Higher Order Spin Resonances in a 2.1-GeV/c Polarized Proton Beam

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Spin resonances can depolarize or spin flip a polarized beam. We studied 1st and higher order spin resonances with stored 2.1 GeV/c vertically polarized protons. The 1st order vertical (ν_y) resonance caused almost full spin flip, while some higher order ν_y resonances caused partial depolarization. The 1st order horizontal (ν_x) resonance caused almost full depolarization, while some higher order ν_x resonances again caused partial depolarization. Moreover, a 2nd order ν_x resonance is about as strong as some 3rd order ν_x resonances, while some 3rd order ν_y resonances are much stronger than a 2nd order ν_y resonance. One thought that ν_y spin resonances are far stronger than ν_x, and that lower order resonances are stronger than higher order; the data do not support this.

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To study the strong interaction's spin dependence with polarized proton beams, one must preserve and control the polarization during acceleration and storage [1–6]. This can be difficult due to many 1st and higher order depolarizing (spin) resonances. For vertically polarized beams in flat accelerators, it was thought that vertical spin resonances should be stronger than horizontal resonances, and lower order resonances should be stronger than higher order resonances [7,8]. There were several theoretical attempts to calculate the strengths of higher order spin resonances [9–11]. Some 2nd order and synchrotron-sideband resonances were seen in electron rings [12] and proton rings [13]. Moreover, a 2nd order proton resonance was studied in detail at IUCF [14]. We used 2.1 GeV/c polarized protons stored in the COSY synchrotron for a detailed experimental study of higher order spin resonances. Our preliminary ν_y data was presented at SPIN 2004 [15], but both the ν_y data and the never-presented ν_x data needed significant reanalysis. The properly reanalyzed data presented here suggest that many higher order spin resonances, both ν_y and ν_x, must be overcome to accelerate polarized protons to high energies.

In flat circular rings, a beam proton's spin precesses around the vertical fields of the ring's dipole magnets. The spin tune $\nu_s = G\gamma$ is the number of spin precessions during one turn around the ring, where $G = (g-2)/2$ is the proton's gyromagnetic anomaly and γ is its Lorentz energy factor. Horizontal magnetic fields can perturb the proton's stable vertical polarization creating a spin resonance [16–19]. Spin resonances occur when

$$\nu_s = k\nu_x + l\nu_y + m, \tag{1}$$

where k, l, and m are integers; ν_x and ν_y are the horizontal and vertical betatron tunes, respectively. Imperfection spin resonances occur when $k = l = 0$. Intrinsic spin resonances occur when either $k \neq 0$ or $l \neq 0$, or both; the sum $|k| + |l|$ defines each resonance's order.

The experiment's apparatus, including the COSY storage ring [20,21], EDDA detector [22,23], electron cooler [24], low energy polarimeter (LEP) [25], injector cyclotron, and polarized ion source [26–28], were shown in Fig. 1 of Ref. [29]. The beam from the polarized H^- ion source was accelerated by the cyclotron to 45 MeV and then strip injected into COSY.

Before this injection, the LEP measured the H^- beam's polarization to monitor its stability. The cylindrical EDDA detector [22,23] measured the beam's polarization in COSY after crossing the resonances. We reduced its systematic errors by cycling the polarized source [26–28]

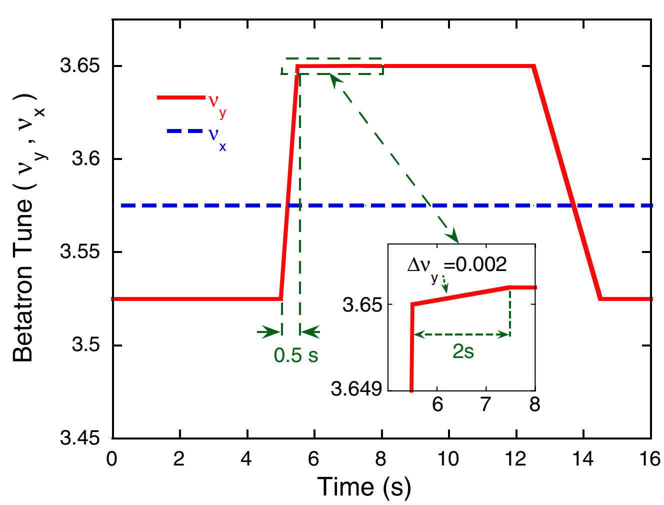

FIG. 1 (color online). Typical ν_y betatron tune ramps during COSY cycle.

between the up and down vertical polarization states. The measured flattop polarization, before crossing any resonances, was typically about 75%.

In the COSY ring, the protons' average circulation frequency f_c was 1.491 85 MHz at 2.1 GeV/c, where their Lorentz energy factor was $\gamma = 2.4514$. For these parameters, the spin tune $\nu_s = G\gamma$ was 4.395. During injection, acceleration, and at the beginning of the flattop, the betatron tunes ν_x and ν_y were kept fixed at 3.575 and 3.525, respectively. This kept both betatron tunes away from any 1st, 2nd, or 3rd order spin resonances on the flattop. After reaching the flattop, we varied the ring quadrupoles' currents to vary either ν_y or ν_x, while keeping the other tune fixed; then we measured the polarization.

Figure 1 shows the betatron tunes' behavior in a typical COSY cycle, during the higher order vertical (ν_y) spin resonance study; we first ramped ν_y rapidly from 3.525 to some value between 3.51–3.71 during 0.5 s. Next, we slowly ramped ν_y through a very small tune range of about 0.002 during 2 s, with ν_x fixed at 3.575; then we measured the polarization. The rapid ramp reduced the effects of the resonances between the injection tune of 3.525 and the start of the slow ramp, while each slow ν_y ramp enhanced the effect of any spin resonance in its very small ν_y range.

The LEP monitored the beam polarization before injection into COSY. The measured LEP asymmetries indicated that the initial polarization changed during the experiment by about 10%. Thus, we normalized each final COSY polarization measured by EDDA to the measured LEP asymmetry for that data run. The typical duration of each EDDA data run was 25 min; thus, the LEP data bin sizes were typically 60 min (± 30 min) to include one high count-rate LEP run before each data run, and one after. When needed, the LEP bin size was increased to include a high count-rate LEP run both before and after each data run.

The measured polarizations for the higher order vertical (ν_y) spin resonance study are plotted against the final measured ν_y values in Fig. 2. The measured slow betatron tune ramps of about 0.002 are shown as a horizontal bar for each data point. We used Eq. (1) to calculate the positions of 1st, 2nd, and 3rd order resonances that could be studied between the half-integer 3.5 and quarter-integer 3.75 beam blowup resonances.

To test the data's reproducibility, we tried to measure polarizations at the same ν_y settings several times. However, when we precisely measured the ν_y values after each setting, we found that the slow ramps were often not exactly identical at the ± 0.0002 level. Thus, Fig. 2 has many partly-overlapping points, which obscure the polarization's behavior near each resonance. We tried to clarify Fig. 2 by combining points with nearby ν_y values, except in the regions where the polarization changed very rapidly (between ν_y values of 3.586 to 3.620). We first combined

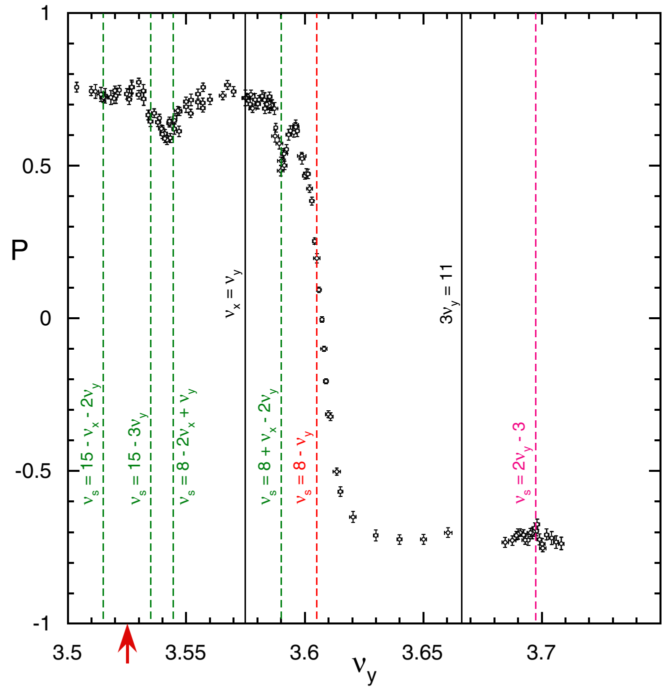

FIG. 2 (color online). Polarization normalized to LEP asymmetry plotted vs ν_y. The measured slow tune ramps of about 0.002 are shown as horizontal bars. The calculated position of each spin resonance is shown by a dashed vertical line and the $\nu_x = \nu_y$ and $3\nu_y = 11$ beam-blowup resonances are shown by solid lines. The arrow shows ν_y at injection.

all pairs of points that had ν_y values within $\delta\nu_y = 0.1 \times 10^{-4}$. To help ensure that this did not bias the results, we combined the data in both the increasing (left-to-right) and decreasing (right-to-left) ν_y directions; the two results were identical. We then sequentially increased the $\delta\nu_y$ intervals in steps of 0.1×10^{-4}; the input data for each step were the output data from the previous step. The error and position of each newly combined point after each step were the properly weighted averages of the errors and positions of the two combined points; each new horizontal bar encompassed the slow ramps of both combined points.

Figure 3 plots polarization vs ν_y for the combination interval of $\delta\nu_y = 7.6 \times 10^{-4}$. The 76 combination steps reduced the number of data points from 131 to 95. The plot shows clear resonance behavior around several 3rd order resonances, but the behavior around the 2nd order resonance is still unclear. When we further increased the combination interval size, the polarization's behavior around the narrow resonances was broadened excessively, as expected.

We observed full spin flip when the 1st order vertical (ν_y) spin resonance was crossed; we also found partial depolarization near several 3rd order resonances and possibly near a 2nd order resonance. The 3rd order $\nu_s = 8 + \nu_x - 2\nu_y$ resonance and the partly overlapping 3rd order $15 - 3\nu_y$ and $8 - 2\nu_x + \nu_y$ resonances appear

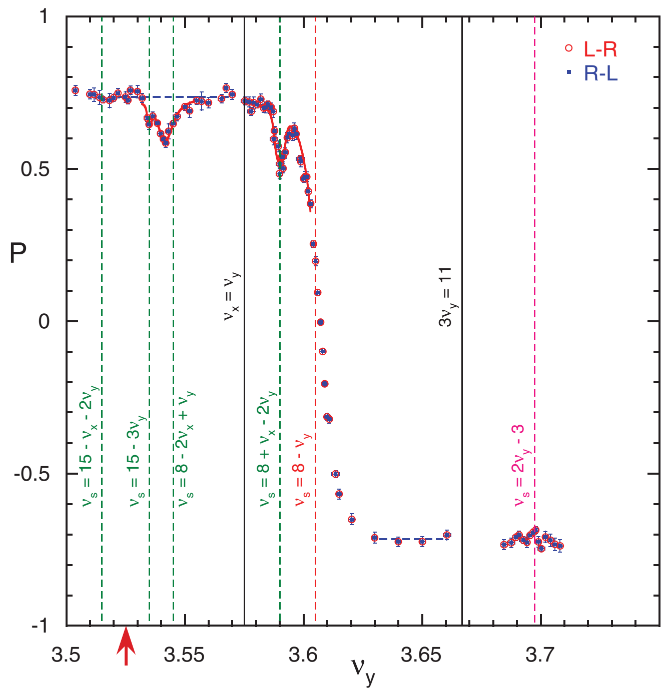

FIG. 3 (color). Polarization normalized to the LEP asymmetry plotted vs ν_y. (See Fig. 2 caption for more details.) The points were combined in steps of 0.1×10^{-4} up to an interval of $\delta\nu_y = 7.6 \times 10^{-4}$, except in the ν_y region of 3.586 to 3.620. The fits to strong 3rd order spin resonances are shown by the solid red curves. The horizontal dashed blue lines show the fits for P_i and P_f for the 1st order ν_y resonance. [See online version with figures expanded by 400%–800% for details.]

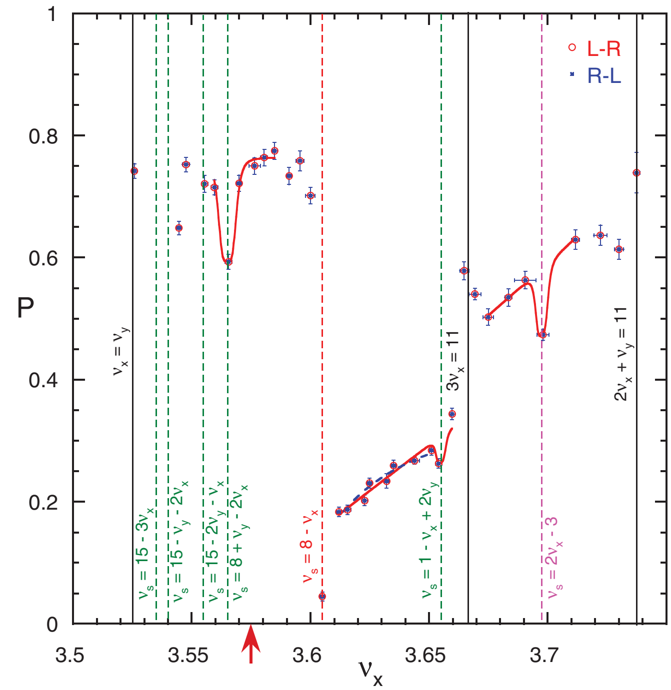

FIG. 4 (color). Polarization normalized to LEP asymmetry plotted vs ν_x. Only 5 pairs of nearby ν_x points were combined. Dashed vertical lines indicate position of each spin resonance. The $\nu_x = \nu_y$, $3\nu_x = 11$, and $2\nu_x + \nu_y = 11$ beam-blowup resonances are shown by solid vertical lines. The fits to strong 2nd and 3rd order spin resonances are shown by the solid red curves; the dashed blue curve shows the 1st order ν_x resonance's fit to Eq. (2). The arrow shows ν_x at injection.

significantly stronger than the 2nd order $2\nu_y - 3$ resonance. This suggests that many significant 3rd and possibly higher order spin resonances must be overcome to accelerate and store polarized protons above 100 GeV.

We also studied the higher order horizontal (ν_x) spin resonances by using ν_x ramps similar to the ν_y ramps in Fig. 1, with ν_y fixed at 3.525. We first rapidly ramped ν_x from 3.575 to a value between 3.525–3.74 in 0.5 s; we next slowly ramped ν_x through a range of about 0.002 in 2 s; then we measured the polarization. The rapid ramp again reduced the effects of the resonances between the injection tune of 3.575 and the start of the slow ramp, while each slow tune ramp enhanced the effect of the resonance in that small ν_x range.

The polarizations are plotted in Fig. 4 against ν_x. The 5 pairs of overlapping points were combined, as earlier described for Fig. 3. Figure 4 shows almost full depolarization at the 1st order spin resonance. Above this resonance, the polarization increased steadily probably because this fairly strong resonance was crossed at increasing $\Delta\nu_x/\Delta t$ rates, which decreased the depolarization [16]; $\Delta\nu_x/\Delta t$ increased because the ramp time Δt was fixed at 0.5 s, while the ramp range $\Delta\nu_x$ was increased. Thus, we found partial depolarization near a 2nd order ν_x resonance and near several 3rd order ν_x resonances; these ν_x resonances all seem about equally strong. Recall that some 3rd order ν_y resonances seem significantly stronger than the 2nd order ν_y resonance.

Also note that the polarization increased significantly at the two ν_x beam-blowup resonances probably because they removed mostly those beam particles with larger betatron amplitudes, as supported by the sharp decrease in the precisely measured count rates in EDDA at each blowup resonance. These outside particles were probably more depolarized [30] when crossing the strong 1st order ν_x spin resonance; thus, removing them increased the beam's polarization while decreasing its intensity.

The measured strengths of the 11 resonances, for which we had adequate data, are listed in Table I. We first obtained the very strong 1st order ν_y resonance's P_i and P_f, respectively, from the left and right horizontal dashed line fits in Fig. 3. We then obtained its strength ε using the measured P_f/P_i and the fast ramp's time Δt of 0.5 s and $\Delta\nu$ of 0.105 in the Froissart-Stora equation [16]:

$$P_f/P_i = 2\exp\left[\frac{-(\pi\varepsilon)^2 f_c}{\Delta\nu/\Delta t}\right] - 1. \quad (2)$$

TABLE I. Summary of observed resonances' strengths.

Type	Order	Resonance	$P_f/P_i(\%)$	$\varepsilon \times 10^{-6}$
ν_y	1st	$8-\nu_y$	-97.2 ± 1.4	>240
ν_y	2nd	$2\nu_y-3$	>95	<1.3
ν_y	3rd	$15-\nu_x-2\nu_y$	>95	<1.3
ν_y	3rd	$15-3\nu_y$	92.5 ± 2.6	1.6 ± 0.4
ν_y	3rd	$8-2\nu_x+\nu_y$	80.1 ± 1.7	2.7 ± 0.2
ν_y	3rd	$8+\nu_x-2\nu_y$	76.9 ± 4.5	2.9 ± 0.4
ν_x	1st	$8-\nu_x$	F-S eq. fit	41 ± 2
ν_x	2nd	$2\nu_x-3$	77.7 ± 2.2	2.8 ± 0.2
ν_x	3rd	$15-3\nu_x$	inadequate data	
ν_x	3rd	$15-2\nu_x-\nu_y$	inadequate data	
ν_x	3rd	$15-\nu_x-2\nu_y$	>95	<1.3
ν_x	3rd	$8-2\nu_x+\nu_y$	77.7 ± 2.2	2.8 ± 0.2
ν_x	3rd	$1-\nu_x+2\nu_y$	85.8 ± 4.7	2.2 ± 0.5

We could only set a lower limit on ε of 240×10^{-6} because the 1st order ν_y resonance was so strong that the spin was fully flipped for our fixed Δt of 0.5 s. For the strong 1st order ν_x resonance, the blue dashed curve in Fig. 4 is the fit of Eq. (2) to the 8 data points just after crossing it, using Δt of 0.5 s and $\Delta\nu$ equal to each point's $\Delta\nu_x$ from the ν_x value at injection.

For each isolated 2nd and 3rd order resonance, we obtained its dip's depth or polarization loss (P_f/P_i) by using a χ^2 minimization fit of a 2nd order Lorentzian to that resonance's data with a baseline obtained from its nearby points. The P_f/P_i values of the two overlapping 3rd order ν_y resonances in Fig. 3 were obtained by a fit using two overlapping Lorentzians and the baseline shown by the horizontal dashed blue line. We simultaneously fit the stronger ($8-2\nu_x+\nu_y$) resonance to a 1st order Lorentzian, with its frequency a variable in the fit, and the weaker ($15-3\nu_y$) resonance to a 2nd order Lorentzian with its frequency held fixed at the calculated value shown by its dashed green line. The fits to all 2nd and 3rd order resonances are shown by the solid red curves in Figs. 3 and 4. Three 2nd and 3rd order resonances had no observable dip at their calculated ν_x or ν_y value; therefore, the lower limits on their P_f/P_i were taken to be 95%, which was 4 times the average error on straight line fits to the data points near these apparently weak resonances. We then phenomenologically used P_f/P_i in Eq. (2) with our fixed experimental Δt of 2 s and $\Delta\nu$ of 0.002 to obtain ε for each 2nd and 3rd order resonance.

There were several theoretical attempts [9–11] to calculate the strengths of higher order spin resonances; one [9] suggests that odd order resonances may be stronger than even order resonances for rings with Siberian snakes. It is not yet clear if these theoretical approaches allow one to explain our experimental results.

In summary, we used 2.1 GeV/c polarized protons stored in the COSY synchrotron to study 1st and higher order spin resonances. We observed almost full spin flip when the 1st order $8-\nu_y$ spin resonance was crossed and partial depolarization near the 2nd and 3rd order spin resonances. We also observed almost full depolarization near the 1st order $8-\nu_x$ spin resonance and partial depolarization near the 2nd and 3rd order spin resonances. It was thought that, for vertically polarized protons in flat accelerators, vertical spin resonances are stronger than horizontal resonances, and lower order resonances are stronger than higher order resonances. The data suggest that many higher order spin resonances, both horizontal and vertical, must be overcome to accelerate polarized protons to high energies; these data may help RHIC to better overcome its snake resonances between 100 and 250 GeV/c.

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