Resolving the Ghost Problem in Nonlinear Massive Gravity

S. F. Hassan and Rachel A. Rosen

Department of Physics & The Oskar Klein Centre, Stockholm University, AlbaNova University Centre, SE-106 91 Stockholm, Sweden

(Received 25 July 2011; published 24 January 2012)

We analyze the ghost issue in the recently proposed models of nonlinear massive gravity in the Arnowitt-Deser-Misner formalism. We show that, in the entire two-parameter family of actions, the Hamiltonian constraint is maintained at the complete nonlinear level and we argue for the existence of a nontrivial secondary constraint. This implies the absence of the pathological Boulware-Deser ghost to all orders. To our knowledge, this is the first demonstration of the existence of a consistent theory of massive gravity at the complete nonlinear level, in four dimensions.

DOI: 10.1103/PhysRevLett.108.041101 PACS numbers: 04.50.Kd

Introduction and summary.—The search for a consistent theory of massive gravity is motivated by both theoretical and observational considerations. Since the construction of a linear theory of massive gravity by Fierz and Pauli in 1939 [1,2], a proof of the existence of a consistent nonlinear generalization has remained elusive, making it a theoretically intriguing problem. On the observational side, the recent discovery of dark energy and the associated cosmological constant problem has prompted investigations into long distance modifications of general relativity. An obvious such modification is massive gravity.

Theories of massive gravity generically suffer from a ghost instability. The origin of this problem can be understood as follows. In general relativity the four constraint equations of the theory along with four general coordinate transformations remove four of the six propagating modes of the metric, where a propagating mode refers to a pair of conjugate variables. The total number of propagating modes is thereby reduced to the physical two modes of the massless graviton. In contrast, in massive gravity the four constraint equations generically remove the four nonpropagating components of the metric while the general covariance is broken. Thus the theory will contain six propagating modes of which only five correspond to the physical polarizations of the massive graviton. The remaining mode is a ghost.

The question then is whether it is possible to construct a theory of massive gravity in which one of the constraint equations and an associated secondary constraint eliminate the propagating ghost mode instead. The linear Fierz-Pauli theory succeeds in eliminating the ghost in this way. But Boulware and Deser [3] showed that the ghost generically reappears at the nonlinear level. More recently, progress was made in [4] by observing that the ghost is related to the longitudinal mode of the Goldstone bosons associated with the broken general covariance. This greatly simplifies the analysis of the ghost problem in the so-called decoupling limit which isolates nonlinear effects in the ghost sector. Based on this approach a procedure was outlined in [4,5] to avoid the ghost order-by-order by tuning the coefficients in an expansion of the mass term in powers of the metric perturbation and of the Goldstone mode. In 2010 de Rham and Gabadadze [6] successfully obtained such an expansion which is ghost-free in the decoupling limit. Later in [7], these perturbative actions were resummed into fully nonlinear actions resulting in a two-parameter family of theories. This was the first successful construction of potentially ghost-free nonlinear actions of massive gravity. Also in [7], one of these resummed actions was analyzed in the Arnowitt-Deser-Misner (ADM) formalism [8] and it was argued to be ghost-free to fourth order in metric perturbations around flat space. In [9] it is claimed that the ghost still appears at the fourth order. (For a review of recent developments in massive gravity; see [10].) The present work addresses the ghost issue at the nonperturbative level.

The systematics and generality of these potentially ghost-free actions are studied in [11]. In particular, they are presented as a two-parameter generalization of a minimal extension of the Fierz-Pauli theory. In this work we show that the entire two-parameter family of actions is ghost-free at the full nonlinear level. Starting with the minimal theory in the ADM formalism, we show that the lapse N is indeed a Lagrange multiplier leading to a Hamiltonian constraint on the propagating modes. We also show that the same analysis extends to the full two-parameter generalization of the minimal theory. We then argue that this Hamiltonian constraint gives rise to a secondary constraint. These are enough to eliminate a single propagating mode, ensuring that the theory contains only five propagating degrees of freedom appropriate for the spin-2 massive graviton. Thus the Boulware-Deser ghost is eliminated.

Nonlinear massive gravity.—In the Fierz-Pauli theory, linearized general relativity in flat space is extended by the addition of a mass term for the metric fluctuations $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$,

$$\frac{m^2}{4}(h_{\mu\nu}h^{\mu\nu} - h^\mu_\mu h^\nu_\nu). \qquad (1)$$

To construct nonlinear generalizations of the Fierz-Pauli mass term, an additional nondynamical metric $f_{\mu\nu}$ is invariably required. In the recently developed potentially ghost-free theories, the basic building block is a matrix of the form $\sqrt{g^{-1}f}$ [7,11], where the square root of the matrix is defined such that $\sqrt{g^{-1}f}\sqrt{g^{-1}f} = g^{\mu\lambda}f_{\lambda\nu}$. In particular, the minimal extension of the Fierz-Pauli action with zero cosmological constant is given by [11],

$$S = M_p^2 \int d^4x\sqrt{-g}[R - 2m^2(\mathrm{tr}\sqrt{g^{-1}f} - 3)]. \quad (2)$$

Our ghost analysis is based on this action for the case of a flat $f_{\mu\nu}$ so that in the physical gauge $f_{\mu\nu} = \eta_{\mu\nu}$. Recent studies of massive gravity have primarily focused on this case [6,7].

However, with this choice of $f_{\mu\nu}$, the minimal action does not have a Vainshtein mechanism [12] and thus exhibits the van Dam–Veltman–Zakharov discontinuity [13,14], as shown in [15]. To be compatible with observations one must consider theories with additional higher order interactions that could induce a Vainshtein mechanism (see, e.g., [15–17]). We will show that our analysis naturally extends to the entire family of such actions without any modification.

The most general nonlinear massive gravity theories that are potentially ghost-free are given by a two-parameter family of actions. Defining a matrix $\mathbb{K}$ so that $\sqrt{g^{-1}f} = \mathbb{1} + \mathbb{K}$, these actions can be written as [7],

$$S = M_p^2 \int d^4x\sqrt{-g}\left[R + 2m^2 \sum_{n=2}^{4} \alpha_n e_n(\mathbb{K})\right], \quad (3)$$

with $\alpha_2 = 1$ and where the e_k are defined in (6) below.

For our purposes it will be easier to work with an equivalent formulation of (3) [11],

$$S = M_p^2 \int d^4x\sqrt{-g}\left[R + 2m^2 \sum_{n=0}^{3} \beta_n e_n(\sqrt{g^{-1}f})\right], \quad (4)$$

where the β_n are given in terms of the α_n of (3) as,

$$\begin{aligned} \beta_0 &= 6 - 4\alpha_3 + \alpha_4, & \beta_1 &= -3 + 3\alpha_3 - \alpha_4, \\ \beta_2 &= 1 - 2\alpha_3 + \alpha_4, & \beta_3 &= \alpha_3 - \alpha_4. \end{aligned} \quad (5)$$

The $e_k(\mathbb{X})$ are elementary symmetric polynomials of the eigenvalues of $\mathbb{X}$. For a generic 4×4 matrix they are given by,

$$\begin{aligned} &e_0(\mathbb{X}) = 1, \quad e_1(\mathbb{X}) = [\mathbb{X}], \quad e_2(\mathbb{X}) = \tfrac{1}{2}([\mathbb{X}]^2 - [\mathbb{X}^2]) \\ &e_3(\mathbb{X}) = \tfrac{1}{6}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]), \\ &e_4(\mathbb{X}) = \tfrac{1}{24}([\mathbb{X}]^4 - 6[\mathbb{X}]^2[\mathbb{X}^2] + 3[\mathbb{X}^2]^2 + 8[\mathbb{X}][\mathbb{X}^3] - 6[\mathbb{X}^4]), \\ &e_k(\mathbb{X}) = 0 \;\text{ for } k > 4, \end{aligned} \quad (6)$$

where the square brackets denote the trace. The action (4) contains terms that are at most third order in $\sqrt{g^{-1}f}$ rather than fourth order as in (3). When $\alpha_3 = \alpha_4 = 0$ one obtains the resummed theory for which the ghost analysis was performed in [7] to fourth order. When $\alpha_3 = \alpha_4 = 1$ one obtains the minimal action (2). After treating the minimal action, we will extend the ghost analysis to the most general case (4), for arbitrary β_n.

The Hamiltonian constraint.—Let us recapitulate the counting of degrees of freedom in standard massless general relativity. In the ADM formulation [8], the ten components of the metric are parametrized as

$$N = (-g^{00})^{-1/2}, \qquad N_i = g_{0i}, \qquad \gamma_{ij} = g_{ij}. \quad (7)$$

The γ_{ij} describe six potentially propagating modes. The action written in terms of canonical variables is linear in the nonpropagating modes N and N_i (collectively, N_μ). Thus the N_μ equations of motion are constraints on the γ_{ij} and their conjugate momenta π^{ij}. Along with the general coordinate transformations they eliminate four out of six propagating modes, a propagating mode referring to a pair of conjugate variables. The N_μ are determined by the remaining equations, thus leaving two propagating modes corresponding to a spin-2 graviton.

In a generic nonlinear extension of massive gravity, the mass term depends nonlinearly (but still algebraically) on the N_μ. The corresponding equations of motion determine these nondynamical variables in terms of γ_{ij} and π^{ij}, keeping all six of the propagating modes undetermined. Five propagating modes of γ_{ij} correspond to the massive graviton, while the sixth one is a ghost, called the Boulware-Deser mode [3]. A ghost-free theory of massive gravity must maintain a single constraint on γ_{ij} and π^{ij} along with an associated secondary constraint to eliminate this ghostlike sixth mode. Below we show that this is indeed the case for the nonlinear massive gravity actions described above.

Let us first consider the minimal massive gravity action (2). In the ADM parameterization the Lagrangian $\mathcal{L}$ is given by,

$$\pi^{ij}\partial_t\gamma_{ij} + NR^0 + N^iR_i - 2m^2\sqrt{\gamma}N(\mathrm{tr}\sqrt{g^{-1}\eta} - 3), \quad (8)$$

where (with $N^i = \gamma^{ij}N_j$),

$$(g^{-1}\eta)^\mu_{\ \nu} = \frac{1}{N^2}\begin{pmatrix} 1 & N^l\delta_{lj} \\ -N^i & (N^2\gamma^{il} - N^iN^l)\delta_{lj} \end{pmatrix}. \quad (9)$$

Here and in what follows we use $\sqrt{\gamma}$ to denote $\sqrt{\det\gamma_{ij}}$.

The action (9) is highly nonlinear in N_μ and thus it might appear that there are no constraint equations for the propagating degrees of freedom. However, if the four N_μ equations only depend on three combinations of N and N_i, the fourth equation can be used to determine the sixth mode of γ_{ij} in terms of remaining modes.

To show that is the case, we start by assuming that three such combinations n^i exist. Then, after writing N^i in terms of n^i, the massive gravity actions should satisfy the following two criteria, (i) The action is linear in N so that the N equation of motion becomes a constraint on the other fields. (ii) The equations of motion for the n^i are independent of N and hence are algebraically solvable for the n^i. Thus the N equation becomes a constraint on the γ_{ij} and π^{ij}. Along with a secondary constraint, this removes the ghost. N itself is nondynamical and is expected to be determined by the remaining equations, as in GR [8]. We will see that when criterion i is satisfied, ii will follow automatically.

Criterion i means that the change of variables must be linear in N, hence we consider,

$$N^i = (\delta^i_j + ND^i{}_j)n^j. \tag{10}$$

The matrix $D \equiv D^i{}_j$ is determined by requiring that the mass term is linear in N. Indeed that will be the case if the square-root matrix has the form,

$$N\sqrt{g^{-1}\eta} = \mathbb{A} + N\mathbb{B}, \tag{11}$$

where matrices $\mathbb{A}$ and $\mathbb{B}$ are independent of N. Then,

$$g^{-1}\eta = \frac{1}{N^2}\mathbb{A}^2 + \frac{1}{N}(\mathbb{A}\mathbb{B} + \mathbb{B}\mathbb{A}) + \mathbb{B}^2. \tag{12}$$

On the other hand, to write $g^{-1}\eta$ in terms of the new variables, let us assemble the n^i into a column vector n, with transpose n^T, and write $\eta = \mathrm{diag}\{-1, \mathbf{I}\}$, where,

$$\mathbf{I} = \delta_{ij}, \qquad \mathbf{I}^{-1} = \delta^{ij}, \quad \text{whereas } \mathbb{1} = \delta^i_j. \tag{13}$$

Then, writing (9) in terms of the variables (10) and identifying the resulting expression with (12), we read off,

$$\mathbb{A} = \frac{1}{\sqrt{1 - n^T\mathbf{I}n}}\begin{pmatrix} 1 & n^T\mathbf{I} \\ -n & -nn^T\mathbf{I}\end{pmatrix}, \tag{14}$$

$$\mathbb{B} = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{(\gamma^{-1} - Dnn^TD^T)\mathbf{I}}\end{pmatrix}, \tag{15}$$

and

$$(\sqrt{1 - n^T\mathbf{I}n})D = \sqrt{(\gamma^{-1} - Dnn^TD^T)\mathbf{I}}. \tag{16}$$

This last equation can be easily solved for $D^i{}_j$. However, for the arguments that follow we only need the equality (16) and not the explicit solution. Note that the transformation (10) contains no time derivatives and can be shown to be invertible.

A crucial property of D is that $D^i{}_l\delta^{lj}$ is symmetric,

$$D\mathbf{I}^{-1} = (D\mathbf{I}^{-1})^T. \tag{17}$$

This can be seen from (16) combined with the relation $(\sqrt{M\mathbf{I}})\mathbf{I}^{-1} = \mathbf{I}^{-1}(\sqrt{\mathbf{I}M})$ which holds for any matrix M^{ij}.

Now in terms of the new variables n^i, the action is linear in N, meeting the first criterion,

$$\begin{aligned}\mathcal{L} = {} & \pi^{ij}\partial_t\gamma_{ij} + NR^0 + R_i(\delta^i_j + ND^i{}_j)n^j \\ & - 2m^2\sqrt{\gamma}[\sqrt{1 - n^T\mathbf{I}n} \\ & + N\,\mathrm{tr}(\sqrt{\gamma^{-1}\mathbf{I} - Dnn^TD^T\mathbf{I}}) - 3N].\end{aligned} \tag{18}$$

The symmetry property of D along with expression (16) can now be used to show that the n^i equations of motion are independent of N as demanded by criterion ii. Indeed, using $\delta\,\mathrm{tr}\sqrt{M} = \frac{1}{2}\,\mathrm{tr}(\sqrt{M}^{-1}\delta M)$ to differentiate the trace term, one gets, after some manipulations, the n^k equation of motion,

$$\left(R_i + \frac{2m^2\sqrt{\gamma}n^l\delta_{li}}{\sqrt{1 - n^r\delta_{rs}n^s}}\right)\left[\delta^i_k + N\frac{\partial}{\partial n^k}(D^i{}_jn^j)\right] = 0.$$

The expression in the square brackets is the Jacobian of the transformation (10) and is nonzero. Hence the n^i equations are,

$$(\sqrt{1 - n^r\delta_{rs}n^s})R_i + 2m^2\sqrt{\gamma}n^l\delta_{li} = 0. \tag{19}$$

These can be readily solved to determine n^i in terms of γ_{ij} and the conjugate momenta π^{ij},

$$n^i = -R_j\delta^{ji}[4m^4\det\gamma + R_k\delta^{kl}R_l]^{-1/2}. \tag{20}$$

This solution implies that $\sqrt{1 - n^T\mathbf{I}n}$ is real.

The N equation of motion is,

$$R^0 + R_iD^i{}_jn^j - 2m^2\sqrt{\gamma}[\sqrt{1 - n^r\delta_{rs}n^s}D^k{}_k - 3] = 0. \tag{21}$$

Using the n^i solution, this clearly becomes a constraint on the 12 components of γ_{ij} and π^{ij}. Note that in the limit that $m^2 \to 0$, (19) and (21) reduce to the four constraints of general relativity.

The general action.—We now extend the analysis of the previous section to the full two-parameter generalization of the minimal theory. First consider the next higher term in $\sqrt{g^{-1}\eta}$ in the action (4), given by,

$$e_2(\sqrt{g^{-1}\eta}) = \frac{1}{2}[\mathrm{tr}(\sqrt{g^{-1}\eta})^2 - \mathrm{tr}\,g^{-1}\eta]. \tag{22}$$

To express this in terms of the variables defined in the previous section, note that the matrix $\mathbb{A}$ has the property $\mathrm{tr}(\mathbb{A}^k) = (\mathrm{tr}\mathbb{A})^k$. The potential (22) then gives,

$$Ne_2 = \tfrac{1}{2}[2(\mathrm{tr}\,\mathbb{A}\,\mathrm{tr}\,\mathbb{B} - \mathrm{tr}\mathbb{A}\mathbb{B}) + N((\mathrm{tr}\mathbb{B})^2 - \mathrm{tr}\mathbb{B}^2)]. \tag{23}$$

This is linear in the lapse N and thus also satisfies our first criterion. Varying with respect to n^k gives,

$$\begin{aligned}\frac{\delta}{\delta n^k}(Ne_2) = {} & -(n^l\delta_{li}D^m{}_m - n^l\delta_{lm}D^m{}_i) \\ & \times\left[\delta^i_k + N\frac{\partial}{\partial n^k}(D^i{}_jn^j)\right].\end{aligned} \tag{24}$$

It is straightforward to show that the next term in the potential, $Ne_3(\sqrt{g^{-1}\eta})$, is also linear in the lapse N, and, through a more involved analysis, determine the corresponding contribution to the n^k equation of motion.

Combining these results with those from the previous sections, the complete equations of motion for n^i are,

$$R_i - 2m^2\sqrt{\gamma}\bigg(\beta_1 \frac{n^l\delta_{li}}{\sqrt{1-n^r\delta_{rs}n^s}} + \beta_2 n^l[\delta_{li}D^k{}_k - \delta_{lk}D^k{}_i] + \beta_3(\sqrt{1-n^r\delta_{rs}n^s})n^l\delta_{lk}\Big[D^k{}_m D^m{}_i - D^k{}_i D^m{}_m + \frac{1}{2}D^m{}_m D^j{}_j\delta^k_i - \frac{1}{2}D^m{}_j D^j{}_m \delta^k_i\Big]\bigg) = 0. \quad (25)$$

As in the previous sections, these equations are independent of N and can be used to eliminate n^i. The N equation is then the constraint on γ_{ij} and π^{ij},

$$R^0 + R_i D^i{}_j n^j + 2m^2\sqrt{\gamma}[\beta_0 + \beta_1 \mathrm{tr}\mathbb{B} + \tfrac{1}{2}\beta_2\{(\mathrm{tr}\mathbb{B})^2 - \mathrm{tr}\mathbb{B}^2\} + \tfrac{1}{6}\beta_3\{(\mathrm{tr}\mathbb{B})^3 - 3\,\mathrm{tr}\mathbb{B}\,\mathrm{tr}\mathbb{B}^2 + 2\,\mathrm{tr}\mathbb{B}^3\}] = 0. \quad (26)$$

The secondary constraint.—We now argue that the Hamiltonian constraint gives rise to a secondary constraint (for a proof, see [18], completed while this work was in review). This implies that the 12 dimensional phase space of the dynamical variables γ_{ij} and π^{ij} has only 10 degrees of freedom, corresponding to the five polarizations of the massive graviton.

We have shown that, upon integrating out the shift N^i, the Lagrangian (18) remains linear in the lapse N,

$$\mathcal{L} = \pi^{ij}\partial_t\gamma_{ij} - \mathcal{H}_0(\gamma_{ij}, \pi^{ij}) + N\mathcal{C}(\gamma_{ij}, \pi^{ij}). \quad (27)$$

A secondary constraint is obtained by demanding that the primary constraint $\mathcal{C}$ is independent of time on the constraint surface. In the Hamiltonian formulation this condition is given in terms of the Poisson bracket, $\{\mathcal{C}, H\} \approx 0$, where $H = \int d^3x(\mathcal{H}_0 - N\mathcal{C})$. If $\{\mathcal{C}(x), \mathcal{C}(y)\} \approx 0$, then this condition is independent of N and thus becomes a constraint on γ_{ij} and π^{ij},

$$\mathcal{C}_{(2)} \equiv \{\mathcal{C}, H_0\} \approx 0, \quad (28)$$

where now $H_0 = \int d^3x \mathcal{H}_0$.

By construction, the Lagrangian (27) reproduces the Fierz-Pauli Lagrangian at lowest order in the fields,

$$\mathcal{H}_0 \simeq \mathcal{H}_0^{\mathrm{FP}} + O(\gamma^3, \pi^3), \quad \mathcal{C} \simeq \mathcal{C}^{\mathrm{FP}} + O(\gamma^2, \pi^2). \quad (29)$$

Hence one can compute,

$$\mathcal{C}_{(2)} \simeq \mathcal{C}_{(2)}^{\mathrm{FP}} + O(\gamma^2, \pi^2), \quad (30)$$

where $\mathcal{C}_{(2)}^{\mathrm{FP}}$ is neither identically zero nor equal to $\mathcal{C}^{\mathrm{FP}}$. Now, in the Fierz-Pauli case, we know that $\{\mathcal{C}^{\mathrm{FP}}(x), \mathcal{C}^{\mathrm{FP}}(y)\} \approx 0$. Thus at lowest order in the fields there exists a nontrivial secondary constraint. As long as $\{\mathcal{C}(x), \mathcal{C}(y)\} \approx 0$ continues to hold at the nonlinear level (for a proof, see [18]), then $\mathcal{C}_{(2)}$ remains a nontrivial secondary constraint at the nonlinear level as well. Moreover, as can be seen from the Fierz-Pauli structure, enforcing $\{\mathcal{C}_{(2)}, H\} \approx 0$ will result in an equation for N, rather than a tertiary constraint. Thus no further degrees of freedom are removed in this way.

Discussion.—This work demonstrates the existence of nonlinear theories of massive gravity that do not suffer from the Boulware-Deser ghost instability. Note that, in order not to violate the constraints found above, the coupling of the metric to matter must also be linear in the lapse and shift functions. The minimal coupling of general relativity automatically satisfies this requirement and hence will not change the arguments presented here.

It should be emphasized that while it is common to discuss the ghost in terms of Stückelberg fields [4–7], the Boulware-Deser instability [3] is, strictly speaking, due to the loss of the Hamiltonian constraint. We have shown that the massive actions (4) precisely avoid this problem.

We would like to thank F. Berkhahn, C. de Rham, S. Folkerts, G. Gabadadze, K. Hinterbichler, S. Hofmann, A. Pritzel, A. Schmidt-May, B. Sundborg, A. Tolley, and N. Wintergerst for discussions and comments on the draft. R. A. R is supported by the Swedish Research Council (VR) through the Oskar Klein Centre.

[1] M. Fierz, Helv. Phys. Acta **12**, 3 (1939).
[2] M. Fierz and W. Pauli, Proc. R. Soc. A **173**, 211 (1939).
[3] D. G. Boulware and S. Deser, Phys. Rev. D **6**, 3368 (1972).
[4] N. Arkani-Hamed, H. Georgi, and M. D. Schwartz, Ann. Phys. (N.Y.) **305**, 96 (2003).
[5] P. Creminelli, A. Nicolis, M. Papucci, and E. Trincherini, J. High Energy Phys. 09 (2005) 003.
[6] C. de Rham and G. Gabadadze, Phys. Rev. D **82**, 044020 (2010).
[7] C. de Rham, G. Gabadadze, and A. J. Tolley, Phys. Rev. Lett. **106**, 231101 (2011).
[8] R. L. Arnowitt, S. Deser, and C. W. Misner, Gen. Relativ. Gravit. **40**, 1997 (2008).
[9] L. Alberte, A. H. Chamseddine, and V. Mukhanov, J. High Energy Phys. 04 (2011) 004.
[10] K. Hinterbichler, arXiv:1105.3735.
[11] S. F. Hassan and R. A. Rosen, J. High Energy Phys. 07 (2011) 009.
[12] A. I. Vainshtein, Phys. Lett. B **39**, 393 (1972).
[13] H. van Dam and M. J. G. Veltman, Nucl. Phys. **B22**, 397 (1970).
[14] V. I. Zakharov, Pis'ma Zh. Eksp. Teor. Fiz. **12**, 447 (1970); [JETP Lett. **12**, 312 (1970)].
[15] K. Koyama, G. Niz, and G. Tasinato, Phys. Rev. D **84**, 064033 (2011).
[16] T. M. Nieuwenhuizen, Phys. Rev. D **84**, 024038 (2011).
[17] G. Chkareuli and D. Pirtskhalava, arXiv:1105.1783.
[18] S. F. Hassan and R. A. Rosen, arXiv:1111.2070.