Master Equations for Correlated Quantum Channels

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We derive the general form of a master equation describing the reduced time evolution of a sequence of subsystems ''propagating'' in an environment which can be described as a sequence of subenvironments. The interaction between subsystems and subenvironments is described in terms of a collision model, with the irreversible dynamics of the subenvironments between collisions explicitly taken into account. In the weak coupling regime, we show that the collisional model produces a correlated Markovian evolution for the joint density matrix of the multipartite system. The associated Lindblad superoperator contains pairwise terms describing cross correlation between the different subsystems. Such a model can describe a broad range of physical situations, ranging from quantum channels with memory to photon propagation in concatenated quantum optical systems.

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The dynamics of the open dynamics can be described either in terms of maps and Kraus operators or in terms of master equations and Lindblad operators, the correspondence between such two descriptions being not always straightforward. In general, determining if a given quantum transformation is compatible with a Lindblad structure is a computationally hard problem [1]. Furthermore, for a multipartite system a Lindbladian structure for the global system $\mathcal{S}$ in general may introduce non-Markovian elements in the dynamics of the subsystems that compose it, which also are far from trivial to characterize [2]. In the study of quantum channels, the description in terms of maps is the natural choice, as one is interested in the consecutive interaction of a series of quantum subsystems acting as information carriers with an environment. For *memoryless* channels, the assumption that each subsystem interacts with its own local environment, i.e., that the channel acts independently on each separate carrier, is made. In recent years, however, the study of *correlated* (or memory) channels has shown that interesting new features emerge when one makes the realistic assumption that the action of the noise acting over consecutive carriers is correlated (e.g., see [3–9], and references therein). Such correlations have been phenomenologically described in terms of a Markov chain which gives the joint probability distribution of the local Kraus operators acting on the elements of $\mathcal{S}$ [3]. Alternatively, they have been effectively represented in terms of local interactions of the carriers with a common multipartite environment which is originally prepared into a correlated (possibly entangled) initial state [7] or with a structured environment composed by local and global components [4–6].

The aim of the present Letter is to provide a continuous time description of correlated quantum channels in terms of a joint master equation (ME) [10,11] for $\mathcal{S}$. This will lead us to identify the structure of the Lindblad generators which are responsible for the arising of specific correlations among the carriers. They are apt to describe those scenarios where a structured, multipartite quantum system interacts with a large environment characterized by (relatively) slow reaction times. As an example, think of a string of particles characterized by some internal degree of freedom (say, spin) that fly into an ionization chamber, exciting the gas that fills it while passing through. If the speed of the particles is sufficiently high, one might expect that, thanks to the mediation of gas, excitations from one particle could be passed to the next one, modifying its internal state. The net result of course is the creation of delocalized excitations over the whole string of particles. The master equations we derive here are perfectly fitted to describe the resulting dynamics.

In our analysis we will adopt a rather pragmatic approach, deriving the dynamical evolution of $\mathcal{S}$ from a collisional model [12,13] in which dissipative effects originate from a sequence of weak but frequent interactions with a collection of uncorrelated particles which mimic the system environment. Consider hence a multipartite quantum system $\mathcal{S}$, consisting of M—not necessarily identical—ordered subsystems $S_1, S_2, \ldots, S_M$ (the information carriers of the model). In what follows, each subsystem is supposed to interact with a multipartite environment $\mathcal{E}$ consisting of a large number of subenvironments $E_1, E_2, \ldots$ via an ordered sequence of pairwise interactions (for a pictorial representation, see Fig. 1). As in Refs. [12,13], the pairwise collision between the subsystem S_m and the subenvironment E_n is described by a local unitary $U_{S_mE_n} = \exp[-igH_{S_mE_n}\Delta t]$ characterized by a collision time Δt and by the intensity parameter g and generated by the Hamiltonian coupling $H_{S_mE_n}$, which (without loss of generality) we write as

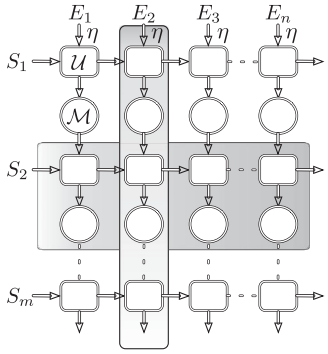

FIG. 1. Schematic of the process. The horizontal lines describe an ordered set of carriers $S_1, S_2, \ldots$ which interact with an ordered set of (possibly infinite) identical local subenvironments $E_1, E_2, \ldots$ via local unitaries $\mathcal{U}_{S_m E_n}$ (η being the initial state of the E_js). Between collisions, each subenvironment evolves according to a map $\mathcal{M}$. The overall dynamics can be described as an ordered sequence of row or of column superoperators (visualized by the rectangular sets in the figure).

$$H_{S_m E_n} := \sum_{\ell} A_{S_m}^{(\ell)} \otimes B_{E_n}^{(\ell)}, \tag{1}$$

with $A_S^{(\ell)}, B_E^{(\ell)} \neq 0$ Hermitian. Accordingly, the mth carrier interacts with the first n elements of the environment $\mathcal{E}$ through the joint unitary

$$U_{S_m \mathcal{E}}^{(n)} := U_{S_m,E_n} U_{S_m,E_{n-1}} \ldots U_{S_m,E_2} U_{S_m,E_1} \tag{2}$$

(the presence of a local free Hamiltonian evolution operating between the collisions can be included in the model by passing into the interaction picture representation and replacing $A_{S_m}^{(\ell)}$ with the corresponding evolved operators). Finally, to account for the internal dynamics of the environment, we assume that between two consecutive collisions each subenvironment evolves according to a completely positive, trace-preserving (CPT) map $\mathcal{M}$. Operationally, $\mathcal{M}$ acts as a "damper" for the information that percolates from one carrier to the subsequent one. It mimics the relaxation processes that may take place within the environment alone (e.g., originating from the mutual interactions between its various parts) and which in principle involve time scales different from those that define the *rate* of the collisional events.

Consider then the case where the M subsystems of $\mathcal{S}$ are initially in a (possibly correlated) state $\rho(0)$ while the subenvironments of $\mathcal{E}$ are all prepared into the same input state η (which, as in Ref. [12], represents some equilibrium state of the particles of the reservoir). For the sake of simplicity, in the following we will work under the hypothesis that

$$\langle B_E^{(\ell)} \mathcal{M}^m(\eta) \rangle_E = 0 \qquad \forall\ \ell, m, \tag{3}$$

where we use the symbol $\langle \ldots \rangle_X$ to represent taking the expectation value with respect to the system X and $\mathcal{M}^m$ to represent the channel obtained by applying m times the map $\mathcal{M}$. The assumption (3) allows us to rigorously define the continuous limit of the model. It is worth noticing, however, that it does not imply any loss of generality, as it can always be enforced by moving into an interaction representation with respect to a rescaled local Hamiltonian for the system $\mathcal{S}$.

After the interactions with the first n elements of $\mathcal{E}$, the global state $R(n)$ of the system and of the environment is obtained from the initial state $\rho(0) \otimes \eta^{\otimes n}$ as $R(n) = \mathcal{W}^{(n,M)}[\rho(0) \otimes \eta^{\otimes n}]$, where $\mathcal{W}^{(n,M)}$ is the superoperator which describes the collisions and the free evolutions of $\mathcal{E}$. As schematically shown in Fig. 1, it can be expressed as a composition of *row* superoperators stacked in series one on top of the other:

$$\mathcal{W}^{(n,M)} = \mathcal{R}_{S_M,\mathcal{E}}^{(n)} \circ \mathcal{R}_{S_{M-1},\mathcal{E}}^{(n)} \circ \cdots \circ \mathcal{R}_{S_2,\mathcal{E}}^{(n)} \circ \mathcal{R}_{S_1,\mathcal{E}}^{(n)}, \tag{4}$$

where $\mathcal{R}_{S_m,\mathcal{E}}^{(n)} := \mathcal{M}^{\otimes n} \circ \mathcal{U}_{S_m,\mathcal{E}}^{(n)}$. Here, given a unitary transformation U, we define $\mathcal{U}(\ldots) = U(\ldots)U^\dagger$. Also we use the symbol "$\circ$" to represent the composition of superoperators and $\mathcal{M}^{\otimes n}$ to represent $\mathcal{M}_{E_1} \circ \cdots \circ \mathcal{M}_{E_n}$, $\mathcal{M}_{E_j}$ being the map $\mathcal{M}$ operating on the jth element E_j of $\mathcal{E}$. The transformation $\mathcal{R}_{S_m,\mathcal{E}}^{(n)}$ describes the evolution of S_m in its interaction with $\mathcal{E}$ plus the subsequent free evolution of the latter induced by the maps $\mathcal{M}$. Alternatively, by exploiting the fact that for $m' \neq m$, $n' \neq n$ the operators U_{S_m,E_n} and $U_{S_{m'},E_{n'}}$ commute, $\mathcal{W}^{(n,M)}$ can also be expressed in terms of *column* superoperators concatenated in series as follows:

$$\mathcal{W}^{(n,M)} = \mathcal{C}_{\mathcal{S},E_n}^{(M)} \circ \mathcal{C}_{\mathcal{S},E_{n-1}}^{(M)} \circ \cdots \circ \mathcal{C}_{\mathcal{S},E_2}^{(M)} \circ \mathcal{C}_{\mathcal{S},E_1}^{(M)}, \tag{5}$$

where, for all $j = 1, \ldots, n$,

$$\mathcal{C}_{\mathcal{S},E_j}^{(M)} := \mathcal{M}_{E_j} \circ \mathcal{U}_{S_M,E_j} \circ \cdots \circ \mathcal{M}_{E_j} \circ \mathcal{U}_{S_1,E_j}. \tag{6}$$

Thanks to Eq. (5), we can now write the following recursive expression for $R(n)$:

$$R(n+1) = \mathcal{C}_{\mathcal{S},E_{n+1}}^{(M)}(R(n) \otimes \eta). \tag{7}$$

The master equation.—For a particular class of interaction unitaries, the authors of Ref. [13] have shown that the collision model leads to a dynamics which can be described by a Lindblad superoperator via direct integration of the equation of motion. Here we introduce an alternative approach which allows one to derive a ME for the reduced dynamics of the many-body system $\mathcal{S}$ in our generalized multipartite collision model. The details of the derivation can be found in Ref. [14]. We simply assume a weak coupling regime where we take a proper expansion with respect to the parameters g and Δt which quantifies the intensity and the duration of the single events. In particular, we work in the regime in which $g\Delta t$ is small enough to allow for the expansion of the dynamical equation (7) up to $O((g\Delta t)^2)$, i.e.,

$$R(n+1) = [\mathcal{I}_{\mathcal{S},E_{n+1}} + \mathcal{C}'_{\mathcal{S},E_{n+1}} g\Delta t + \mathcal{C}''_{\mathcal{S},E_{n+1}} (g\Delta t)^2] \times [R(n) \otimes \eta] + O[(g\Delta t)^3], \quad (8)$$

where $\mathcal{I}_{\mathcal{S},E_{n+1}}$ is the identity superoperator while $\mathcal{C}'_{\mathcal{S},E_{n+1}}$ and $\mathcal{C}''_{\mathcal{S},E_{n+1}}$ are the first and second expansion terms in $g\Delta t$ of the superoperator $\mathcal{C}^{(M)}_{\mathcal{S},E_{n+1}}$, respectively. Tracing over the degree of freedom of the environment, the resulting equation defines the incremental evolution of the density matrix $\rho(n) := \langle R(n) \rangle_{\mathcal{E}}$ of $\mathcal{S}$ when passing from the nth to the $(n+1)$th collision. The continuous limit is finally taken by sending Δt to zero while g and n explode in such a way that $n\Delta t$ and $g^2\Delta t$ remains finite, i.e.,

$$\lim_{\Delta t \to 0^+} n\Delta t = t < \infty, \qquad \lim_{\Delta t \to 0^+} g^2 \Delta t = \gamma < \infty. \quad (9)$$

Notice that, while the first condition is necessary to properly define the axis of time, the second is needed to guarantee that $\mathcal{S}$ fills the interactions with $\mathcal{E}$. Indeed, one easily verifies that the linear terms in g do not enter in the dynamical evolution of $\mathcal{S}$ since $\langle \mathcal{C}'_{\mathcal{S},E_{n+1}} (R(n) \otimes \eta) \rangle_{\mathcal{E}} = 0$ due to the assumption (3).

Defining hence $\rho(t) = \lim_{\Delta t \to 0^+} \rho(n)$ the reduced density matrix of $\mathcal{S}$ at time t and $\dot{\rho}(t) := \lim_{\Delta t \to 0^+} \frac{\rho(n+1) - \rho(n)}{\Delta t}$ its time derivative, from Eq. (9) we get the following ME:

$$\dot{\rho}(t) = \sum_{m=1}^{M} \mathcal{L}_m(\rho(t)) + \sum_{m' > m} \mathcal{D}^{(\rightarrow)}_{m,m'}(\rho(t)). \quad (10)$$

This is mathematically equivalent to the standard derivation of a Markovian ME for a system interacting with a large environment, in which one assumes that the overall system-environment density operator at any given time t of the evolution factorizes as in $\rho(t) \otimes \eta$, where η is the environment density operator. The two scenarios are, however, different. In the standard case, the reason for which the environment state is unchanged is because it is big. In our scenario, consistently with the collisional model, the environment state is constant because, as we said, each subsystem collides briefly with a sequence of subenvironments all initially in the same state. Of course, one expects a strongly non-Markovian behavior if a given subsystem interacts repeatedly with *the same* subenvironment [15].

The ME (10) contains both *local* Lindblad terms (i.e., Lindblad terms which act locally on the mth carrier) and *two-body nonlocal terms* which couple the m carrier to the $m' > m$. More precisely, the mth local term is the superoperator

$$\mathcal{L}_m(\ldots) = \frac{1}{2} \sum_{\ell,\ell'} \gamma^{(\ell,\ell')}_m [2A^{(\ell')}_{S_m}(\ldots)A^{(\ell)}_{S_m} - A^{(\ell)}_{S_m}A^{(\ell')}_{S_m}(\ldots) - (\ldots)A^{(\ell)}_{S_m}A^{(\ell')}_{S_m}], \quad (11)$$

where the non-negative matrix $\gamma^{(\ell,\ell')}_m$ is given by

$$\gamma^{(\ell,\ell')}_m := \gamma \langle B^{(\ell)}_E B^{(\ell')}_E \mathcal{M}^{m-1}(\eta) \rangle_E, \quad (12)$$

with γ as in Eq. (9). Equation (12) defines the correlation matrix of the subenvironment operators $B^{(\ell)}_E$ and $B^{(\ell')}_E$ evaluated (for the infinitesimal time interval Δt) on the density matrix $\mathcal{M}^{m-1}(\eta)$, which describes the state of the subenvironment after $m-1$ free evolution steps [16]. For $m' > m$ the cross terms of Eq. (10) are defined instead as

$$\mathcal{D}^{(\rightarrow)}_{m,m'}(\ldots) = \sum_{\ell,\ell'} \gamma^{(\ell,\ell')}_{m,m'} A^{(\ell)}_{S_m}[(\ldots), A^{(\ell')}_{S_{m'}}]_- - \sum_{\ell,\ell'} [\gamma^{(\ell,\ell')}_{m,m'}]^* \times [(\ldots), A^{(\ell')}_{S_{m'}}]_- A^{(\ell)}_{S_m} \quad (13)$$

with $[\ldots,\ldots]_-$ being the commutation matrix and $\gamma^{(\ell,\ell')}_{m,m'}$ being the complex matrix [17]

$$\gamma^{(\ell,\ell')}_{m,m'} := \gamma \langle B^{(\ell')}_E \mathcal{M}^{m'-m}(B^{(\ell)}_E \mathcal{M}^{m-1}(\eta)) \rangle_E. \quad (14)$$

The coefficients $\gamma^{(\ell,\ell')}_{m,m'}$ introduce cross correlation among the carriers and depend upon their *distance* $m' - m$. Furthermore, similarly to the terms of Eq. (12), they also depend on $m-1$ due to the fact that the model admits a *first* carrier. However, if we assume that for large m the sequence $\mathcal{M}^m(\eta)$ converges to a final point η_0, then we can reach a stationary configuration where (for $m \gg 1$) $\gamma^{(\ell,\ell')}_{m,m'}$ depends only upon the distance $m' - m$ while $\gamma^{(\ell,\ell')}_m$ becomes constant in m, i.e.,

$$\gamma^{(\ell,\ell')}_{m,m'} \simeq \langle B^{(\ell')}_E \mathcal{M}^{m'-m}(B^{(\ell)}_E \eta_0) \rangle_E, \quad (15)$$

$$\gamma^{(\ell,\ell')}_m \simeq \langle B^{(\ell')}_E B^{(\ell)}_E \eta_0 \rangle_E. \quad (16)$$

A similar behavior is obtained also if we assume η to be a fixed point for $\mathcal{M}$ (a reasonable hypothesis if $\mathcal{E}$ is supposed to describe an environment in its stationary configuration). In this case, Eqs. (15) and (16) hold exactly for all m and m', with η_0 being replaced by η. Finally, a case of particular interest is the one in which $\mathcal{M}$ is the channel which sends every input state into η (this is the extremal version of the last two examples). Under this condition, the "damping" action of $\mathcal{M}$ is extremely efficient (the environmental subsystems are immediately reset to their initial state after each collision), and one expects that no correlations between the various carriers can be established. Indeed, in this case we have $\mathcal{M}(\theta) = \langle \theta \rangle_E \eta$ for all operators θ, which, thanks to Eq. (3), yields $\gamma^{(\ell,\ell')}_{m,m'} = \gamma \langle B^{(\ell')}_E \eta \rangle_E \langle B^{(\ell)}_E \eta \rangle_E = 0$ and hence $\mathcal{D}^{(\rightarrow)}_{m,m'} = 0$.

Correlations.—Equation (13) obeys proper time-ordering rules which guarantee that the dynamical evolution of S_m is *not* influenced by the subsystems that follow it in the sequence, while it might depend in a nontrivial way on the carriers that precede it. Indeed, when traced over the degree of freedom of the second carrier $\mathcal{S}_{m'}$, the cross term $\mathcal{D}^{(\rightarrow)}_{m,m'}$ nullifies, i.e.,

$$\langle \mathcal{D}^{(\rightarrow)}_{m,m'}(\ldots) \rangle_{S_{m'}} = 0, \quad (17)$$

while in general it does not disappear when tracing over S_m (it *does* disappear, however, if all the coefficients $\gamma_{m,m'}^{(\ell,\ell')}$ are real; see below). The evolution described by Eq. (10) is thus *nonanticipatory* [18] or, in the jargon introduced in Ref. [19], *semicausal* with respect to the ordering of the channels used. To see this explicitly, consider the evolution of the reduced density matrix $\rho_{1,2}(t)$ of the first two carriers obtained by taking the partial trace of Eq. (10) over all elements of $\mathcal{S}$ but S_1 and S_2. Noticing that $\langle \mathcal{L}_m(\ldots)\rangle_{S_m} = 0$ and exploiting Eq. (17), we get

$$\dot{\rho}_{1,2}(t) = \mathcal{L}_1(\rho_{1,2}(t)) + \mathcal{L}_2(\rho_{1,2}(t)) + \mathcal{D}_{1,2}^{(\rightarrow)}(\rho_{1,2}(t)). \quad (18)$$

The resulting dynamics is purely Markovian in full agreement with the fact that S_1 and S_2 couple weakly and sequentially with subenvironments $\mathcal{E}$ which have not interacted yet with other carriers. Tracing over S_2, we can then derive the dynamical equation for S_1, i.e., $\dot{\rho}_1(t) = \mathcal{L}_1(\rho_1(t))$, which again is Markovian. Vice versa, the dynamics of S_2 cannot be expressed in terms of a close differential equation for $\rho_2(t)$ alone. Indeed, by taking the partial trace of Eq. (18) over S_1, we get

$$\dot{\rho}_2(t) = \mathcal{L}_2(\rho_2(t)) - 2i\sum_{\ell,\ell'} \mathrm{Im}[\gamma_{1,2}^{(\ell,\ell')}][A_{S_2}^{\ell'}, \langle A_{S_1}(t), \rho_{1,2}(t)\rangle_{S_1}]_-, \quad (19)$$

where the last term explicitly depends upon the joint density matrix of S_2 *and* S_1 [20]. This formally shows that in general S_1 acts as a controller for S_2, while no backaction is allowed in the model.

A case of special interest is represented by those situations in which the matrices $\gamma_{m,m'}^{(\ell,\ell')}$ are real. When this happens, also the partial trace over S_m of $\mathcal{D}_{m,m'}^{(\rightarrow)}$ nullifies, i.e., $\langle \mathcal{D}_{m,m'}^{(\rightarrow)}(\ldots)\rangle_{S_m} = 0$. Accordingly, the evolution of *any* subset of $\mathcal{S}$ is independent from the evolution of the remaining carriers. In this case, hence our model becomes nonanticipatory with respect to all possible ordering of the carriers, describing hence a *nonsignaling* evolution [19] in which the reduced density matrix of each carrier evolves independently from the others. For instance, in Eq. (19), the second term disappears, yielding a Markovian equation also for $\rho_2(t)$, i.e., $\dot{\rho}_2(t) = \mathcal{L}_2(\rho_2(t))$.

Example.—As an application, we focus on the case in which the carriers and $\mathcal{E}$ form two sets of independent bosonic modes. In particular, defining a_m and b_n to be annihilation operators of the modes S_m and E_n, respectively, we consider the Hamiltonians $H_{S_m,E_n} = a_m \otimes b_n^\dagger + a_m^\dagger \otimes b_n$. We also take η as the vacuum state of E_n and $\mathcal{M}$ as a lossy bosonic quantum channel of transmissivity κ. Notice that with these choices the Hermitian operators $A_{S_m}^{(\ell)}$ and $B_{E_n}^{(\ell)}$ entering in Eq. (1) are just quadrature operators of the fields and that Eq. (3) is automatically verified for all m since $\mathcal{M}(\eta) = \eta$. The resulting model describes a correlated quantum channel analogous to that of Ref. [9], which mimics the transmission of a sequence of optical pulses along an attenuating optical fiber characterized by finite relaxation times. The corresponding local $\mathcal{L}_m(\ldots)$ and cross term $\mathcal{D}_{m,m'}^{(\rightarrow)}$ entering in the final ME (10) become, respectively, $\frac{\gamma}{2}\{2a_m(\ldots)a_m - a_m^\dagger a_m(\ldots) - (\ldots)a_m^\dagger a_m\}$ and $\gamma\kappa^{m'-m/2}\{[a_m(\ldots), a_{m'}^\dagger]_- - [(\ldots)a_m^\dagger, a_{m'}]_-\}$, which exhibit an attenuation of the signals and an exponential decaying in the correlations [in particular, $\mathcal{D}_{m,m'}^{(\rightarrow)}(\ldots)$ coincides with the cross term derived in Ref. [21] for a collection of QED cavity modes coupled in cascade].

Conclusions and perspectives.—In deriving the ME (10), we assumed a specific ordering for the carriers of the model which implies that each element in the sequence $S_1, S_2, \ldots, S_M$ can influence only the dynamical evolution of those which follow. This assumption was specifically introduced to account for the causal correlations that are present in many memory quantum channel models [18]. The collisional model, however, can be generalized to include more general correlations. For instance, cyclical correlations can be accounted by identifying S_1 with the $(M+1)$th element of the set of carriers in such a way that S_M can influence its dynamics. To do so, it is sufficient to add an independent set $\mathcal{F}$ of subenvironments $F_1, F_2, \ldots, F_N$ which couple to $\mathcal{S}$ following a new ordering in which (say) all the carriers are shifted by one position (i.e., the element of $\mathcal{F}$ first interacts with S_2, then with $S_3, S_4, \ldots, S_N$, and finally with S_1). Apart from the new ordering, the new couplings are assumed to share the same properties of those that apply to $\mathcal{E}$ [in particular, we require that identities analogous to those in Eqs. (3) and (9) hold]. Under these conditions (and by assuming no direct interaction between $\mathcal{E}$ and $\mathcal{F}$), the ME (10) will acquire new extra terms which directly couple each carrier to all the others. Specifically, given $m' > m$, we will have both a standard contribution of the form $\mathcal{D}_{m,m'}^{(\rightarrow)}$ as in Eq. (10) but also a contribution in which the role of m and m' are exchanged (i.e., something like $\mathcal{D}_{m',m}^{(\rightarrow)}$) that originates from the couplings with $\mathcal{F}$. From this example, it should be clear that, by increasing the number of subenvironmental sets and by properly tuning their interactions with $\mathcal{S}$, any sort of correlations can be built in dynamical evolution of the system.

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[1] M. M. Wolf, J. Eisert, T. S. Cubitt, and J. I. Cirac, Phys. Rev. Lett. **101**, 150402 (2008); T. S. Cubitt, J. Eisert, and M. M. Wolf, arXiv:0908.2128; arXiv:1005.0005.

[2] J. Piilo *et al.*, Phys. Rev. Lett. **100**, 180402 (2008); H.-P. Breuer and B. Vacchini, *ibid.* **101**, 140402 (2008); H.-P. Breuer, E. M. Laine, and J. Piilo, Phys. Rev. Lett. **103**,

210401 (2009); D. Chruściński and A. Kossakowski, *ibid.* **104**, 070406 (2010).

[3] C. Macchiavello and G. M. Palma, Phys. Rev. A **65**, 050301(R) (2002); C. Macchiavello, G. M. Palma, and S. Virmani, *ibid.* **69**, 010303(R) (2004).

[4] G. Bowen and S. Mancini, Phys. Rev. A **69**, 012306 (2004).

[5] D. Kretschmann and R. F. Werner, Phys. Rev. A **72**, 062323 (2005).

[6] V. Giovannetti, J. Phys. A **38**, 10 989 (2005).

[7] V. Giovannetti and S. Mancini, Phys. Rev. A **71**, 062304 (2005); M. B. Plenio and S. Virmani, Phys. Rev. Lett. **99**, 120504 (2007); New J. Phys. **10**, 043032 (2008); D. Rossini, V. Giovannetti, and S. Montangero, *ibid.* **10**, 115009 (2008); F. Caruso, V. Giovannetti, and G. M. Palma, Phys. Rev. Lett. **104**, 020503 (2010).

[8] G. Benenti, A. D'Arrigo, and G. Falci, Phys. Rev. Lett. **103**, 020502 (2009); New J. Phys. **9**, 310 (2007).

[9] C. Lupo, V. Giovannetti, and S. Mancini, Phys. Rev. Lett. **104**, 030501 (2010); Phys. Rev. A **82**, 032312 (2010).

[10] G. Lindblad, Commun. Math. Phys. **48**, 119 (1976); V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, J. Math. Phys. (N.Y.) **17**, 821 (1976).

[11] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University, Oxford, 2007).

[12] V. Scarani *et al.*, Phys. Rev. Lett. **88**, 097905 (2002); M. Ziman *et al.*, Phys. Rev. A **65**, 042105 (2002).

[13] M. Ziman, P. Štelmachovič, and V. Bužek, J. Opt. B **5**, S439 (2003); Open Syst. Inf. Dyn. **12**, 81 (2005); M. Ziman and V. Bužek, Phys. Rev. A **72**, 022110 (2005).

[14] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.108.040401 for mathematical details.

[15] G. Benenti and G. M. Palma, Phys. Rev. A **75**, 052110 (2007).

[16] Equation (11) can also be cast in a more traditional form [10,11] by diagonalizing the matrix $\gamma_m^{(\ell,\ell')}$: This allows one to identify the decay rates of the system with the non-negative eigenvalues $r_m^{(\ell)}$ of $\gamma_m^{(\ell,\ell')}$ and the associated Lindblad operators $L_{S_m}^{(\ell)}$ with a proper linear combinations of the $A_{S_m}^{(\ell)}$.

[17] Equation (14) can be written as $\gamma\langle\mathcal{M}_{\mathrm{H}}^{m'-m}(B_E^{(\ell')})\times B_E^{(\ell)}\mathcal{M}^{m-1}(\eta)\rangle_E$, where $\mathcal{M}_{\mathrm{H}}$ is the Heisenberg adjoint of $\mathcal{M}$. In this form $\gamma_{m,m'}^{(\ell,\ell')}$ appears to be a generalized two-time correlation function of the environment operators $B_E^{(\ell')}$ and $B_E^{(\ell)}$ with respect to the density operator $\mathcal{M}^{m-1}(\eta)$.

[18] R. G. Gallager, *Information Theory and Reliable Communication* (Wiley, New York, 1968).

[19] D. Beckman, D. Gottesman, M. A. Nielsen, and J. Preskill, Phys. Rev. A **64**, 052309 (2001); M. Piani, M. Horodecki, P. Horodecki, and R. Horodecki, *ibid.* **74**, 012305 (2006); T. Eggeling, D. M. Schlingemann, and R. F. Werner, Europhys. Lett. **57**, 782 (2002).

[20] An approximate equation for $\rho_2(t)$ alone can be obtained by requiring $\rho_{1,2}(t) \simeq \rho_1(t) \otimes \rho_2(t)$ for each time t (this is equivalent to considering S_1 as part of an effective environment which is weakly coupled to S_2). In this case, Eq. (19) becomes $\dot{\rho}_2(t) \simeq -i[H_2^{(\mathrm{eff})}(t), \rho_2(t)]_- + \mathcal{L}_2(\rho_2(t))$, with $H_2^{(\mathrm{eff})}(t) := 2\sum_{\ell,\ell'}\mathrm{Im}[\gamma_{1,2}^{(\ell,\ell')}]\times\langle A_{S_1}^{(\ell)}\rho_1(t)\rangle_{S_1}A_{S_2}^{(\ell')}$ being an effective time-dependent Hamiltonian of S_2.

[21] C. W. Gardiner, Phys. Rev. Lett. **70**, 2269 (1993); H. J. Carmichael, *ibid.* **70**, 2273 (1993); C. W. Gardiner and A. S. Parkins, Phys. Rev. A **50**, 1792 (1994).