar/\mu\omega_\perp}$ of the modified potential.

simple picture this can be explained as follows: If one particle tunnels through the barrier the interaction energy is released as kinetic energy, which leaves the other particle in the unperturbed ground state of the potential. This state has a tunneling time scale much larger than the duration of the experiment. Thus we can fit exponentials of the form $N(t) = N_{\mathrm{tunnel}} e^{-(t/\tau)} + N_{\mathrm{remain}}$ to the mean particle number to deduce the tunneling time constant τ for different magnetic fields. The mean numbers of tunneled (N_{tunnel}) and remaining particles (N_{remain}) are expected to be unity. However, due to the finite preparation fidelity they are slightly lower. In Fig. 4 we show the determined tunneling time constants of a system of two interacting fermions for different interaction energies as a function of the magnetic field. We observe a decrease in the tunneling time constant over 2 orders of magnitude for increasing magnetic field due to the gain in interaction energy caused by the CIR.

For a direct comparison of the properties of the two interacting distinguishable fermions with those of two identical fermions we perform the same measurement with two fermions in state $|\uparrow\uparrow\rangle$ in the same potential [Fig. 1(c)]. The results of these reference measurements are shown in Figs. 3 and 4 (green [light gray] points). As the identical fermions are noninteracting we find no dependence of the tunneling time constant on the magnetic field in this measurement. Comparing the results of the two systems we find that the tunneling time constant for the interacting system decreases monotonically with increasing magnetic field and crosses the magnetic field independent tunneling time constant of the two identical fermions. Thus there is one magnetic field value where the tunneling time constants of both systems are equal. At this point both systems must have the same energy. For a 1D system with given energy there is only one unique solution for the square modulus of the wave function. Therefore, right at the observed crossing point of the tunneling time constants the energy and the square modulus of the wave function $|\psi(z_1, z_2)|^2$ of the two interacting distinguishable fermions and the two noninteracting identical fermions must be equal. Hence, exactly at this crossing point the system of two distinguishable fermions is fermionized. As predicted by theory [6,10] we find the position of the fermionization

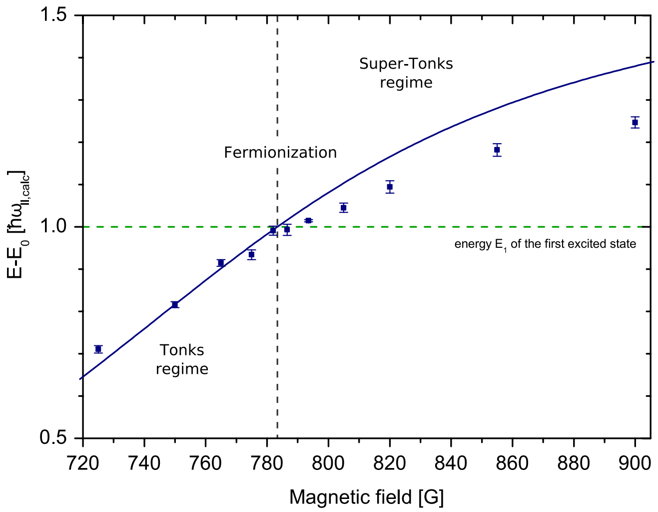

FIG. 5 (color online). Interaction energy of two fermions for different interaction regimes. By using a WKB based calculation we can determine the energy of two distinguishable fermions at different interaction strengths (blue points) from the tunneling time constants presented in Fig. 4(a). The blue curve shows the expected energy shift for a harmonically trapped system (dashed rectangle in Fig. 2).

at the magnetic field value where $g_{|\uparrow\downarrow\rangle}$ diverges due to the confinement-induced resonance.

For magnetic field values below the CIR we have realized the two-particle limit of a Tonks-Girardeau gas [6]. Above the CIR we have created a super-Tonks state consisting of two particles. The super-Tonks state is a strongly correlated metastable state above the attractive ground state branch [see Fig. 2(b)]. In a system with particle numbers ≥ 3 inelastic three-body collisions lead to a fast decay of the metastable super-Tonks-Girardeau gas [4]. In contrast, our two-particle super-Tonks state is stable against collisional losses since there is no third particle available to undergo an inelastic three-body event. To determine the energy of the two interacting fermions from the measured tunneling time constants we use a WKB calculation (see Supplemental Material [13]). This requires knowledge of the potential shape. The parameters of the optical potential are determined by precise measurements of the level spacings in the potential. The final parameter to determine the barrier height is fixed by the measured tunneling time constant of two identical fermions (see [13]). The energies obtained from the tunneling time constants of two distinguishable fermions are shown in Fig. 5.

We compare these energies to the analytic theory for a harmonic potential [10] (see Fig. 2). This theory needs two input parameters, the coupling strength and the level spacing. For the coupling strength we use $g_{|\uparrow\downarrow\rangle}$ of our system shown in Fig. 4(b). For the level spacing we use the energy difference $\hbar\omega_{\|\text{calc}} = E_0 - E_1 = 2\pi\hbar \times 743$ Hz between the ground and first excited state of the potential which we calculate using the WKB method. With this approximation the energy obtained from the tunneling measurements and the energy obtained from the analytic theory [10] are the same at the CIR. For the Tonks regime we find excellent agreement of the experimentally determined energy with the theoretical prediction for a harmonic trap. Above the CIR the harmonic theory is not applicable because the second excited state is not bound in our potential. Additionally, we expect deviations for larger energies due to the limited validity of the WKB approximation for energies close to the continuum threshold. A more precise description could be achieved by adapting the theory described in [10] to our nonharmonic potential and by using a more accurate theory for the tunneling process [20,21].

In summary, we have measured the interaction energy of two distinguishable fermions as a function of the interaction strength and identified the point of fermionization. The good agreement between our results and theoretical predictions shows that our experiment has the capability to simulate strongly correlated few-body quantum systems. Using the experimental methods established in this work it is straightforward to extend our studies to more complex systems. Simply adding a third particle either in one of the present spin states [22] or a different spin state [23,24] allows us to study a highly nontrivial system where no analytical solution exists. In a few-body system with defined particle number and attractive interaction we could investigate pairing phenomena and thus work towards studying superfluidity in finite systems. This has already been investigated in the context of nuclear physics [25]. By dynamically changing the shape of the trapping potential we could simulate a vast amount of different time-dependent quantum systems. A feasible experiment would be to periodically modulate the strength of the magnetic field gradient. This would allow us to study ionizationlike excitations in the strong-field regime [26] which have been studied in ultrafast physics [27].

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