Preferred States of Decoherence under Intermediate System-Environment Coupling

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(Received 26 April 2011; revised manuscript received 9 September 2011; published 17 February 2012)

The notion that decoherence rapidly reduces a superposition state to an incoherent mixture implicitly adopts a special representation, namely, the representation of preferred (pointer) states (PS). For weak or strong system-envrionment interaction, the behavior of PS is well known. Via a simple dynamical model that simulates a two-level system interacting with few other degrees of freedom as its environment, it is shown that even for intermediate system-environment coupling, approximate PS may still emerge from the coherent quantum dynamics of the whole system in the absence of any thermal averaging. The found PS can also continuously deform to expected limits for weak or strong system-environment coupling. Computational results are also qualitatively explained. The findings should be useful towards further understanding of decoherence and quantum thermalization processes.

DOI: 10.1103/PhysRevLett.108.070403 PACS numbers: 03.65.Ta, 03.65.Yz, 03.67.Mn, 05.45.Mt

Introduction.—As illustrated by the Schrödinger cat paradox, there is a clash between the quantum superposition principle and the way we perceive the macroscopic reality. So how does a classical world emerge from the quantum? One promising solution to this profound question is decoherence, i.e., the loss of quantum coherence due to the interaction of a system of interest with its environment [1–3]. Decoherence may rapidly reduce a coherent superposition state of the system to an incoherent mixture. During this process the environment singles out special basis states, often called ''preferred (pointer) states'' (PS), of which a classical probabilistic description becomes sufficient to describe the system and the bizarre superposition state of the PS is out of the picture. That is, in the PS representation the reduced density matrix (RDM) of the system becomes diagonal as time evolves.

Such a decoherence perspective is not expected to resolve all conceptual issues regarding quantum weirdness vs classical reality. Nevertheless, it is highly useful as it implies the environmental dependence of the quantum-classical transition and the representation-dependent nature of decoherence issues. For example, different environments may select different PS, and a quantum system decohered in one PS representation may still possess certain quantum coherence in other representations. Going further, one may envision the possibilities of environment engineering to form desired PS [4], such that system properties are robust to decoherence.

PS have been identified in several cases. If the system-environment interaction is in the adiabatic limit or if it commutes with the system's self-Hamiltonian, then populations on the energy eigenstates of the system's self-Hamiltonian do not change but their relative phases are destroyed by the environment. The energy eigenstates then form the PS [5–10]. Analogous to this, if the system-environment coupling is sufficiently weak, then the energy relaxation time scale is typically much longer than the pure-dephasing time scale. As a result, the energy eigenstates still form the PS before relaxation sets in [11–13]. Again, for weak system-environment interaction but for a longer time scale, a model of quantum Brownian motion reveals that coherent states localized in both position and momentum turn out to be the PS [1,2,10,14]. In the opposite situation, the system-environment coupling is strong and the system's self-Hamiltonian becomes negligible within a certain time period. In this case, the eigenstates of the system-environment interaction Hamiltonian form the approximate PS [5–7].

Little has been said about the possible existence of PS for a generic system-environment coupling (i.e., not commutable with system's self-Hamiltonian) of intermediate strength. Under such a situation, the widely used quantum-master-equation approach or other perturbative approaches may not be applicable in analyzing the existence of PS. Also motivated by the ongoing investigations of quantum thermalization processes [15,16], we choose to work with a simple dynamical model to address the issue of PS. That is, within a single isolated quantum system composed of interacting quantum subsystems, will the concept of PS still work well in describing the decoherence of one subsystem due to its interaction with other subsystems [17]?

We start from a computationally intuitive definition of PS. We then show interesting evidence that PS may still exist for intermediate system-environment coupling and further explain why this is possible. The found PS, neither the system's energy eigenstates nor the eigenstates of the system-environment interaction Hamiltonian, undergo continuous deformation as the system-environment coupling strength varies. These findings show that decoherence-induced superselection rule can be twofold:

superposition states of the PS are destroyed but PS themselves can be rich superposition states for intermediate system-environment coupling. Equally interesting, it can be concluded that the concept of PS is still important in understanding quantum dynamical processes in the absence of any thermal averaging.

Identifying PS from the time-evolving RDM.—If PS exists, then the RDM will gradually become diagonal in the PS representation. On the other hand, the same RDM is always diagonal in its own eigenrepresentation. Therefore, if we computationally track the eigenstates of the RDM, then we can see clearly whether or not a well-defined set of PS can emerge from a decoherence process. That is, if after a certain period the eigenstates of the RDM are found to evolve closely around a fixed basis set, then this fixed set of states can be defined as the PS, at least approximately. This computational definition of PS extends the PS criterion used by Diósi and Kiefer [8]. Note also that such a definition of PS gives up their precise analytical form. Consistent with this picture, the off-diagonal elements of the RDM in the PS representation must be also small when compared with the difference of its diagonal elements. Indeed, were the RDM diagonal elements almost degenerate, then a small fluctuation in the off-diagonal elements can still cause a drastic rotation of the RDM eigenstates, a fact that would contradict with the existence of PS [18]. We hence mainly work in the parameter regimes where an appreciable difference between the diagonal elements of RDM can emerge from the dynamics. These preliminaries also make it clear that even a stationary RDM does not necessarily mean the existence of PS.

Consider now a two-level system S interacting with its environment $\mathcal{E}$, with a total Hamiltonian $H = H_S + H_I + H_\mathcal{E}$, where H_S and $H_\mathcal{E}$ are the Hamiltonians of S and $\mathcal{E}$, and H_I is the system-environment interaction Hamiltonian, with $[H_S, H_I] \neq 0$. Eigenstates of H_S are denoted by $|E_k\rangle$, with $k = 0, 1$. Throughout, we use $|\Psi\rangle$ to denote a state vector for the whole system-environment combination, denoted by $S + \mathcal{E}$ and isolated from any thermal bath. The time-evolving RDM for S is given by $\rho^s(t) \equiv \mathrm{Tr}_\mathcal{E}(|\Psi(t)\rangle\langle\Psi(t)|)$, where $|\Psi(t)\rangle = e^{-iHt/\hbar}|\Psi(0)\rangle$ coherently evolves according to Schrödinger equation for $S + \mathcal{E}$.

Eigenstates of the RDM $\rho^s(t)$ are represented by $|\rho_k(t)\rangle$, with eigenvalues $\rho_k(t)$, i.e., $\rho^s(t)|\rho_k(t)\rangle = \rho_k(t)|\rho_k(t)\rangle$, with $k = 0, 1$. The two states $|\rho_0(t)\rangle$ and $|\rho_1(t)\rangle$ also form an orthonormal basis set for the Hilbert subspace associated with H_S. The distance between this basis set and another basis set $|\eta_{k'}\rangle$ ($k' = 0, 1$) for the same subspace may be measured by $D(|\rho_k(t)\rangle, |\eta_{k'}\rangle) = 1 - |\langle\rho_k(t)|\eta_{k'}\rangle|^2$, with k and k' determined by the condition $|\langle\rho_k|\eta_{k'}\rangle|^2 \geq 1/2$. A time-averaged distance d over a period $[t_a, t_b]$ can then be defined as $d(|\eta_{k'}\rangle) = [1/(t_b - t_a)] \int_{t_a}^{t_b} dt' D(|\rho_k(t')\rangle, |\eta_{k'}\rangle)$. If, for a particular basis set $\{|\tilde{\rho}_{k'}\rangle\}$ defined below, $d(|\tilde{\rho}_{k'}\rangle)$ is small for sufficiently large t_a, t_b, and $t_b - t_a$, then $\rho^s(t)$ becomes almost diagonal in the $\{|\tilde{\rho}_0\rangle, |\tilde{\rho}_1\rangle\}$ representation, and hence $\{|\tilde{\rho}_0\rangle, |\tilde{\rho}_1\rangle\}$ can be computationally identified as the PS.

To find $\{|\tilde{\rho}_0\rangle, |\tilde{\rho}_1\rangle\}$ that may reflect the average behavior of $|\rho_k(t)\rangle$ with acceptable fluctuations, we calculate the time-evolving RDM eigenstates $|\rho_k(t)\rangle$, average the density matrix $|\rho_k(t)\rangle\langle\rho_k(t)|$ over time (value of $k = 0$ or $k = 1$ is chosen to maintain a continuity), and then obtain a time-averaged density matrix $\overline{\rho}$. Finally, the eigenstates of $\bar{\rho}$ are defined as the basis states $\{|\tilde{\rho}_0\rangle, |\tilde{\rho}_1\rangle\}$ [19]. If, in the $\{|\tilde{\rho}_0\rangle, |\tilde{\rho}_1\rangle\}$ representation, $|\rho^s_{01}(t)| \ll |\rho^s_{00}(t) - \rho^s_{11}(t)|$ for sufficiently large times t, then $d(|\tilde{\rho}_{k'}\rangle)$ is small and PS can hence be identified [19]. If this is not the case, then PS fails to emerge from the dynamics.

Model.—We now turn to a concrete model. To reflect the fact that typically a small system S is not directly coupled to the whole of its environment $\mathcal{E}$, we let S be directly coupled to a small component A of $\mathcal{E}$, and then let A be further coupled to the rest part B of $\mathcal{E}$, with $\mathcal{E} = A + B$. For convenience, A is also assumed to be a two-level system. Such kind of coupling scheme was also considered recently [20] to model a nonlinear system-environment coupling. It can be also qualitatively argued that our coupling scheme can yield much less fluctuation in the RDM than a full coupling between S and $\mathcal{E}$ does. The B part of $\mathcal{E}$ is simulated by a quantum kicked rotor on a torus with only 1 degree of freedom, whose classical limit is fully chaotic [21,22]. The irregular motion of B due to quantum chaos, instead of many noninteracting degrees of freedom of a thermal bath, is responsible for decoherence in S. In terms of standard Pauli matrices and operators for a kicked rotor in dimensionless units, the Hamiltonians for the system, the environment, and their coupling are

$$\begin{aligned} &H_S = \omega_x\sigma_x^S + \omega_z\sigma_z^S, \qquad H_I = \varepsilon\sigma_z^S \otimes \sigma_z^A, \\ &H_\mathcal{E} = H_A + H_B + H_{AB}. \end{aligned} \tag{1}$$

Here, $H_A = \omega_A\sigma_x^A$, $H_B = \frac{p^2}{2} + v\cos\gamma\sum_j \delta(t - jT)$, and $H_{AB} = \lambda\sigma_z^A\cos\gamma\sum_j\delta(t - jT)$, where p and γ are momentum and coordinate operators of the kicked rotor. Since the system-environment coupling is already of the σ_z^S type, for generality H_S is made to contain both σ_x^S and σ_z^S terms [23]. The unitary propagator associated with one period T is (with $\hbar = 1$).

$$\begin{aligned} \hat{U}_T = {} & e^{-iT(\omega_x\sigma_x^S + \omega_z\sigma_z^S + \omega_A\sigma_x^A + \varepsilon\sigma_z^S\otimes\sigma_z^A)} \\ & \times e^{-iT(p^2/2)}e^{-iv\cos\gamma}e^{-i\lambda\sigma_z^A\cos\gamma}. \end{aligned} \tag{2}$$

The initial state is chosen as $|\Psi(0)\rangle = |\psi_0^S\rangle \otimes |0\rangle_A \otimes |\varphi_0\rangle_B$, where $|\psi_0^S\rangle$ and $|\varphi_0\rangle_B$ are vectors in the Hilbert spaces of S and of B, and $|0\rangle_A$ is an eigenstate of H_A. The quantum kicked rotor is quantized on a phase space torus with a Hilbert space dimension $N = 4096$, whose initial state is taken as a randomly generated vector from its Hilbert space (quantum recurrence time is already sufficiently large). The kicking period T is taken as $2\pi/N$.

Typical values of ω_x, ω_z, and ω_A are set around 10^3 in dimensionless units, such that within one kicking period T the characteristic phase evolution of the two-level systems are of the order of unity. Many initial states were studied but in Fig. 1 we only report representative results for one initial state. Note also that one key parameter is ε, which represents the strength of system-environment coupling.

As shown in Ref. [11], if the system-environment coupling strength ε is below a threshold ε_p, then the off-diagonal elements of the RDM in the eigenrepresentation of H_S will show a Gaussian-type decay. The dephasing time of energy eigenstates (T_2) then scales as ε^{-1}, whereas the population relaxation time (T_1) goes as ε^{-2} (obtained from Fermi's golden rule). In our model we find $\varepsilon_p \sim 1/N$. So for $\varepsilon < 1/N$, $T_2 \ll T_1$, and hence the energy eigenstates $|E_k\rangle$ form the PS. Detailed calculations from our present model confirm this and also reveal something interesting. As shown in Fig. 1 (empty squares), $|E_k\rangle$ are found to agree well with the computationally found PS (i.e., very small values of θ) for ε as large as 10^1. Though our previous work [11] did not rule out the possibility of $|E_k\rangle$ being the PS for $\varepsilon > \varepsilon_p$, it is remarkable to see that $|E_k\rangle$ here still form the PS even for $\varepsilon \gg \varepsilon_p$. This should be related to the fact that here the system (S) is only directly coupled with a small component (A) of the environment.

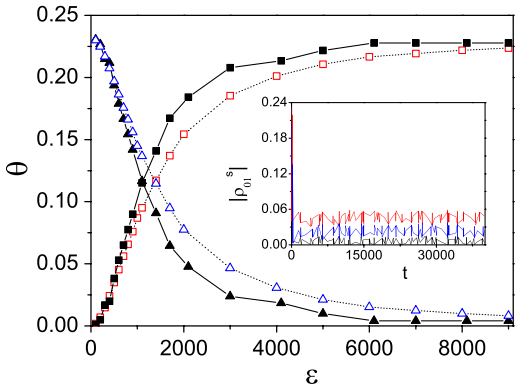

FIG. 1 (color online). Angle θ (in unit of radiant) to be rotated to reach state $|\tilde{\rho}_1\rangle$ (empty symbols) from one eigenstate of H_S (square) or of H_I (triangle), for a wide range of ε. State $|\tilde{\rho}_1\rangle$ is one numerically found eigenstate of $\bar{\rho}$ (a time-averaged RDM for $t \in [30\,000T, 40\,000T]$). For comparison, the θ values to reach the state $|\alpha\rangle$ (filled symbols) determined theoretically [details after Eq. (6)] by maximizing $\|\Delta H\|$, are also presented. The initial state of the system S is placed in a superposition state $0.8\exp(5i)|x_+\rangle + 0.6|x_-\rangle$, where $|x_+\rangle$ and $|x_-\rangle$ represent spin-up and spin-down states along the x direction. For the initial state of the environment, A is placed in an eigenstate of H_A and the kicked-rotor state is chosen randomly. Other system parameters are $\omega_x = 0.5 \times 10^3$, $\omega_z = 1.0 \times 10^3$, $\omega_A = 1.5 \times 10^3$, $v = 90/T$, $N = 2^{12}$, $\lambda = 0.1$. Inset: Decay of the off-diagonal element of RDM with time, in $|E_k\rangle$ representation (upper red curve), in eigenrepresentation of H_I (middle blue curve), and in representation of the PS identified here (bottom dotted curve), for $\varepsilon = 2000$. Note that the decay is essentially done within about 600 T.

Results for larger values of ε are also detailed in Fig. 1. Consider first ε values approaching 10^4, i.e., the right end of the curve shown with empty triangles. In these cases, the computationally found states $|\tilde{\rho}_k\rangle$ are rotated from the H_I eigenstates by essentially a zero angle. Hence, the eigenstates of H_I can be regarded as the PS in this strong coupling case, even for a time scale much larger than the characteristic scale of H_S. Next we turn to intermediate cases with $\varepsilon \in [10^2, 5 \times 10^3]$. As seen from Fig. 1, states $|\tilde{\rho}_k\rangle$ can notably deviate from $|E_k\rangle$ as well as the H_I eigenstates. As we tune up the value of ε, states $|\tilde{\rho}_k\rangle$ exhibit a clear and smooth transition from being close to $|E_k\rangle$ to being close to the H_I eigenstates. Further, as a consistency check, the inset of Fig. 1 shows the decay of the off-diagonal elements of the RDM in three representations, for $\varepsilon = 2000$, as an example. It is seen that only in the $|\tilde{\rho}_k\rangle$ representation, the off-diagonal elements decay to small values with some fluctuations [24].

It is yet to be shown that at sufficiently later times the eigenstates $|\rho_k(t)\rangle$ of the time-evolving RDM only slightly fluctuate around $|\tilde{\rho}_k\rangle$. The upper panel of Fig. 2 depicts the distance $d(|\tilde{\rho}_{k'}\rangle)$ vs ε (solid line) (i.e., for the PS identified in Fig. 1). It is seen that for the entire considered regime of ε, d remains impressively small. For intermediate values of ε, it is much smaller than the same d distance between $|\rho_k(t)\rangle$ and eigenstates of H_S (dashed red line) or between $|\rho_k(t)\rangle$ and eigenstates of H_I (dotted blue line). Therefore, after an initial period the time-evolving RDM eigenstates $\{|\rho_0(t)\rangle, |\rho_1(t)\rangle\}$ do remain close to $\{|\tilde{\rho}_0\rangle, |\tilde{\rho}_1\rangle\}$, suggesting

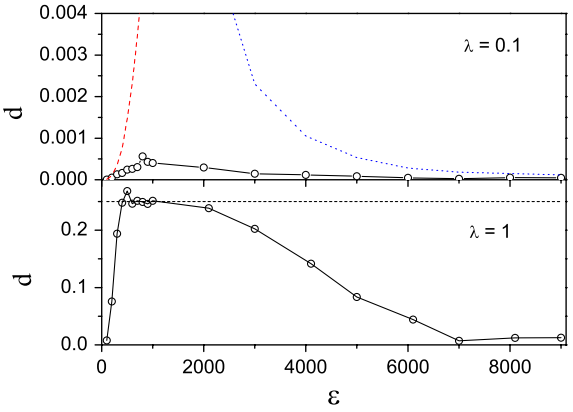

FIG. 2 (color online). Upper panel: Behavior of the time-evolving eigenstates of the system's RDM, as described by a time-averaged distance d from the states $(|\tilde{\rho}_0\rangle, |\tilde{\rho}_1\rangle)$ found computationally in Fig. 1 (circles), from eigenstates of H_S (dashed red line), and from eigenstates of H_I (dotted blue line), for a wide range of ε. Other system parameters are the same as in Fig. 1. Bottom panel: distance d between time-evolving eigenstates of the system's RDM and states $(|\tilde{\rho}_0\rangle, |\tilde{\rho}_1\rangle)$ found computationally in the case of $\lambda = 1.0$. Large d values in the bottom panel indicate the loss of PS.

that the RDM becomes almost diagonal in the $|\tilde{\rho}_k\rangle$ representation, even for intermediate values of ε. We finally infer that the states $|\tilde{\rho}_k\rangle$, whose behavior is shown in Fig. 1, are indeed excellent PS emerging from the coherent quantum dynamics of $S + \mathcal{E}$. Other detailed calculations also indicate that PS does not always exist. For example, if the coupling between A and B is also very strong (e.g., $\lambda = 1.0$, bottom panel of Fig. 2), then the diagonal elements of RDM become very close, d can reach as large as 0.25 (dashed line) (a value that can be estimated theoretically [18]), and consequently PS is lost.

Theoretical insights.—We shall now develop some insights into our computational results. We rewrite the total state for $S + \mathcal{E}$ as

$$|\Psi(t)\rangle = |\alpha\rangle|\phi_\alpha(t)\rangle + |\beta\rangle|\phi_\beta(t)\rangle, \tag{3}$$

where $(|\alpha\rangle, |\beta\rangle)$ is a chosen time-independent orthonormal basis set in the Hilbert subspace for S, $|\phi_\alpha(t)\rangle$ and $|\phi_\beta(t)\rangle$ are the associated "expansion states" living the Hilbert subspace for $\mathcal{E}$. The time dependence of the off-diagonal element of the system's RDM is then given by

$$\rho^s_{\alpha\beta} \equiv \langle\alpha|\rho^s|\beta\rangle = \langle\phi_\beta(t)|\phi_\alpha(t)\rangle. \tag{4}$$

With this notation, seeking PS then becomes the search for $(|\alpha\rangle, |\beta\rangle)$, such that the evolution of $|\phi_\alpha(t)\rangle$ is as different as possible from $|\phi_\beta(t)\rangle$. This insight motivates us to examine the time evolution of the states $|\phi_\alpha(t)\rangle$ and $|\phi_\beta(t)\rangle$ used in Eq. (3). To that end, we first define $H^S_{\alpha\beta} \equiv \langle\alpha|H_S|\beta\rangle$, $H^I_{\alpha\beta} \equiv \langle\alpha|H_I|\beta\rangle$ (and those by $\alpha \leftrightarrow \beta$). Note that $H^S_{\alpha\beta}$ thus defined is a scalar, but $H^I_{\alpha\beta}$ is still an operator on the Hilbert subspace for $\mathcal{E}$. Just to have a rather compact Schrödinger-like equation for $|\phi_\alpha(t)\rangle$ and $|\phi_\beta(t)\rangle$, we introduce more operators on the $\mathcal{E}$ subspace, i.e., $H_{\alpha\alpha} \equiv H_\mathcal{E} + H^I_{\alpha\alpha} + H^S_{\alpha\alpha}$, $H_{\alpha\beta} \equiv H^S_{\alpha\beta} + H^I_{\alpha\beta}$, $K_\alpha \equiv H_{\alpha\beta}H_{\beta\beta}H^{-1}_{\alpha\beta}$, and $J_\alpha \equiv H_{\alpha\beta}H_{\beta\alpha}$. Using these definitions and the Schrödinger equation for $S + \mathcal{E}$, we obtain

$$i\frac{d}{dt}|\phi_\alpha\rangle = H_{\alpha\alpha}|\phi_\alpha\rangle + i|\xi_\alpha\rangle, \tag{5}$$

where $|\xi_\alpha\rangle \equiv -iH_{\alpha\beta}|\phi_\beta\rangle$, with

$$i\frac{d}{dt}|\xi_\alpha\rangle = K_\alpha|\xi_\alpha\rangle - iJ_\alpha|\phi_\alpha\rangle. \tag{6}$$

An analogous equation for $|\phi_\beta(t)\rangle$ is obtained by exchanging α and β.

Equation (5) indicates that the difference between the evolution of $|\phi_\alpha(t)\rangle$ and that of $|\phi_\beta(t)\rangle$ is caused by the difference between the operators $H_{\alpha\alpha}$ and $H_{\beta\beta}$ and by the difference between $|\xi_\alpha\rangle$ and $|\xi_\beta\rangle$. Further, Eq. (6) shows that $|\xi_\alpha\rangle$ and $|\xi_\beta\rangle$ evolve differently due to the difference between K_α and K_β and between J_α and J_β. To quantify these operator differences we define $\Delta H \equiv H_{\alpha\alpha} - H_{\beta\beta}$, $\Delta K \equiv K_\alpha - K_\beta$, and $\Delta J \equiv J_\alpha - J_\beta$. In our model, because H_I is a direct product of two spin operators, one finds $\Delta J = 0$ [18]. Furthermore, for ε much less or much larger than the energy scale of H_S, we find $\Delta K \approx -\Delta H$ [18]. For cases with intermediate values of ε, a more detailed analysis [18] gives again that $\Delta K \approx -\Delta H$, at least for those states $(|\alpha\rangle, |\beta\rangle)$ that maximize $\|\Delta H\|$, where $\|\Delta H\|$ represents the Frobenius-2 norm, a simple measure of ΔH. Putting these observations together, we intuitively expect (not a proof) that for the entire considered regime of ε, the basis states $(|\alpha\rangle, |\beta\rangle)$ maximizing $\|\Delta H\|$ may approximately give the most substantial difference between $|\phi_\alpha(t)\rangle$ and $|\phi_\beta(t)\rangle$ and hence the most significant decay of $|\rho^s_{\alpha\beta}|$. As such, the basis states $(|\alpha\rangle, |\beta\rangle)$ theoretically determined by maximizing $\|\Delta H\|$ should agree with the PS computationally obtained above.

In Fig. 1 we compare the PS (empty symbols) found from the decoherence dynamics with the states $(|\alpha\rangle, |\beta\rangle)$ (filled symbols) determined directly by maximizing $\|\Delta H\|$. In terms of their relation with the eigenstates of H_S and of H_I, nice agreement is obtained for the whole regime of ε.

It is also interesting to discuss the implication of the term $i|\xi_\alpha\rangle$ in Eq. (5). Note that this term does not preserve the norm $\langle\phi_\alpha|\phi_\alpha\rangle$. Hence, populations on the basis states $(|\alpha\rangle, |\beta\rangle)$, even when they are identified as the PS, can still fluctuate with time. This constitutes a crucial difference from a pure-dephasing problem. Unlike in a pure-dephasing problem, here the decay of $\rho^s_{\alpha\beta} = \langle\phi_\beta(t)|\phi_\alpha(t)\rangle$ cannot be interpreted as that of the overlap of two independent environment histories $|\phi_\alpha(t)\rangle$ and $|\phi_\beta(t)\rangle$. Instead, these two evolution histories are mingled together through population transitions between them. Difference from a pure-dephasing picture is also made evident by the role of the term $\Delta H^S \equiv H^S_{\alpha\alpha} - H^S_{\beta\beta}$ as one component of $\Delta H = \Delta H^S + H^I_{\alpha\alpha} - H^I_{\beta\beta}$. For a pure-dephasing problem, i.e., if the term $i|\xi_\alpha\rangle$ is switched off, then the component ΔH^S becomes irrelevant: it is a c number for the environmental Hilbert subspace, and hence cannot cause the decay of $|\langle\phi_\beta(t)|\phi_\alpha(t)\rangle|$. By contrast, in our model the term ΔH^S in ΔH is found to be necessary for predicting the PS with intermediate system-environment coupling. That is, without this term, maximization of $\|\Delta H\|$ would incorrectly predict that eigenstates of H_I are the PS regardless of the value of ε.

Conclusion.—The concept of PS may still apply if an environment with many degrees of freedom (like a thermal bath) is replaced by an environment with very few degrees of freedom. Approximate PS are shown to exist for intermediate system-environment coupling and can continuously deform to expected limits. Such types of PS emerging from quantum dynamics alone are of importance to understanding decoherence and thermalization processes.

W. G. was supported by the Natural Science Foundation of China under Grants No. 10775123 and No. 10975123 and the National Fundamental Research Programme of China Grant No. 2007CB925200. J. G. was supported by the NUS YIA R-144-000-195-101.

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[18] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.108.070403 for discussions on conditions for PS to be identified computationally, for a distance d obtained by averaging over the Hilbert space, and for a detailed analysis in properties of ΔH, ΔK, and ΔJ in the model studied in this Letter.
[19] Certainly this convention is just one possibility for picking out a good reference basis set $\{|\tilde{\rho}_0\rangle, |\tilde{\rho}_1\rangle\}$ to which the time-evolving states $|\rho_k(t)\rangle$ will be compared. What really matters here is whether PS can be found, i.e., whether the eigenstates of the RDM can indeed stay close to some time-independent basis states.
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[23] If $\omega_z = 0$, then H_S is quantized along x and H_I is quantized along the perpendicular direction z. For this case, it was found that fluctuations around the computed intermediate PS are much larger due to close diagonal elements of the reduced density matrix.
[24] Because both pointer states were found to have appreciable populations (with populations around 0.4 and 0.6 in the case of the inset of Fig. 1), they are not at all fixed-point solutions of the dynamics.