General Second-Order Scalar-Tensor Theory and Self-Tuning

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Starting from the most general scalar-tensor theory with second-order field equations in four dimensions, we establish the unique action that will allow for the existence of a consistent self-tuning mechanism on Friedmann-Lemaître-Robertson-Walker backgrounds, and show how it can be understood as a combination of just four base Lagrangians with an intriguing geometric structure dependent on the Ricci scalar, the Einstein tensor, the double dual of the Riemann tensor, and the Gauss-Bonnet combination. Spacetime curvature can be screened from the net cosmological constant at any given moment because we allow the scalar field to break Poincaré invariance on the self-tuning vacua, thereby evading the Weinberg no-go theorem. We show how the four arbitrary functions of the scalar field combine in an elegant way opening up the possibility of obtaining nontrivial cosmological solutions.

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In a little known paper published in 1974, Horndeski presented the most general scalar-tensor theory with second-order field equations in four dimensions [1]. Given the amount of research into modified gravity over the last ten years or so (see [2] for a review), it seems appropriate to revisit Horndeski's work. Scalar-tensor models of modified gravity range from Brans-Dicke gravity [3] to the recent models [4,5] inspired by Galilean theory [6], the latter being examples of higher order scalar-tensor Lagrangians with second-order field equations. Each of these models represents a special case of Horndeski's panoptic theory.

In this Letter, we study Horndeski's theory on Friedmann-Lemaître-Robertson-Walker (FLRW) backgrounds. In particular, we ask whether or not there are subclasses of [1] giving a viable self-tuning mechanism for solving the (old) cosmological constant problem. In other words, we ask if one can completely screen the spacetime curvature from the net cosmological constant. Naively one might expect this to be impossible on account of Weinberg's no-go theorem. Given certain assumptions, this theorem states that there exists no model of self-adjusting fields that is able to screen the spacetime curvature from a nontrivial vacuum energy [7]. However, as Weinberg himself emphasizes, for model builders the power of a theorem often lies in identifying which assumptions one might wish to relax in order to evade its clutches. Here we note that Weinberg not only assumes Poincaré invariance at the level of the spacetime curvature but also at the level of the self-adjusting fields. Here we follow a route similar to [8] and allow our scalar field to break Poincaré invariance on the self-tuning vacua, while maintaining a flat spacetime geometry.

By demanding that the self-tuning mechanism continues to work through phase transitions that cause the vacuum energy to jump, we are able to impose some powerful restrictions on Horndeski's theory. Consistent with Einstein's equivalence principle (EEP), we assume that matter is only minimally coupled to the metric and then pass the model through our self-tuning filter. This reduces it to four base Lagrangians each depending on an arbitrary function of the scalar only. These are

$$\mathcal{L}_1 = \sqrt{-g}V_1(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi, \tag{1}$$

$$\mathcal{L}_2 = \sqrt{-g}V_2(\phi)P^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi, \tag{2}$$

$$\mathcal{L}_3 = \sqrt{-g}V_3(\phi)R, \tag{3}$$

$$\mathcal{L}_4 = \sqrt{-g}V_4(\phi)\hat{G}, \tag{4}$$

where $\hat{G} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss-Bonnet combination, $\varepsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita tensor, and $P^{\mu\nu\alpha\beta} = \frac{1}{4}\varepsilon^{\mu\nu\lambda\sigma}R_{\lambda\sigma\gamma\delta}\varepsilon^{\alpha\beta\gamma\delta}$ is the double dual of the Riemann tensor [9].

Our results prove that any self-tuning scalar-tensor theory (satisfying EEP) must be built from these four Lagrangians. The weakest of the four is $\mathcal{L}_4$ since this cannot give rise to self-tuning without a little help from $\mathcal{L}_1$ and/or $\mathcal{L}_2$. When this is the case, $\mathcal{L}_4$ does have a nontrivial effect on the cosmological dynamics but does not spoil self-tuning. $\mathcal{L}_3$ also has difficulties in going solo: when $V_3 =$ const, we just have general relativity and no self-tuning, whereas when $V_3 \neq$ const, we have Brans-Dicke gravity with $w = 0$, which does self-tune but is immediately ruled out by solar system constraints. Thus it is best to consider the four base Lagrangians as combining to give a single theory, as opposed to four different theories in their own right. In particular, we expect that one should always include $\mathcal{L}_1$ and/or $\mathcal{L}_2$ for the reasons given above, and because their nontrivial derivative interactions might give rise to Vainshtein effects [10] that would help in passing solar system tests. Chameleon effects [11] may also play an important role in this regard [12].

Horndeski's theory.—The most general second-order scalar-tensor theory is

$$S = S_H[g_{\mu\nu}, \phi] + S_m[g_{\mu\nu}; \Psi_n], \tag{5}$$

where the Horndeski action, $S_H = \int d^4x\sqrt{-g}\mathcal{L}_H$, is obtained from Eq. (4.21) of [1],

$$\begin{aligned}\mathcal{L}_H = {}& \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}[\kappa_1 \nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} - \frac{4}{3}\kappa_{1,\rho}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi + \kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma} - 4\kappa_{3,\rho}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi] \\ & + \delta^{\alpha\beta}_{\mu\nu}[(F+2W)R_{\alpha\beta}{}^{\mu\nu} - 4F_{,\rho}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi] - 3[2(F+2W)_{,\phi} \\ & + \rho\kappa_8]\nabla_\mu\nabla^\mu\phi + \kappa_9(\phi,\rho),\end{aligned} \tag{6}$$

with $\rho = \nabla_\mu\phi\nabla^\mu\phi$ and $\delta^{\nu_1\nu_2\ldots\nu_n}_{\mu_1\mu_2\ldots\mu_n} = n!\delta^{[\nu_1}_{\mu_1}\delta^{\nu_2}_{\mu_2}\ldots\delta^{\nu_n]}_{\mu_n}$. Greek indices are taken to run over the four spacetime dimensions, and we express partial derivatives with commas, e.g., $F_{,\rho} = \frac{\partial F}{\partial\rho}$. We have four arbitrary functions of ϕ and ρ, $\kappa_m = \kappa_m(\phi,\rho)$ for $m = 1, 3, 8, 9$, as well as $F = F(\phi,\rho)$, which is constrained so that $F_{,\rho} = \kappa_{1,\phi} - \kappa_3 - 2\rho\kappa_{3,\rho}$. Note that $W = W(\phi)$, which means that it can be absorbed into a redefinition of $F(\phi,\rho)$. The matter part of the action is given by $S_m[g_{\mu\nu}; \Psi_n]$, where we require that the matter fields Ψ_n are all minimally coupled to the metric $g_{\mu\nu}$. This follows (without further loss of generality) from assuming that there is no violation of Einstein's equivalence principle [13]. This reasoning is consistent with the original construction of Brans-Dicke gravity [3].

Here we are interested in Horndeski's theory on FLRW backgrounds, for which we have a homogeneous scalar, $\phi = \phi(t)$, and a homogeneous and isotropic metric,

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega_{(2)}\right], \tag{7}$$

with κ being a (positive or negative) constant, specifying the spatial curvature. Plugging this into (6), we obtain an effective Horndeski Lagrangian in the minisuperspace approximation

$$L_H^{\rm eff}(a,\dot a,\phi,\dot\phi) = a^3\sum_{n=0}^{3}\left(X_n - Y_n\frac{\kappa}{a^2}\right)H^n, \tag{8}$$

where $H = \dot a/a$ is the Hubble parameter, and we have,

$$\begin{aligned}&X_0 = -\tilde{Q}_{7,\phi}\dot\phi + \kappa_9,\\ &X_1 = 3(2\kappa_8\dot\phi^3 - 4F_{,\phi}\dot\phi + \tilde{Q}_{7,\dot\phi}\dot\phi - \tilde{Q}_7),\\ &X_2 = -12(F + F_{,\rho}\dot\phi^2), \quad X_3 = 8\kappa_{1,\rho}\dot\phi^3,\\ &Y_0 = \tilde{Q}_{1,\phi}\dot\phi + 12\kappa_3\dot\phi^2 - 12F, \quad Y_1 = \tilde{Q}_1 - \tilde{Q}_{1,\dot\phi}\dot\phi,\end{aligned}$$

where we have introduced $\tilde{Q}_1$ and $\tilde{Q}_7$, which are given implicitly by $\partial\tilde{Q}_1/\partial\dot\phi = -12\kappa_1$ and $\partial\tilde{Q}_7/\partial\dot\phi = 6F_{,\phi} - 3\dot\phi^2\kappa_8$. In a cosmological setting, the matter action contributes a homogeneous and isotropic fluid with energy density ρ_m and pressure p_m, satisfying the usual conservation law $\dot\rho_m + 3H(\rho_m + p_m) = 0$.

The generalized Friedmann equation follows in the standard manner by computing the Hamiltonian density for the Horndeski Lagrangian, and identifying it with the energy density, ρ_m, as follows

$$\mathcal{H}(a,\dot a,\phi,\dot\phi) = \frac{1}{a^3}\left[\dot a\frac{\partial L_H^{\rm eff}}{\partial\dot a} + \dot\phi\frac{\partial L_H^{\rm eff}}{\partial\dot\phi} - L_H^{\rm eff}\right] = -\rho_m. \tag{9}$$

Since matter only couples directly to the metric, and not the scalar, the scalar equation of motion is given by

$$\mathcal{E}(a,\dot a,\ddot a,\phi,\dot\phi,\ddot\phi) = \frac{d}{dt}\left[\frac{\partial L_H^{\rm eff}}{\partial\dot\phi}\right] - \frac{\partial L_H^{\rm eff}}{\partial\phi} = 0. \tag{10}$$

Note that this equation is always linear in both $\ddot a$ and $\ddot\phi$.

Self-tuning.—Since our ultimate goal is to identify those corners of Horndeski's theory that exhibit self-tuning, we first ask what it means to self-tune, in a relatively model independent way. Consider our cosmological background in vacuum. The matter sector is expected to contribute a constant vacuum energy density, which we identify with the cosmological constant, $\langle\rho_m\rangle_{\rm vac} = \rho_\Lambda$. In a self-tuning scenario, this should not impact on the curvature, so whatever the value of ρ_Λ, we have a Minkowski spacetime [14], with $H^2 + \kappa/a^2 = 0$. This should remain true even when the matter sector goes through a phase-transition, changing the overall value of ρ_Λ by a constant amount. This extra requirement will place the strongest constraints on our theory.

In order to proceed we shall take these transitions to be instantaneous, thereby assuming that ρ_Λ evolves in a piecewise constant fashion. Now consider a self-tuning solution, $H^2 + \kappa/a^2 = 0$, $\phi = \phi_\Lambda(t)$, satisfying the "on-shell-in-a" [15] equations of motion for the metric

$$\bar{\mathcal{H}}(\phi_\Lambda,\dot\phi_\Lambda,a) = -\rho_\Lambda \tag{11}$$

and the scalar,

$$\bar{\mathcal{E}}(\phi_\Lambda,\dot\phi_\Lambda,\ddot\phi_\Lambda,a) = \ddot\phi_\Lambda f(\phi_\Lambda,\dot\phi_\Lambda,a) + g(\phi_\Lambda,\dot\phi_\Lambda,a) = 0. \tag{12}$$

Suppose that a phase transition occurs at some arbitrary time $t = t_*$, so that $\rho_\Lambda(t_*^-) \neq \rho_\Lambda(t_*^+)$. We require that the scalar field is continuous at the transition, $\phi_\Lambda(t_*^-) = \phi_\Lambda(t_*^+)$, but allow its derivative to jump, $\dot{\phi}_\Lambda(t_*^-) \neq \dot{\phi}_\Lambda(t_*^+)$. We first consider Eq. (11). This is discontinuous on the right-hand side, so it must also be discontinuous on the left, which means that $\bar{\mathcal{H}}$ must have some nontrivial $\dot{\phi}_\Lambda$ dependence. Next consider Eq. (12). As $\dot{\phi}_\Lambda$ is discontinuous, $\ddot{\phi}_\Lambda$ must run into a delta function at $t = t_*$. This is not supported on the right-hand side of Eq. (12), and since t_* can be chosen arbitrarily, we deduce that f must vanish independently of g, so that (12) actually splits into two equations

$$f(\phi_\Lambda, \dot{\phi}_\Lambda, a) = 0, \qquad g(\phi_\Lambda, \dot{\phi}_\Lambda, a) = 0. \tag{13}$$

Focusing on the former it is clear that if f has nontrivial dependence of $\dot{\phi}_\Lambda$ then it may be discontinuous at the transition. Since it is constrained to vanish either side of the transition we deduce that $\frac{\partial f}{\partial \dot{\phi}_\Lambda} = 0$, or equivalently $f = f(\phi_\Lambda, a)$. Using this simplified form for f, we now take derivatives, staying on-shell-in-a, so that we have

$$\frac{df}{dt}(\phi_\Lambda, \dot{\phi}_\Lambda, a) = \frac{\partial f}{\partial \phi_\Lambda}\dot{\phi}_\Lambda + \frac{\partial f}{\partial a}\sqrt{-\kappa} = 0. \tag{14}$$

Again, applying similar logic we now conclude that $\frac{\partial f}{\partial \phi_\Lambda} = 0$ or equivalently $f = f(a)$. An identical line of argument implies that $g = g(a)$. What this tells us is that the on-shell-in-a scalar equation of motion (12) has lost all dependence on the scalar field ϕ_Λ and its derivatives. $\phi_\Lambda(t)$ is fixed by the gravity equation (11), and must necessarily retain some nontrivial time dependence even away from transitions in order to evade the clutches of Weinberg's theorem. More generally, in order to cope with transitions the on-shell-in-a gravity equation (11) must depend on $\dot{\phi}_\Lambda$. The scalar equation (10) should vanish identically on a flat spacetime, and must therefore have the schematic form

$$\mathcal{E} = \sum_{n\geq 1}\left[A_n + \tilde{A}_n\frac{d}{dt}\right]\Delta_n, \tag{15}$$

where $A_n = A_n(\phi, \dot{\phi}, \ddot{\phi}, a)$, $\tilde{A}_n = \tilde{A}_n(\phi, \dot{\phi}, \ddot{\phi}, a)$ are generic functions and we define

$$\Delta_n = H^n - \left(\frac{\sqrt{-\kappa}}{a}\right)^n, \tag{16}$$

which vanishes on-shell-in-a for $n > 0$. Since the scalar equation (10) ultimately forces self-tuning, it should not be trivial. Furthermore, for a remotely viable cosmology it should be dynamical in the sense that we can evolve towards $H^2 + \kappa/a^2 = 0$ rather than having it be true at all times. This imposes the condition that at least one of the $\tilde{A}_n$ should be nonvanishing. Note that the sum does not include $n = 0$, which is absolutely crucial in order to force self-tuning.

Let us now apply the self-tuning filters to Horndeski's theory. Using Eq. (10) we can infer the following form of the minisuperspace Lagrangian in a self-tuning setup,

$$L^{\text{eff}}_{\text{self-tun}} = a^3\left[c(a) + \sum_{n=1}^{3} Z_n(\phi, \dot{\phi}, a)\Delta_n\right], \tag{17}$$

where $c(a)$ and Z_n, $n = 1, 2, 3$ are arbitrary functions, the former depending only on the scale factor, the latter depending also on ϕ and $\dot{\phi}$. In order for the on-shell-in-a gravity equation (11) to retain dependence on $\dot{\phi}_\Lambda$ we demand that $\sum_{n=1}^{3} nZ_{n,\dot{\phi}}(\frac{\sqrt{-\kappa}}{a})^n \neq 0$. By requiring (8) to take the form (17) up to a total derivative, we find that we must have $\kappa < 0$, and that

$$\kappa_1 = 2V_4'(\phi)[1 + \tfrac{1}{2}\ln(|\rho|)] - \tfrac{3}{8}V_2(\phi)\rho,$$
$$\kappa_3 = V_4''(\phi)\ln(|\rho|) - \tfrac{1}{8}V_2'(\phi)\rho - \tfrac{1}{4}V_1(\phi)[1 - \ln(|\rho|)],$$
$$\kappa_8 = \tfrac{1}{2}V_1'(\phi)\ln(|\rho|), \qquad \kappa_9 = -\rho_\Lambda^{\text{bare}} - 3V_3''(\phi)\rho,$$
$$F = \tfrac{1}{2}V_3(\phi) - \tfrac{1}{4}V_1(\phi)\rho\ln(|\rho|),$$

with $V_3' \equiv 0$ allowed, if and only if there exist other nonvanishing potentials. It follows that the self-tuning version of Horndeski's theory must take the form

$$S^{\text{self-tun}} = \int d^4x[\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 - \sqrt{-g}\rho_\Lambda^{\text{bare}}] + S_m[g_{\mu\nu}; \Psi_n], \tag{18}$$

where the base Lagrangians are built from the four base Lagrangians (1) to (4). Note also the presence of the bare cosmological constant term $\rho_\Lambda^{\text{bare}}$ which can always be absorbed into a renormalization of the vacuum energy (contained within S_m). This serves as a good consistency check of our derivation. Such a term had to be allowed by the self-tuning theories—if it had not been there it would have amounted to fine tuning the vacuum energy.

Cosmology of the self-tuning theory.—We shall now briefly present the cosmological equations for the general self-tuning theory (18). To this end, we note that the minisuperspace Lagrangians for the four base Lagrangians have the desired structure given by Eq. (17), and that the Friedmann equations describing this cosmology are

$$\mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_4 = -[\rho_\Lambda + \rho_{\text{matter}}], \tag{19}$$

where we have absorbed $\rho_\Lambda^{\text{bare}}$ into the vacuum energy contribution ρ_Λ, and

$$\mathcal{H}_1 = 3V_1(\phi)\dot{\phi}^2\left(3H^2 + \frac{\kappa}{a^2}\right),$$
$$\mathcal{H}_2 = -3V_2(\phi)\dot{\phi}^3 H\left(5H^2 + 3\frac{\kappa}{a^2}\right),$$
$$\mathcal{H}_3 = -6V_3(\phi)\left[\left(H^2 + \frac{\kappa}{a^2}\right) + H\dot{\phi}\frac{V_3'}{V_3}\right],$$
$$\mathcal{H}_4 = -24V_4'(\phi)\dot{\phi}H\left(H^2 + \frac{\kappa}{a^2}\right).$$

The scalar equations of motion are $\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \mathcal{E}_4 = 0$ where

$$\mathcal{E}_1 = 6\frac{d}{dt}[a^3 V_1(\phi)\dot{\phi}\Delta_2] - 3a^3 V_1'(\phi)\dot{\phi}^2\Delta_2,$$
$$\mathcal{E}_2 = -9\frac{d}{dt}[a^3 V_2(\phi)\dot{\phi}^2 H\Delta_2] + 3a^3 V_2'(\phi)\dot{\phi}^3 H\Delta_2,$$
$$\mathcal{E}_3 = -6\frac{d}{dt}[a^3 V_3'(\phi)\Delta_1] + 6a^3 V_3''(\phi)\dot{\phi}\Delta_1 + 6a^3 V_3'(\phi)\Delta_1^2,$$
$$\mathcal{E}_4 = -24V_4'(\phi)\frac{d}{dt}\left[a^3\left(\frac{\kappa}{a^2}\Delta_1 + \frac{1}{3}\Delta_3\right)\right].$$

We see that on-shell-in-a, $H^2 = -\kappa/a^2$, $\mathcal{L}_4$'s contribution to the Friedmann equation loses its dependence on $\dot{\phi}$. This explains why $\mathcal{L}_4$ cannot self-tune by itself. We should emphasize that $\mathcal{L}_4$ does not spoil self-tuning when $\mathcal{L}_1$ and/or $\mathcal{L}_2$ are also present, even though it does alter the cosmological dynamics. Note also that if $V_3' = 0$, and all the other potentials are vanishing, then the scalar equation of motion becomes trivial and does not force self-tuning.

For a generic combination of the four base Lagrangians that includes $\mathcal{L}_1$ and/or $\mathcal{L}_2$, we have a scalar-tensor model of self-tuning. The self-tuning is forced by the scalar equation of motion, while the gravity equation links phase transitions in vacuum energy to discontinuities in the temporal derivative of the scalar field. On self-tuning vacua, the scalar field is explicitly time dependent, as it must be in order to evade Weinberg's theorem [7]. A detailed study of the self-tuning cosmology will be presented elsewhere.

Discussion.—In this Letter we have resurrected Horndeski's theory that describes the most general scalar-tensor theory with second-order field equations. We have asked which corners of this theory admit a consistent self-tuning mechanism for solving the (old) cosmological constant problem. Remarkably, this reduces the theory down to a combination of four base Lagrangians. Self-tuning is made possible by breaking Poincaré invariance in the scalar sector.

There are hints at some deep underlying structure in this theory. This merits further investigation, but for now we note that each of the four base Lagrangians can be associated with a dimensionally enhanced Euler density. This is immediately evident for $\mathcal{L}_3$ and $\mathcal{L}_4$, whereas for $\mathcal{L}_1$ and $\mathcal{L}_2$ we note that they can both be written in the form $V(\phi)\nabla_\mu\phi\nabla_\nu\phi\frac{\delta W}{\delta g^{\mu\nu}}$, with $W_1 = \int d^4x\sqrt{-g}R$ and $W_2 = -\frac{1}{4}\int d^4x\sqrt{-g}\phi\hat{G}$.

Have we really solved the cosmological constant problem? We have certainly evaded Weinberg's theorem, but there is plenty more to consider. Can our Lagrangians combine to give a gravity theory that is phenomenologically consistent, in particular, at the level of both cosmology and solar system tests? This is a work in progress, but there are reasons to be guardedly optimistic, especially when one considers the fact that $\mathcal{L}_1$ and $\mathcal{L}_2$ contain nontrivial derivative interactions that may give rise to a successful Vainshtein effect. Relevant work involving three of the four base Lagrangians was carried out in [16].

We should also ask whether or not the self-tuning property of the four base Lagrangians is spoiled by radiative corrections. Although we expect that it is spoiled by matter loops, it is interesting to note that the self-tuning is imposed by the scalar equation of motion, and the scalar does not couple directly to matter. Indeed, we have performed a rough calculation that suggests that radiative corrections on the self-tuning background can be suppressed provided the (possibly time dependent) cutoff Λ_{UV} satisfies the inequality $\sqrt{\mathcal{G}\rho_\Lambda} < \Lambda_{UV} < \rho_\Lambda^{1/4}$, where $\mathcal{G}$ is the (possibly time dependent) strength of the gravitational coupling to matter, in the linearized regime. The intriguing geometric properties of the four Lagrangians may also play a role in a more detailed analysis, but such considerations are beyond the scope of this Letter.

In any event, the ethos behind our approach is not to make any grandiose claims regarding a solution of the cosmological constant problem but to ask what can be achieved in this direction at the level of a scalar-tensor theory. Given that our starting point was the most general scalar-tensor theory, we should be in a position to make some reasonably general statements. As we have shown, Weinberg's theorem alone is not enough to rule out possible self-tuning mechanisms, so even if our self-tuning theory is ultimately ruled out by other considerations we should be able to say we have learned something about the obstacles towards solving the cosmological constant problem and how one might think about extending the scope of Weinberg's theorem.

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[12] The Vainshtein mechanism occurs when the additional light fields have strong derivative self-interactions that kick in at larger than expected distances from a heavy source, weakening their effective coupling to matter, and diluting their field lines relative to those of the graviton. The chameleon mechanism works rather differently in that the additional light fields become more massive in dense environments, shutting down the range of their interaction.

[13] For EEP to hold all matter must be minimally coupled to the same metric, $\tilde{g}_{\mu\nu}$, and this should only be a function of $g_{\mu\nu}$ and ϕ. Dependence on derivatives is not allowed since it would result in the gravitational coupling to matter being momentum dependent, leading to violations of EEP. Given $\tilde{g}_{\mu\nu} = \tilde{g}_{\mu\nu}(g_{\alpha\beta}, \phi)$, we simply compute $g_{\alpha\beta} = g_{\alpha\beta}(\tilde{g}_{\mu\nu}, \phi)$, and substitute back into the action (5), before dropping the tildes. Since this procedure will not generate any additional derivatives in the equations of motion, it simply serves to redefine the Horndeski potentials, $\kappa_i(\phi, \rho)$.

[14] Different values of κ represent different slicings of Minkowksi space: $\kappa = 0$ corresponds to Minkowski coordinates; $\kappa < 0$ corresponds to Milne coordinates; $\kappa > 0$ is not permitted since one cannot foliate Minkowski space with spherical spatial sections.

[15] By "on-shell-in-a" we mean that we have set $H^2 = -\kappa/a^2$. We shall see later that this is indeed a consistent solution to our system.

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