Charge Transport in Weyl Semimetals

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We study transport in Weyl semimetals with N isotropic Weyl nodes in the presence of Coulomb interactions or disorder at temperature T. In the interacting clean limit, we determine the conductivity $\sigma(\omega, T)$ by solving a quantum Boltzmann equation within a "leading log" approximation and find it to be proportional to T, up to logarithmic factors arising from the flow of couplings. In the noninteracting disordered case, we compute the Kubo conductivity and show that it behaves differently for $\omega \ll T$ and $\omega \gg T$: in the former regime we recover a previous result, of a finite dc conductivity and a Drude width vanishing as NT^2; in the latter, we find that $\sigma(\omega, T)$ vanishes linearly with ω with a leading term as $T \to 0$ equal to the clean, free-fermion result: $\sigma_0^{(N)}(\omega, T = 0) = N\frac{e^2}{12h}\frac{|\omega|}{v_F}$. We compare our results to transport data on $Y_2Ir_2O_7$ and comment on the possible relevance to recent experiments on $Eu_2Ir_2O_7$.

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There has been a surge of recent activity studying Dirac excitations in two-dimensional media, most famously graphene [1]. A natural question is whether there are analogs in three dimensions, with a vanishing density of states at the chemical potential and linearly dispersing excitations. It has long been known that touchings between a pair of nondegenerate bands are stable in three dimensions, and typically have linear dispersion. Near these, electronic excitations are described by an analog of the Weyl equation of particle physics, which describes two-component chiral fermions [2–4]. Hence these states have been dubbed *Weyl semimetals* (WSMs) [5].

To remove a band touching (or Weyl node) one necessarily must connect to another node. This is in contrast with two dimensions: graphene's nodes can be gapped by different intranode perturbations that break inversion ($\mathcal{I}$) or time reversal ($\mathcal{T}$) symmetry. The enhanced protection in three dimensions is due to a topological property of the nodes—they are sources (monopoles) of Chern flux in the Brillouin zone (BZ). This momentum space topology is associated with several physical phenomena. In particular, it was recently realized [5] that unusual surface states will result as a consequence of the band topology. These take the form of Fermi arcs that connect the projections of the nodes onto the surface BZ. Such topological properties are sharply defined as long as one can distinguish band touching associated with opposite Chern flux. The presence of translation symmetry, and hence conserved crystal momenta, is sufficient to protect these defining properties since the nodes are separated in the BZ. In principle one needs perfect crystalline order to define these phases; in practice, smooth disorder that only weakly mixes nodes is expected to have little effect. Other manifestations of the band topology include an anomalous Hall effect [6,7] that is tied to the momentum space displacement between nodes, and magnetoresistance arising from the Adler-Bell-Jackiw anomaly of Weyl fermions [4,8].

Physical realizations of WSMs require nondegenerate bands to touch; therefore, spin degeneracy must be lifted (by either spin-orbit interactions or magnetic order), and either $\mathcal{T}$ or $\mathcal{I}$ must be broken: otherwise, all bands would be doubly degenerate. We further require that the Fermi "surface" consists exactly of the Weyl nodes. In $\mathcal{T}$-breaking realizations where $\mathcal{I}$ is unbroken, a simple "parity criterion" applied to eight $\mathcal{T}$-invariant momenta in the BZ can be used to diagnose the existence of Weyl nodes [9]. In [5], certain pyrochlore iridates $A_2Ir_2O_7$ ($A = Y$ or Eu) were proposed to be magnetically ordered WSMs, with $N = 24$ Weyl points, all at the Fermi energy; [10] reached similar conclusions but with $N = 8$. Alternate proposals include $HgCr_2Se_4$ in the ferromagnetic state [11] and topological insulator-ferromagnet heterostructures [7], with $N = 2$, the minimum allowed.

Motivated by the availability of transport data on the iridates [12,13], we study the electrical conductivity of an idealized model of a WSM, with an even number N of isotropic Weyl nodes characterized by the same dispersion, with $N/2$ nodes of each chirality as required by topology [5,14]. The leading behavior of the conductivity σ provides insight into the dominant scattering mechanism in the system, as in three dimensions, σ has dimensions of inverse length in units of e^2/h and the appropriate length scale is set by the quasiparticle mean free path. In the absence of impurities and interactions we expect the free-fermion result, $\sigma_0^{(N)}(\omega) = N\frac{e^2}{12h}\frac{|\omega|}{v_F}$; we demonstrate how this is modified in two cases.

(i) In clean undoped systems with Coulomb interactions, current is carried equally by counterpropagating electrons and holes and can be relaxed via interactions alone. Solving a quantum Boltzmann equation (QBE) we find a finite conductivity proportional to the temperature T (up to logarithmic factors), as expected of a quantum critical system [15], where T is the sole energy scale,

$$\sigma_{\rm dc}^{(N)}(T) = \frac{e^2}{h}\frac{k_B T}{\hbar v_F(T)}\frac{1.8}{\alpha_T^2 \ln \alpha_T^{-1}}. \quad (1)$$

Here $v_F(T) = v_F(\alpha_0/\alpha_T)^{2/N+2}$ and $\alpha_T = \alpha_0[1 + \frac{(N+2)\alpha_0}{3\pi}\ln(\frac{\hbar\Lambda}{k_B T})]^{-1}$ are the Fermi velocity and fine structure constant renormalized to the scale of the temperature T, Λ is a momentum cutoff set by the separation between the Weyl nodes, and v_F and $\alpha_0 = e^2/\varepsilon\hbar v_F$ are the corresponding ''bare'' values at the microscopic scale [16].

(ii) In the presence of impurities, power counting shows that white-noise disorder is an irrelevant perturbation, and a naive expectation is that the clean result $\sigma_0^{(N)}$ is reproduced. However, the result is more interesting: by evaluating a standard Kubo formula, we find that the finite-frequency conductivity exhibits different behaviors for $\omega \ll T$ and $\omega \gg T$: in the former regime we find in agreement with [7] a finite Drude-like response with a peak width vanishing as NT^2; in the latter, we recover $\sigma_0^{(N)}$ as the leading behavior, which is universal and independent of disorder. We also determine the manner in which the conductivity interpolates between these limits.

Previous studies of 3D Dirac points have assumed Lorentz invariance [17] or worked at a topological phase transition between insulators [18]. Although our work differs from both of these situations–instantaneous Coulomb interactions break Lorentz invariance, and we study a stable phase–there are sufficient parallels that a similar ''leading log'' approximation suffices to solve the QBE. Coulomb interactions also lead to a finite dc conductivity in clean graphene–the 2D analog of a WSM–but the leading log approximation fails here and more analysis is needed [15,19].

Model.—In a WSM, the electronic dispersion about a Weyl node is generically of the form $H_{\rm Weyl} = \boldsymbol{u}\cdot\boldsymbol{k}1 + \sum_{a=1}^{3} \boldsymbol{v}^a\cdot\boldsymbol{k}\sigma^a$, where σ^a are the Pauli matrices. The velocities satisfy $\boldsymbol{v}^1\cdot(\boldsymbol{v}^2\times\boldsymbol{v}^3) \neq 0$, and the Chern number ± 1 (''chirality'') associated with the Weyl node is $\mathrm{sgn}(\boldsymbol{v}^1\cdot(\boldsymbol{v}^2\times\boldsymbol{v}^3))$. For simplicity, we shall drop the term proportional to identity and assume isotropic dispersion; relaxing this assumption should only produce small corrections. The Hamiltonian for a system of N identically dispersing Weyl nodes (''flavors'') with Coulomb interactions and disorder may then be written as $H = H_0 + H_I + H_D$, with (repeated indices summed)

$$\begin{aligned} H_0 &= \sum_a H_a = \sum_a \int_{\boldsymbol{k}} \psi^\dagger_{\boldsymbol{k},a}(\chi_a v_F \boldsymbol{k}\cdot\boldsymbol{\sigma})\psi_{\boldsymbol{k},a}, \\ H_I &= \frac{1}{2}\int_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{q}} V(\boldsymbol{q})\psi^\dagger_{\boldsymbol{k}_2-\boldsymbol{q},a\sigma}\psi_{\boldsymbol{k}_2,a\sigma}\psi^\dagger_{\boldsymbol{k}_2+\boldsymbol{q},b\sigma'}\psi_{\boldsymbol{k}_1 b\sigma'}, \quad (2) \\ H_D &= \int_r \sum_a \psi^\dagger_a(\boldsymbol{r})U(\boldsymbol{r})\psi_a(\boldsymbol{r}), \end{aligned}$$

where $\psi_{\boldsymbol{k},a}$ is a two-component spinor in the (pseudo)spin indices σ, σ', $a, b = 1, \ldots, N$ index the flavors, v_F is the Fermi velocity, which we set to unity, $\chi_a = \pm 1$ is the chirality of the ath Weyl node, $V(\boldsymbol{q}) = \frac{4\pi e^2}{\varepsilon q^2}$ describes the Coulomb interaction in a material with dielectric constant ε, $U(\boldsymbol{r})$ is a random potential with white-noise correlations $\langle\langle U(\boldsymbol{r})U(\boldsymbol{r}')\rangle\rangle = n_i v_0^2\delta(\boldsymbol{r}-\boldsymbol{r}')$, where v_0 characterizes the strength of the individual impurities and n_i their concentration, $\int_{\boldsymbol{k}} \equiv \int\frac{d^3k}{(2\pi)^3}$, and we have written H_0 assuming that the Fermi level is at the Weyl nodes, which is the only case studied in this Letter. Here and below we set $\hbar = k_B = |e| = 1$ and define $\beta = 1/T$.

Conductivity with interactions.—Critical systems–such as graphene and the WSM at neutrality–are exceptions to the rule that disorder is essential for a finite conductivity, since they support current-carrying states in which particles and holes transport charge with no net momentum by moving exactly opposite to each other. In contrast to conventional finite-momentum charge transport, such deviations from equilibrium can relax in the presence of interactions alone, leading to a finite conductivity.

We study transport in an interacting WSM by solving a QBE for the thermal distribution function of quasiparticle states. In doing so, it is convenient to first calculate the current from a single node (but interacting with all the nodes) before making the leap to the current carried by all N nodes. We focus on a node with flavor a, which we take to have $\chi_a = 1$. The single-quasiparticle states are obtained by diagonalizing H_a: $\psi_{\boldsymbol{k},a} \to W^\dagger\psi_{\boldsymbol{k},a} \equiv \gamma_{\boldsymbol{k},a}$, $H_a \to WH_aW^{-1} = \int_{\boldsymbol{k}} \lambda v_F k \gamma^\dagger_{\boldsymbol{k}\lambda a}\gamma_{\boldsymbol{k}\lambda a}$, and are labeled by their helicity λ (the eigenvalue of $\sigma\cdot\hat{\boldsymbol{p}}$). From now on we will suppress the index a. In general, operators corresponding to various transport properties are *not* diagonal in the helicity; diagonal contributions correspond to motion of particles and holes in the applied field and may be characterized by appropriate distribution functions $f_\lambda(\boldsymbol{k}, t) = \langle\gamma^\dagger_{\boldsymbol{k}\lambda}\gamma_{\boldsymbol{k}\lambda}\rangle$, while the off-diagonal terms $(\gamma^\dagger_{\boldsymbol{k}\pm}\gamma_{\boldsymbol{k}\mp})$ describe the motion of particle-hole pairs. For $\omega \ll T$, contributions of the latter to transport are expected to be suppressed, essentially by Pauli exclusion[16], and we drop them forthwith. In this approximation, it is therefore sufficient to solve the QBE for quasiparticle distribution functions $f_\lambda(\boldsymbol{k}, t)$, subject to an external force $\boldsymbol{F}$,

$$\left(\frac{\partial}{\partial t} + \boldsymbol{F}\cdot\nabla_{\boldsymbol{k}}\right) f_\lambda(\boldsymbol{k}, t) = -w[f_\lambda(\boldsymbol{k}, t)], \quad (3)$$

where w is the rate at which quasiparticles scatter out of the state $(\lambda, \boldsymbol{k})$ at time t and captures the effect of interactions. Our goal will be to determine the steady-state form of the nonequilibrium quasiparticle distribution function. We will restrict ourselves to linear response in $\boldsymbol{F}$; i.e., we assume that the deviation of f_λ from equilibrium is small. The result is a linear functional equation which may be recast as a variational problem. We solve the latter approximately by identifying leading log contributions, which dominate the relaxation of the observable under consideration. As mentioned, we assume that the constants that enter the solution

of (3) are renormalized to the energy scale of interest, namely, T.

Neglecting particle-hole pair contributions, the current is $\boldsymbol{J}(t) = -\int_{\boldsymbol{k}} \langle \psi^\dagger_{\boldsymbol{k}} \boldsymbol{\sigma} \psi_{\boldsymbol{k}} \rangle_t = -\sum_{\lambda=\pm} \int_{\boldsymbol{k}} \lambda \hat{\boldsymbol{k}} f_\lambda(\boldsymbol{k}, t)$. For a weak applied electric field $\boldsymbol{E}(t)$, the deviation of $f_\lambda(\boldsymbol{k}, \omega) = \int dt f(\boldsymbol{k}, t) e^{i\omega t}$ from the equilibrium distribution function $f^0_\lambda(k) = (1 + e^{\lambda\beta k})^{-1}$, and hence the conductivity $\sigma(\omega, T)$, can be parametrized [20] in terms of a dimensionless, isotropic function $g(k, \omega)$:

$$f_\lambda(\boldsymbol{k}, \omega) = 2\pi\delta(\omega) f^0_\lambda(k) + \lambda\beta^2 \hat{\boldsymbol{k}} \cdot \boldsymbol{E}(\omega) \times [f^0_\lambda(k) f^0_{-\lambda}(k)] g(k, \omega), \quad (4)$$

$$\sigma(\omega, T) = 2\beta^2 \int_{\boldsymbol{k}} \left[\frac{k_x^2}{k^2} [f^0_+(k) f^0_-(k)] g(k, \omega) \right]. \quad (5)$$

It therefore remains only to determine the function $g(k, \omega)$, to which we now turn. Inserting (4) into (3), and working to linear order in $\boldsymbol{E}$, we find $-(i\beta\omega g(k, \omega) + 1) f^0_+(k) f^0_-(k) \hat{\mathbf{k}} = \hat{\mathcal{C}}[g(k, \omega)\hat{\boldsymbol{k}}]$, where $\hat{\mathcal{C}}$ is the collision operator, a linear functional of $g(k, \omega)\hat{\boldsymbol{k}}$ given in [16]. This is equivalent to the variational problem of extremizing the quadratic functional [15,17–19]

$$\mathcal{Q}[g] \equiv \int_{\boldsymbol{k}} \left[\frac{1}{2} g(k, \omega) \hat{\boldsymbol{k}} \cdot (\hat{\mathcal{C}}[g(k, \omega)\hat{\boldsymbol{k}}]) + f^0_+(k) f^0_-(k) \left(i\omega \frac{g^2(\omega, k)}{2} + g(\omega, k) \right) \right], \quad (6)$$

in which we have rescaled all momenta and frequencies by T. A key simplification, known as the leading log approximation (LLA) stems from the power-law nature of the Coulomb interaction: as a result of this, logarithmically divergent small-momentum scattering dominates $\hat{\mathcal{C}}$. We may write $\hat{\mathcal{C}} = \hat{\mathcal{C}}_0 + \hat{\mathcal{C}}_1$, which when thought of as linear functionals of $g\hat{\boldsymbol{k}}$ have eigenvalues of $\mathcal{O}(\alpha^2 \log\alpha)$ and $\mathcal{O}(\alpha^2)$, respectively. In the LLA we approximately optimize $\mathcal{Q}$ by choosing $g\hat{\boldsymbol{k}}$ in the space spanned by eigenstates of $\hat{\mathcal{C}}_0$; as shown in [16] the choice $g = k\xi(\omega)$ yields

$$\mathcal{Q}[k\xi(\omega)] \approx \frac{4}{\varepsilon^2} \left[i\omega [\xi(\omega)]^2 \frac{7\pi^4}{30} + 9\xi(\omega)\zeta(3) \right] - \frac{4\pi^3}{9\varepsilon^2} [\xi(\omega)]^2 N\alpha^2 \ln\alpha^{-1}, \quad (7)$$

optimized by $\xi(\omega) = \frac{81\zeta(3)}{2\pi^3} (-i\omega \frac{21\pi}{10} + N\alpha^2 \ln\alpha^{-1})^{-1}$. Finally, we observe that the flipped chirality of half the nodes is unimportant as they all give the same contribution to σ; thus, using the result for $\xi(\omega)$ in (5) and multiplying by N we find the result for N nodes [21],

$$\sigma^{(N)}(\omega, T) = N \frac{e^2}{h} \frac{1.8}{-i\frac{\hbar\omega}{k_B T} 6.6 + N\alpha^2 \ln\alpha^{-1}} \left(\frac{k_B T}{\hbar v_F} \right). \quad (8)$$

Note that in the case of graphene, the LLA fails because the log divergence stems from a phase space effect due to enhanced scattering of collinear particles, which cannot relax a current. Thus, the eigenstates of $\hat{\mathcal{C}}_0$ do not contribute to the relaxation, which therefore occurs only via subleading, noncollinear scattering, i.e., $\hat{\mathcal{C}}_1$ [15,19]. In 3D, $\hat{\mathcal{C}}_0$ includes noncollinear and thus current-relaxing processes, so that the LLA analysis is sufficient [17,18].

In the dc limit, (8) reduces to (1), which we may rationalize using the Einstein relation, $\sigma_{\rm dc} = e^2 D \frac{\partial n}{\partial \mu}$, where $D = v_F^2 \tau$ is the diffusion constant, which depends on the scattering time τ and $\frac{\partial n}{\partial \mu} \sim NT^2/v_F^3$ is the density of states, at energy $\epsilon = T$, up to numerical factors. We may estimate τ from three observations: the scattering rate τ^{-1} is proportional to (i) N, the number of flavors contributing to the scattering, (ii) α^2, which is essentially the cross section for scattering, and (iii) T, which is the single energy scale in the dc limit. Thus, $\tau^{-1} \sim N\alpha^2 T$, which gives (1), modulo logarithms. This provides an estimate of the frequencies over which transport is collision dominated and the preceding calculation is valid: in order for collisions to produce relaxation, we require $\omega \ll \tau^{-1}$, which occurs for $\hbar\omega/k_B T \ll N\alpha^2$.

Conductivity with impurities.—We turn now to the conductivity of the noninteracting, disordered system. We restrict to the case of scattering off random point impurities, characterized by $v_i(\boldsymbol{r}) \sim v_0^2 \delta(\boldsymbol{r})$ and the locations of which we shall assume are uncorrelated, $\langle\langle \rho_i(\boldsymbol{r}) \rho_i(\boldsymbol{r}') \rangle\rangle \propto \delta(\boldsymbol{r} - \boldsymbol{r}')$. With these assumptions, we are led to H_D in (2) with $U(\boldsymbol{r}) \equiv \int d\boldsymbol{r}' v_i(\boldsymbol{r} - \boldsymbol{r}') \rho_i(\boldsymbol{r}')$. As before, we first compute the conductivity for a single node. Assuming that the impurities are sufficiently dilute that the Born approximation is valid, the quasiparticle lifetime due to impurity scattering from a single node is given by $\frac{1}{\tau(\omega)} = -2\,\mathrm{Im}\Sigma^{\rm ret}(\omega, \boldsymbol{k})$, where $\Sigma^{\rm ret}_\lambda(\omega, \boldsymbol{k}) = n_i v_0^2 \int \frac{d^3k'}{(2\pi)^3} \mathcal{F}_{\lambda\lambda'}(\boldsymbol{k}, \boldsymbol{k}') G^{(0)}_{\lambda'}(\omega, \boldsymbol{k}')$ is the retarded self-energy, $G^{(0)}_\lambda(\omega, \boldsymbol{k}) = (\omega + i\delta - \lambda v_F k)^{-1}$ is Green's function for a noninteracting Weyl fermion with helicity λ, and the form factor from the overlap of helicity eigenspinors, $\mathcal{F}_{\lambda\lambda'}(\boldsymbol{k}, \boldsymbol{k}') = \frac{1}{2}(1 + \lambda\lambda' \cos\theta_{\boldsymbol{k}\boldsymbol{k}'})$ to leading order. We find [16] $\frac{1}{\tau(\omega)} \equiv 2\pi\gamma g(\omega)$, where $g(\omega) = \frac{\omega^2}{2\pi^2 v_F^3}$ is the density of states and $\gamma = \frac{1}{2} n_i v_0^2$ characterizes the strength of the impurity potential.

To evaluate the conductivity we use the Kubo formula,

$$\sigma(\omega, T) = -\frac{1}{\omega} \lim_{q \to 0} \mathrm{Im}\Pi^{\rm ret}_{xx}(\omega, |\boldsymbol{q}|), \quad (9)$$

where $\Pi^{\rm ret}_{\mu\nu}(\omega, \boldsymbol{q})$ is the retarded response function which for a system of linear dimension L is defined to be

$$\Pi^{\rm ret}_{\mu\nu}(\omega, \boldsymbol{q}) = -\frac{i}{L^3} \int_0^\infty dt e^{i\omega t} \langle [J_\mu(-\boldsymbol{q}, t), J_\nu(\boldsymbol{q}, 0)] \rangle, \quad (10)$$

with $x_\mu = (t, \boldsymbol{r})$, $p_\mu = (\omega, \boldsymbol{p})$, and $J_\mu = (-\psi^\dagger \psi, \boldsymbol{J})$. From gauge invariance $\Pi^{\rm ret}_{\mu\nu}(\omega, \boldsymbol{q}) = \Pi^{\rm ret}(\omega, |\boldsymbol{q}|)(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})$, so

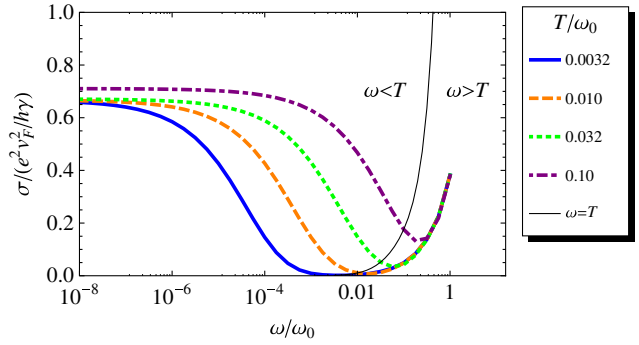

FIG. 1 (color online). Frequency-dependent conductivity of a single Weyl node with disorder (constants defined in the text.)

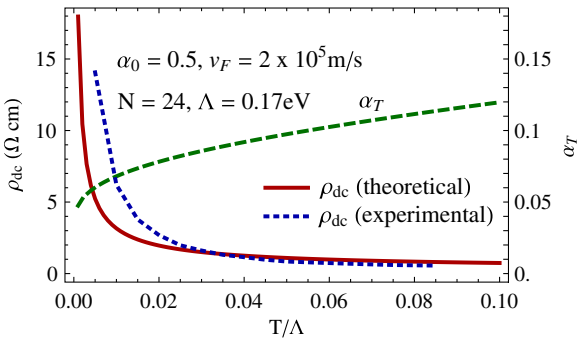

FIG. 2 (color online). $\rho_{\rm dc} = \sigma_{\rm dc}^{-1}$ and α_T (defined in the text) for the inset parameter values compared to experimental data from [12].

that (suppressing $\boldsymbol{q} = 0$), $\sigma(\omega, T) = -\frac{1}{\omega}\,\mathrm{Im}\Pi^{\rm ret}(\omega) = -\frac{1}{3\omega}\,\mathrm{Im}\Pi^{\rm ret}_{\mu\mu}(\omega)$. Some algebra yields [16]

$$\sigma(\omega, T) = \frac{4}{3}e^2 v_F^2 \int \frac{d\epsilon}{2\pi}\,\frac{[f_T(\epsilon) - f_T(\epsilon+\omega)]}{\omega} \times \sum_{\lambda,\lambda'} \int \frac{d^3k}{(2\pi)^3}\,\mathrm{Im}G^{\rm ret}_{\lambda}(\epsilon+\omega, k)\,\mathrm{Im}G^{\rm ret}_{\lambda'}(\epsilon, k), \tag{11}$$

where $f_T(\omega) = [e^{\omega/T} + 1]^{-1}$ is the Fermi-Dirac function and we have used the retarded helicity-basis Green's function dressed with disorder lines, $G^{\rm ret}_{\lambda}(\omega, \boldsymbol{k}) = [\omega - \lambda v_F k + i/2\tau(\omega)]^{-1}$. After a tedious calculation, we may write $\sigma(\omega, T) = \frac{e^2 v_F^2}{h\gamma}\mathcal{J}(\hat{\omega}, \hat{T})$, where $\hat{T} = T/\omega_0$, $\hat{\omega} = \omega/\omega_0$, so that $f_T(\omega) = f_{\hat{T}}(\hat{\omega})$, $\omega_0 = 2\pi v_F^3/\gamma$ is a characteristic scale set by the disorder strength, and $\mathcal{J}(\hat{\omega}, \hat{T}) = \frac{4}{3}\int \frac{d\hat{\epsilon}}{2\pi}\,\frac{[f_{\hat{T}}(\hat{\epsilon}) - f_{\hat{T}}(\hat{\epsilon}+\hat{\omega})]}{\hat{\omega}}\,\mathcal{I}(\hat{\epsilon}+\hat{\omega}, \hat{\epsilon})$, with $\mathcal{I}$ a complicated rational function [16].

In our model disorder can scatter between nodes, so $1/\tau(\omega)$ acquires a factor of N when $N > 1$; in common with the interacting case, σ also has an overall prefactor of N. From these it is easy to show that for N nodes,

$$\sigma^{(N)}(\omega, T) = \frac{e^2 v_F^2}{h\gamma}\,\mathcal{J}\left(N\frac{\omega}{\omega_0}, N\frac{T}{\omega_0}\right), \tag{12}$$

which is identical to the $N = 1$ result (Fig. 1) upon rescaling $\omega_0 \to \omega_0/N$.

While in general we integrate (12) numerically, in certain limits an analytic treatment is feasible. For $\omega \ll T$, $f_{\hat{T}}(\hat{\epsilon}) - f_{\hat{T}}(\hat{\epsilon}+\hat{\omega}) \approx -\hat{\omega} f'(\hat{\epsilon})$. Expanding $\mathcal{I}$ in powers of $\hat{\omega}$ and resumming only terms dominant as $\hat{\epsilon} \to 0$, we recover the result of Burkov and Balents [Eq. (15) of [7]]: namely, a Drude-like response with a width vanishing as NT^2, and a finite dc limit of $\frac{2e^2 v_F^2}{3h\gamma}$.

In the opposite limit, $T \to 0$, at finite ω we may replace the Fermi functions by step functions, which yields

$$\sigma^{(N)}(\omega) \approx N\frac{e^2}{12h}\,\frac{\omega}{v_F}\left[1 - \frac{16N\gamma\omega}{15\pi^2 v_F^3} + \mathcal{O}\left(\frac{N^2\omega^2}{\omega_0^2}\right)\right]. \tag{13}$$

The leading term is universal and independent of disorder, and is simply $\sigma_0^{(N)}$. Both regimes are captured in Fig. 1, which shows $\sigma(\omega, T)$ for $\omega \lesssim \omega_0$, beyond which the Born approximation is insufficient.

Experiments.—In [12] the dc resistivity of polycrystalline $Y_2Ir_2O_7$ was found to vary with temperature as $\rho_{\rm dc}T \approx 130\ \Omega\cdot\mathrm{cm}\cdot\mathrm{K}$ over $10\ \mathrm{K} \lesssim T \lesssim 170\ \mathrm{K}$, which is reminiscent of our result with interactions (1). Accordingly, we compare this data with a model of a clean WSM with $N = 24$ [5], as shown in Fig. 2. We find rather good agreement with experimental data for physically reasonable parameter choices, shown inset. Very recently, transport in single crystals of another pyrochlore iridate, $Eu_2Ir_2O_7$, has been studied [13] under pressure for $2\ \mathrm{K} \lesssim T \lesssim 300\ \mathrm{K}$; at low pressures ~2.06–6.06 GPa, $\rho_{\rm dc}(T)$ resembles Fig. 2, consistent with WSM behavior.

Conclusions.—The conductivity of WSMs thus exhibits a rich variety of behavior on varying frequency and temperature, in both the interacting clean and noninteracting disordered limits, as shown in Figs. 1 and 2. In particular, its nontrivial dependence on N is sensitive to the strength of the interactions; with just disorder, we find a striking difference between the $\omega \ll T$ and $\omega \gg T$ regimes, with the $T \to 0$ ac response dominated by a universal, disorder-independent contribution. While the limited existing dc conductivity data on the candidate iridates broadly agrees with our theory in the clean limit, we caution that more dc and ac conductivity measurements on single crystals with controlled disorder are required to make a rigorous comparison. Theoretically, the interplay of disorder and interactions, and corrections to the isotropic node approximation, still need to be considered. In particular, it would be striking if the distinct behavior of the disordered system across the different frequency regimes survives the inclusion of interactions. A simultaneous treatment of disorder and interactions is, as always, challenging and is left open for future work.

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[20] We have used the fact that at particle-hole symmetry the deviation from equilibrium is proportional to the sign of λ, and $f^0_\lambda(k) = 1 - f^0_{-\lambda}(k)$.
[21] As mentioned in the introduction, α and v_F take on their renormalized values at temperature T.