# Quantum Absorption Refrigerator

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A quantum absorption refrigerator driven by noise is studied with the purpose of determining the limitations of cooling to absolute zero. The model consists of a working medium coupled simultaneously to hot, cold, and noise baths. Explicit expressions for the cooling power are obtained for Gaussian and Poisson white noise. The quantum model is consistent with the first and second laws of thermodynamics. The third law is quantified; the cooling power $\mathcal{J}_c$ vanishes as $\mathcal{J}_c \propto T_c^\alpha$, when $T_c \to 0$, where $\alpha = d + 1$ for dissipation by emission and absorption of quanta described by a linear coupling to a thermal bosonic field, where $d$ is the dimension of the bath.

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The absorption chiller is a refrigerator which employs a heat source to replace mechanical work for driving a heat pump [1]. The first device was developed in 1850 by the Carré brothers which became the first useful refrigerator. In 1926, Einstein and Szilárd invented an absorption refrigerator with no moving parts [2]. This idea has been incorporated recently to an autonomous quantum absorption refrigerator with no external intervention [3,4]. The present study is devoted to a quantum absorption refrigerator driven by noise; for an experimental realization, cf. [5]. The objective is to study the scaling of the optimal cooling power when the absolute zero temperature is approached.

This study is embedded in the field of quantum thermodynamics, the study of thermodynamical processes within the context of quantum dynamics. Historically, consistence with thermodynamics led to Planck's law, the basics of quantum theory. Following the ideas of Planck on black body radiation, Einstein five years later (1905) quantized the electromagnetic field [6]. Quantum thermodynamics is devoted to unraveling the intimate connection between the laws of thermodynamics and their quantum origin [3,4,7–22]. In this tradition, the present study is aimed toward the quantum study of the third law of thermodynamics [23,24], in particular, quantifying the unattainability principle [25]: What is the scaling of the cooling power $\mathcal{J}_c$ of a refrigerator when the cold bath temperature approaches the absolute zero $\mathcal{J}_c \propto T_c^\alpha$ when $T_c \to 0$?

*The quantum trickle.*—The minimum requirement for a quantum thermodynamical device is a system connected simultaneously to three reservoirs [26]. These baths are termed hot, cold, and work reservoir as described in Fig. 1. A quantum description requires a representation of the dynamics working medium and the three heat reservoirs. A reduced description is employed in which the dynamics of the working medium is described by the Heisenberg equation for the operator $\hat{\mathbf{O}}$ for open systems [27,28]:

$$\frac{d}{dt}\hat{\mathbf{O}} = \frac{i}{\hbar}[\hat{\mathbf{H}}_s, \hat{\mathbf{O}}] + \frac{\partial \hat{\mathbf{O}}}{\partial t} + \mathcal{L}_h(\hat{\mathbf{O}}) + \mathcal{L}_c(\hat{\mathbf{O}}) + \mathcal{L}_w(\hat{\mathbf{O}}), \tag{1}$$

where $\hat{\mathbf{H}}_s$ is the system Hamiltonian and $\mathcal{L}_g$ are the dissipative completely positive superoperators for each bath ($g = h, c, w$). A minimal Hamiltonian describing the essence of the quantum refrigerator is composed of three interacting oscillators:

$$\begin{aligned} \hat{\mathbf{H}}_s &= \hat{\mathbf{H}}_0 + \hat{\mathbf{H}}_{\text{int}}, \\ \hat{\mathbf{H}}_0 &= \hbar\omega_h \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} + \hbar\omega_c \hat{\mathbf{b}}^\dagger \hat{\mathbf{b}} + \hbar\omega_w \hat{\mathbf{c}}^\dagger \hat{\mathbf{c}}, \\ \hat{\mathbf{H}}_{\text{int}} &= \hbar\omega_{\text{int}}(\hat{\mathbf{a}}^\dagger \hat{\mathbf{b}}\, \hat{\mathbf{c}} + \hat{\mathbf{a}} \hat{\mathbf{b}}^\dagger \hat{\mathbf{c}}^\dagger). \end{aligned} \tag{2}$$

$\hat{\mathbf{H}}_{\text{int}}$ represents an annihilation of excitations on the work and cold bath simultaneous with creating an

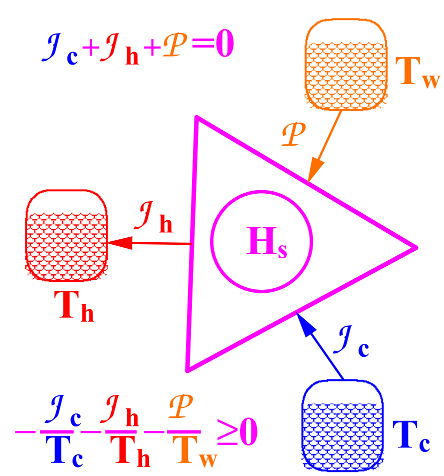


FIG. 1 (color online). The quantum trickle: A quantum heat pump designated by the Hamiltonian $\hat{\mathbf{H}}_s$ is coupled to a work reservoir with temperature $T_w$, a hot reservoir with temperature $T_h$, and a cold reservoir with temperature $T_c$. The heat and work currents are indicated. In the steady state, $\mathcal{J}_h + \mathcal{J}_c + \mathcal{P} = 0$.

 

excitation in the hot bath. In an open quantum system, the superoperators $\mathcal{L}_g$ represent a thermodynamic isothermal partition allowing heat flow from the bath to the system. Such a partition is equivalent to the weak coupling limit between the system and bath [11]. The superoperators $\mathcal{L}_g$ are derived from the Hamiltonian:

$$\hat{\mathbf{H}} = \hat{\mathbf{H}}_s + \hat{\mathbf{H}}_h + \hat{\mathbf{H}}_c + \hat{\mathbf{H}}_w + \hat{\mathbf{H}}_{sh} + \hat{\mathbf{H}}_{sc} + \hat{\mathbf{H}}_{sw}, \quad (3)$$

where $\hat{\mathbf{H}}_g$ are bath Hamiltonians and $\hat{\mathbf{H}}_{sg}$ represent system bath coupling. Each of the oscillators is linearly coupled to a heat reservoir, for example, for the hot bath: $\hat{\mathbf{H}}_{sh} = \lambda_{sh}(\hat{\mathbf{a}}\hat{\mathbf{A}}_h^\dagger + \hat{\mathbf{a}}^\dagger\hat{\mathbf{A}}_h)$. Each reservoir individually should equilibrate the working medium to thermal equilibrium with the reservoir temperature. In general, the derivation of a thermodynamically consistent master equation is technically very difficult [29]. Typical problems are approximations that violate the laws of thermodynamics. We therefore require that the master equations fulfill the thermodynamical laws. Under steady state conditions of operation, they become:

$$\mathcal{J}_h + \mathcal{J}_c + \mathcal{P} = 0, \qquad -\frac{\mathcal{J}_h}{T_h} - \frac{\mathcal{J}_c}{T_c} - \frac{\mathcal{P}}{T_w} \geq 0, \quad (4)$$

where $\mathcal{J}_k = \langle \mathcal{L}_k(\hat{\mathbf{H}})\rangle$. The first equality represents conservation of energy (first law) [8,9], and the second inequality represents positive entropy production in the universe $\Sigma_u \geq 0$ (second law). For refrigeration, $T_w \geq T_h \geq T_c$. From the second law, the scaling exponent $\alpha \geq 1$ [12].

*Gaussian-noise-driven refrigerator.*—In the absorption refrigerator, the noise source replaces the work bath and its contact $\hbar\omega_w\hat{\mathbf{c}}^\dagger\hat{\mathbf{c}}$, leading to

$$\hat{\mathbf{H}}_{\text{int}} = f(t)(\hat{\mathbf{a}}^\dagger\hat{\mathbf{b}} + \hat{\mathbf{a}}\hat{\mathbf{b}}^\dagger) = f(t)\hat{\mathbf{X}}, \quad (5)$$

where $f(t)$ is the noise field. $\hat{\mathbf{X}} = (\hat{\mathbf{a}}^\dagger\hat{\mathbf{b}} + \hat{\mathbf{a}}\hat{\mathbf{b}}^\dagger)$ is the generator of a swap operation between the two oscillators and is part of a set of $SU(2)$ operators, $\hat{\mathbf{Y}} = i(\hat{\mathbf{a}}^\dagger\hat{\mathbf{b}} - \hat{\mathbf{a}}\hat{\mathbf{b}}^\dagger)$, $\hat{\mathbf{Z}} = (\hat{\mathbf{a}}^\dagger\hat{\mathbf{a}} - \hat{\mathbf{b}}^\dagger\hat{\mathbf{b}})$ and the Casimir operator $\hat{\mathbf{N}} = (\hat{\mathbf{a}}^\dagger\hat{\mathbf{a}} + \hat{\mathbf{b}}^\dagger\hat{\mathbf{b}})$.

We first study a Gaussian source of white noise characterized by zero mean $\langle f(t)\rangle = 0$ and delta time correlation $\langle f(t)f(t')\rangle = 2\eta\delta(t-t')$. The Heisenberg equation for a time-independent operator $\hat{\mathbf{O}}$ reduced to

$$\frac{d}{dt}\hat{\mathbf{O}} = i[\hat{\mathbf{H}}_s, \hat{\mathbf{O}}] + \mathcal{L}_n(\hat{\mathbf{O}}) + \mathcal{L}_h(\hat{\mathbf{O}}) + \mathcal{L}_c(\hat{\mathbf{O}}), \quad (6)$$

where $\hat{\mathbf{H}}_s = \hbar\omega_h\hat{\mathbf{a}}^\dagger\hat{\mathbf{a}} + \hbar\omega_c\hat{\mathbf{b}}^\dagger\hat{\mathbf{b}}$. The noise dissipator for Gaussian noise is $\mathcal{L}_n(\hat{\mathbf{O}}) = -\eta[\hat{\mathbf{X}}, [\hat{\mathbf{X}}, \hat{\mathbf{O}}]]$ [30]. The same master equation is obtained for a system subject to a weak quantum measurement of the operator $\hat{\mathbf{X}}$ [28]. The next step is to derive the quantum master equation of each reservoir. We assume that the reservoirs are uncorrelated and also uncorrelated with the driving noise. These conditions simplify the derivation of $\mathcal{L}_h$, which become the standard energy relaxation terms driving oscillator $\omega_h\hat{\mathbf{a}}^\dagger\hat{\mathbf{a}}$ to thermal equilibrium with temperature $T_h$, and $\mathcal{L}_c$ drives oscillator $\hbar\omega_b\hat{\mathbf{b}}^\dagger\hat{\mathbf{b}}$ to equilibrium $T_c$ [28]:

$$\begin{aligned}\mathcal{L}_h(\hat{\mathbf{O}}) &= \Gamma_h(N_h+1)(\hat{\mathbf{a}}^\dagger\hat{\mathbf{O}}\,\hat{\mathbf{a}} - \tfrac{1}{2}\{\hat{\mathbf{a}}^\dagger\hat{\mathbf{a}}, \hat{\mathbf{O}}\}) \\ &\quad + \Gamma_h N_h(\hat{\mathbf{a}}\,\hat{\mathbf{O}}\,\hat{\mathbf{a}}^\dagger - \tfrac{1}{2}\{\hat{\mathbf{a}}\hat{\mathbf{a}}^\dagger, \hat{\mathbf{O}}\}), \\ \mathcal{L}_c(\hat{\mathbf{O}}) &= \Gamma_c(N_c+1)(\hat{\mathbf{b}}^\dagger\hat{\mathbf{O}}\,\hat{\mathbf{b}} - \tfrac{1}{2}\{\hat{\mathbf{b}}^\dagger\hat{\mathbf{b}}, \hat{\mathbf{O}}\}) \\ &\quad + \Gamma_c N_c(\hat{\mathbf{b}}\,\hat{\mathbf{O}}\,\hat{\mathbf{b}}^\dagger - \tfrac{1}{2}\{\hat{\mathbf{b}}\hat{\mathbf{b}}^\dagger, \hat{\mathbf{O}}\}).\end{aligned} \quad (7)$$

In the absence of the stochastic driving field, these equations drive oscillators $a$ and $b$ separately to thermal equilibrium provided that $N_h = [\exp(\frac{\hbar\omega_h}{kT_h}) - 1]^{-1}$ and $N_c = [\exp(\frac{\hbar\omega_c}{kT_c}) - 1]^{-1}$. The kinetic coefficients $\Gamma_{h/c}$ are determined from the bath density functions [11].

The equations of motion are closed to the $SU(2)$ set of operators. To derive the cooling current $\mathcal{J}_c = \langle \mathcal{L}_c(\hbar\omega_c\hat{\mathbf{b}}^\dagger\hat{\mathbf{b}})\rangle$, we solve for stationary solutions of $\hat{\mathbf{N}}$ and $\hat{\mathbf{Z}}$, obtaining

$$\mathcal{J}_c = \hbar\omega_c\frac{(N_c - N_h)}{(2\eta)^{-1} + \Gamma_h^{-1} + \Gamma_c^{-1}}. \quad (8)$$

Cooling occurs for $N_c > N_h \Rightarrow \frac{\omega_h}{T_h} > \frac{\omega_c}{T_c}$. The coefficient of performance (COP) for the absorption chiller is defined by the relation $\text{COP} = \frac{\mathcal{J}_c}{\mathcal{J}_n}$; with the help of Eq. (8), we obtain the Otto cycle COP [31]:

$$\text{COP} = \frac{\omega_c}{\omega_h - \omega_c} \leq \frac{T_c}{T_h - T_c}. \quad (9)$$

A different viewpoint starts from the high temperature limit of the work bath $T_w$ based on the weak coupling limit in Eqs. (2) and (3); then

$$\begin{aligned}\mathcal{L}_w(\hat{\mathbf{O}}) &= \Gamma_w(N_w+1)(\hat{\mathbf{a}}^\dagger\hat{\mathbf{b}}\,\hat{\mathbf{O}}\,\hat{\mathbf{b}}^\dagger\hat{\mathbf{a}} - \tfrac{1}{2}\{\hat{\mathbf{a}}^\dagger\hat{\mathbf{a}}\,\hat{\mathbf{b}}\,\hat{\mathbf{b}}^\dagger, \hat{\mathbf{O}}\}) \\ &\quad + \Gamma_w N_w(\hat{\mathbf{a}}\hat{\mathbf{b}}^\dagger\hat{\mathbf{O}}\hat{\mathbf{a}}^\dagger\hat{\mathbf{b}} - \tfrac{1}{2}\{\hat{\mathbf{a}}\hat{\mathbf{a}}^\dagger\hat{\mathbf{b}}^\dagger\hat{\mathbf{b}}, \hat{\mathbf{O}}\}),\end{aligned} \quad (10)$$

where $N_w = [\exp(\frac{\hbar\omega_w}{kT_h}) - 1]^{-1}$. At a finite temperature, $\mathcal{L}_w(\hat{\mathbf{O}})$ does not lead to a closed set of equations. But in the limit of $T_w \to \infty$ it becomes equivalent to the Gaussian noise generator: $\mathcal{L}_w(\hat{\mathbf{O}}) = -\eta/2([\hat{\mathbf{X}}, [\hat{\mathbf{X}}, \hat{\mathbf{O}}]] + [\hat{\mathbf{Y}}, [\hat{\mathbf{Y}}, \hat{\mathbf{O}}]])$, where $\eta = \Gamma_w N_w$. This noise generator leads to the same current $\mathcal{J}_c$ and COP as Eqs. (8) and (9). We conclude that Gaussian noise represents the singular bath limit equivalent to $T_w \to \infty$. As a result, the entropy generated by the noise is zero.

The solutions are consistent with the first and second laws of thermodynamics. The COP is restricted by the Carnot COP. For low temperatures, the optimal cooling current can be approximated by $\mathcal{J}_c \simeq \omega_c\Gamma_c N_c$. Coupling to a thermal bosonic field such as an electromagnetic or acoustic phonon field implies $\Gamma_c \propto \omega_c^d$, where $d$ is the heat bath dimension. Optimizing the cooling current with respect to $\omega_c$, one obtains that the exponent $\alpha$ quantifying the third law $\mathcal{J}_c \propto T_c^\alpha$ is given by $\alpha = d + 1$.

*Poisson-noise-driven refrigerator.*—Poisson white noise can be referred to as a sequence of independent random pulses with exponential interarrival times. These impulses drive the coupling between the oscillators in contact with the hot and cold bath leading to [32]

$$\frac{d\hat{\mathbf{O}}}{dt} = (i/\hbar)[\tilde{\mathbf{H}}, \hat{\mathbf{O}}] - (i/\hbar)\lambda\langle\xi\rangle[\hat{\mathbf{X}}, \hat{\mathbf{O}}] + \lambda\left(\int_{-\infty}^{\infty} d\xi P(\xi) e^{(i/\hbar)\xi\hat{\mathbf{X}}}\hat{\mathbf{O}}e^{(-i/\hbar)\xi\hat{\mathbf{X}}} - \hat{\mathbf{O}}\right), \quad (11)$$

where $\tilde{\mathbf{H}}$ is the total Hamiltonian including the baths. $\lambda$ is the rate of events, and $\xi$ is the impulse strength averaged over a distribution $P(\xi)$. Using the Hadamard lemma and the fact that the operators form a closed $SU(2)$ algebra, we can separate the noise contribution to its unitary and dissipation parts, leading to the master equation

$$\frac{d\hat{\mathbf{O}}}{dt} = (i/\hbar)[\tilde{\mathbf{H}}, \hat{\mathbf{O}}] + (i/\hbar)[\hat{\mathbf{H}}', \hat{\mathbf{O}}] + \mathcal{L}_n(\hat{\mathbf{O}}). \quad (12)$$

The unitary part is generated with the addition of the Hamiltonian $\hat{\mathbf{H}}' = \hbar\epsilon\hat{\mathbf{X}}$ with the interaction

$$\epsilon = -\frac{\lambda}{2}\int d\xi P(\xi)[2\xi/\hbar - \sin(2\xi/\hbar)].$$

This term can cause a direct heat leak from the hot to cold bath. The noise generator $\mathcal{L}_n(\hat{\boldsymbol{\rho}})$ can be reduced to the form $\mathcal{L}_n(\hat{\mathbf{O}}) = -\eta[\hat{\mathbf{X}}, [\hat{\mathbf{X}}, \hat{\mathbf{O}}]]$, with a modified noise parameter:

$$\eta = \frac{\lambda}{4}\left(1 - \int d\xi P(\xi)\cos(2\xi/\hbar)\right).$$

The Poisson noise generates an effective Hamiltonian which is composed of $\tilde{\mathbf{H}}$ and $\hat{\mathbf{H}}'$, modifying the energy levels of the working medium. This new Hamiltonian structure has to be incorporated in the derivation of the master equation; otherwise, the second law will be violated. The first step is to rewrite the system Hamiltonian in its dressed form. A new set of bosonic operators is defined:

$$\hat{\mathbf{A}}_1 = \hat{\mathbf{a}}\cos(\theta) + \hat{\mathbf{b}}\sin(\theta), \quad \hat{\mathbf{A}}_2 = \hat{\mathbf{b}}\cos(\theta) - \hat{\mathbf{a}}\sin(\theta). \quad (13)$$

The dressed Hamiltonian is given by

$$\hat{\mathbf{H}}_s = \hbar\Omega_+\hat{\mathbf{A}}_1^\dagger\hat{\mathbf{A}}_1 + \hbar\Omega_-\hat{\mathbf{A}}_2^\dagger\hat{\mathbf{A}}_2, \quad (14)$$

where $\Omega_\pm = \frac{\omega_h+\omega_c}{2} \pm \sqrt{[(\omega_h-\omega_c)/2]^2+\epsilon^2}$ and $\cos^2(\theta) = \frac{\omega_h-\Omega_-}{\Omega_+-\Omega_-}$ Eq. (14) impose the restriction $\Omega_\pm > 0$, which can be translated to $\omega_h\omega_c > \epsilon^2$. The master equation in the Heisenberg representation becomes

$$\frac{d\hat{\mathbf{O}}}{dt} = (i/\hbar)[\hat{\mathbf{H}}_s, \hat{\mathbf{O}}] + \mathcal{L}_h(\hat{\mathbf{O}}) + \mathcal{L}_c(\hat{\mathbf{O}}) + \mathcal{L}_n(\hat{\mathbf{O}}), \quad (15)$$

where

$$\begin{aligned}\mathcal{L}_h(\hat{\mathbf{O}}) = {}& \gamma_1^h\mathbf{c}^2(\hat{\mathbf{A}}_1\hat{\mathbf{O}}\hat{\mathbf{A}}_1^\dagger - \tfrac{1}{2}\{\hat{\mathbf{A}}_1\hat{\mathbf{A}}_1^\dagger, \hat{\mathbf{O}}\}) + \gamma_2^h\mathbf{c}^2(\hat{\mathbf{A}}_1^\dagger\hat{\mathbf{O}}\hat{\mathbf{A}}_1 \\ & - \tfrac{1}{2}\{\hat{\mathbf{A}}_1^\dagger\hat{\mathbf{A}}_1, \hat{\mathbf{O}}\}) + \gamma_3^h\mathbf{s}^2(\hat{\mathbf{A}}_2\hat{\mathbf{O}}\hat{\mathbf{A}}_2^\dagger - \tfrac{1}{2}\{\hat{\mathbf{A}}_2\hat{\mathbf{A}}_2^\dagger, \hat{\mathbf{O}}\}) \\ & + \gamma_4^h\mathbf{s}^2(\hat{\mathbf{A}}_2^\dagger\hat{\mathbf{O}}\hat{\mathbf{A}}_2 - \tfrac{1}{2}\{\hat{\mathbf{A}}_2^\dagger\hat{\mathbf{A}}_2, \hat{\mathbf{O}}\}), \\ \mathcal{L}_c(\hat{\mathbf{O}}) = {}& \gamma_1^c\mathbf{s}^2(\hat{\mathbf{A}}_1\hat{\mathbf{O}}\hat{\mathbf{A}}_1^\dagger - \tfrac{1}{2}\{\hat{\mathbf{A}}_1\hat{\mathbf{A}}_1^\dagger, \hat{\mathbf{O}}\}) + \gamma_2^c\mathbf{s}^2(\hat{\mathbf{A}}_1^\dagger\hat{\mathbf{O}}\hat{\mathbf{A}}_1 \\ & - \tfrac{1}{2}\{\hat{\mathbf{A}}_1^\dagger\hat{\mathbf{A}}_1, \hat{\mathbf{O}}\}) + \gamma_3^c\mathbf{c}^2(\hat{\mathbf{A}}_2\hat{\mathbf{O}}\hat{\mathbf{A}}_2^\dagger - \tfrac{1}{2}\{\hat{\mathbf{A}}_2\hat{\mathbf{A}}_2^\dagger, \hat{\mathbf{O}}\}) \\ & + \gamma_4^c\mathbf{c}^2(\hat{\mathbf{A}}_2^\dagger\hat{\mathbf{O}}\hat{\mathbf{A}}_2 - \tfrac{1}{2}\{\hat{\mathbf{A}}_2^\dagger\hat{\mathbf{A}}_2, \hat{\mathbf{O}}\}), \end{aligned} \quad (16)$$

where $\mathbf{s} = \sin(\theta)$ and $\mathbf{c} = \cos(\theta)$, and the noise generator

$$\mathcal{L}_n(\hat{\mathbf{O}}) = -\eta[\hat{\mathbf{W}}, [\hat{\mathbf{W}}, \hat{\mathbf{O}}]], \quad (17)$$

where $\hat{\mathbf{W}} = \sin(2\theta)\hat{\mathbf{Z}} + \cos(2\theta)\hat{\mathbf{X}}$ and a new set of operators which form an $SU(2)$ algebra is defined: $\hat{\mathbf{X}} = (\hat{\mathbf{A}}_1^\dagger\hat{\mathbf{A}}_2 + \hat{\mathbf{A}}_2^\dagger\hat{\mathbf{A}}_1)$, $\hat{\mathbf{Y}} = i(\hat{\mathbf{A}}_1^\dagger\hat{\mathbf{A}}_2 - \hat{\mathbf{A}}_2^\dagger\hat{\mathbf{A}}_1)$, and $\hat{\mathbf{Z}} = (\hat{\mathbf{A}}_1^\dagger\hat{\mathbf{A}}_1 - \hat{\mathbf{A}}_2^\dagger\hat{\mathbf{A}}_2)$. The total number of excitations is accounted for by the operator $\hat{\mathbf{N}} = (\hat{\mathbf{A}}_1^\dagger\hat{\mathbf{A}}_1 + \hat{\mathbf{A}}_2^\dagger\hat{\mathbf{A}}_2)$. The generalized heat transport coefficients become $\zeta_+^k = \gamma_2^k - \gamma_1^k$ and $\zeta_-^k = \gamma_4^k - \gamma_3^k$ for $k = h, c$. Applying the Kubo relation [33,34] $\gamma_1^k = e^{-\hbar\Omega_+\beta_k}\gamma_2^k$ and $\gamma_3^k = e^{-\hbar\Omega_-\beta_k}\gamma_4^k$ leads to the detailed balance relation

$$\frac{\gamma_1^k}{\zeta_+^k} = \frac{1}{e^{\hbar\Omega_+\beta_k} - 1} \equiv N_+^k, \qquad \frac{\gamma_3^k}{\zeta_-^k} = \frac{1}{e^{\hbar\Omega_-\beta_k} - 1} \equiv N_-^k.$$

In general, $\zeta_\pm^k$ is temperature-independent and can be calculated specifically for different choices of spectral density of the baths. For an electromagnetic or acoustic phonon field, $\zeta_\pm^k \propto \Omega_\pm^d$. The heat currents $\mathcal{J}_h$, $\mathcal{J}_c$, and $\mathcal{J}_n$ are calculated by solving the equation of motion for the operators at steady state and at the regime of low temperature, where $\cos^2(\theta) \approx 1$ and $\sin^2(\theta) \approx 0$:

$$\begin{aligned}\frac{d\hat{\mathbf{N}}}{dt} &= -\frac{1}{2}(\zeta_+^h + \zeta_-^c)\hat{\mathbf{N}} - \frac{1}{2}(\zeta_+^h - \zeta_-^c)\hat{\mathbf{Z}} + (\zeta_+^h N_+^h + \zeta_-^c N_-^c), \\ \frac{d\hat{\mathbf{Z}}}{dt} &= -\frac{1}{2}(\zeta_+^h + \zeta_-^c)\hat{\mathbf{Z}} - \frac{1}{2}(\zeta_+^h - \zeta_-^c)\hat{\mathbf{N}} + (\zeta_+^h N_+^h - \zeta_-^c N_-^c) - 4\eta\hat{\mathbf{Z}}. \end{aligned} \quad (18)$$

Once the set of linear equations is solved, the exact expression for the heat currents is extracted: $\mathcal{J}_h = \langle\mathcal{L}_h(\hat{\mathbf{H}}_s)\rangle$, $\mathcal{J}_c = \langle\mathcal{L}_c(\hat{\mathbf{H}}_s)\rangle$, and $\mathcal{J}_n = \langle\mathcal{L}_n(\hat{\mathbf{H}}_s)\rangle$. For simplicity, the distribution of impulses in Eq. (11) is chosen as $P(\xi) = \delta(\xi - \xi_0)$. Then the effective noise parameter becomes

$$\eta = \frac{\lambda}{4}[1 - \cos(2\xi_0/\hbar)]. \quad (19)$$

The energy shift is controlled by

$$\epsilon = -\frac{\lambda}{2}[2\xi_0/\hbar - \sin(2\xi_0/\hbar)]. \quad (20)$$

Figure 2 shows a periodic structure of the heat current $\mathcal{J}_c$ and the entropy production $\Sigma_c = -\mathcal{J}_c/T_c$ with the impulse $\xi_0$. The second law of thermodynamics is obtained by the balance of the large entropy generation on the hot bath compensating for the negative entropy generation of cooling the cold bath. The COP for the Poisson-driven refrigerator is restricted by the Otto and Carnot COP:

$$\mathrm{COP} = \frac{\Omega_-}{\Omega_+ - \Omega_-} \leq \frac{\omega_c}{\omega_h - \omega_c} \leq \frac{T_c}{T_h - T_c}. \quad (21)$$

The heat current $\mathcal{J}_c$ is given by

$$\mathcal{J}_c \approx \hbar\Omega_- \frac{N_-^c - N_+^h}{(2\eta)^{-1} + (\zeta_+^h)^{-1} + (\zeta_-^c)^{-1}}. \quad (22)$$

The scaling of the optimal cooling rate is now accounted for. The heat flow is maximized with respect to the impulse $\xi_0$ by maximizing $\eta$ [Eq. (19)], which occurs for $\xi_0 = n\frac{\pi}{2}$ ($n = 1, 2, \ldots$). On the other hand, the energy shift $\epsilon^2$ [Eq. (20)] should be minimized. The optimum is obtained when $\xi_0 = \frac{\pi}{2}$. The cooling power of the Poisson noise case [Eq. (22)] is similar to the Gaussian one [Eq. (8)]. In the Poisson case, also the noise driving parameter $\eta$ is restricted by $\omega_c$. This is because $\epsilon$ is restricted by $\Omega_- \geq 0$, and therefore $\lambda$ is restricted to scale with $\omega_c$. In total, when $T_c \to 0$, $\mathcal{J}_c \propto T_c^{d+1}$.

The optimal scaling relation $\mathcal{J}_c \propto T_c^{\alpha}$ of the autonomous absorption refrigerators should be compared to the scaling of the discrete four-stroke Otto refrigerators [35]. In the driven discrete case, the scaling depends on the external control scheduling function on the expansion stroke. For a scheduling function determined by a constant frictionless nonadiabatic parameter, the optimal cooling rate scaled with $\alpha = 2$. Faster frictionless scheduling procedures were found based on a bang-bang type of optimal control solutions. These solutions led to a scaling of $\alpha = 3/2$ when positive frequencies were employed and $\mathcal{J}_c \propto -T_c/\log T_c$ when negative imaginary frequencies were allowed [36,37]. $\mathcal{J}_c \propto T_c$ was obtained in the limit of large energy levels for a swap-based Otto cycle [38]. The drawback of the externally driven refrigerators is that their analysis is complex. The optimal scaling assumes that the heat conductivity $\Gamma \gg \omega_c$ and that noise in the controls does not influence the scaling. For this reason, an analysis based on the autonomous refrigerators is superior.

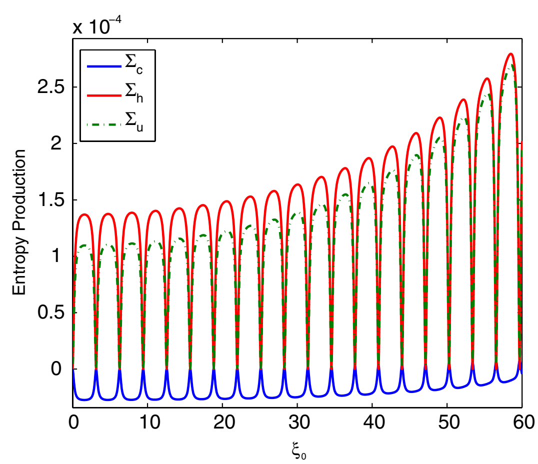


FIG. 2 (color online). Entropy production $\Sigma_k = -\mathcal{J}_k/T_k$ as a function of impulse $\xi_0$ for the cold $\Sigma_c$, hot $\Sigma_h$, and the total entropy production $\Sigma_u = \Sigma_h + \Sigma_c$. $T_c = 10^{-3}$, $T_h = 2$, $\omega_c = T_c$, $\omega_h = 10$, $\lambda = \omega_c$, and $\zeta_{\pm}^k = \omega_c/10$ ($\hbar = k = 1$).

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