No Evidence of Magnitude Clustering in an Aftershock Sequence of Nano- and Picoseismicity

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One of the hallmarks of our current understanding of seismicity as highlighted by the epidemic-type-aftershock sequence model is that the magnitudes of earthquakes are independent of one another and can be considered as randomly drawn from the Gutenberg-Richter distribution. This assumption forms the basis of many approaches for forecasting seismicity rates and hazard assessment. Recently, it has been suggested that the assumption of independent magnitudes is not valid. It was subsequently argued that this conclusion was not supported by the original earthquake data from California. One of the main challenges is the lack of completeness of earthquake catalogs. Here, we study an aftershock sequence of nano- and picoseismicity as observed at the Mponeng mine, for which the issue of incompleteness is much less pronounced. We show that this sequence does not exhibit any significant evidence of magnitude correlations.

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One prominent feature of seismicity is the clustering of earthquakes in space and time. This is evident, for example, from the observation that the local rate of activity after a large earthquake is much higher than before. Starting with the Omori law [1], there has been a significant amount of research dedicated to quantify, characterize, and understand this spatiotemporal clustering (see, for example, Refs. [2–12] and references therein). The scientific effort is partly driven by the desire to predict or forecast earthquakes. To this end, the Omori law and other empirical observations including the Gutenberg-Richter (GR) law have been used to formulate stochastic models of seismicity [13–16]. Despite some success forecasting aftershock activity and seismicity rates [17–21], we are, however, far away from reliably predicting the occurrence of large earthquakes ahead of time [22,23], which potentially could even be an unreachable goal [24].

Recent evidence for the latter hypothesis comes from attempts to determine the magnitude of an earthquake from seismic signals *before* the rupture terminates. The current conclusion is that this is not possible [25–29]. Thus, it came as a surprise when it was suggested that statistical correlations exist between the magnitudes of earthquakes [30,31]. If indeed true, it would imply that one could predict the magnitude of a future earthquake based on the magnitudes of previously observed earthquakes. In a subsequent paper, it was argued, however, that the existence of nontrivial correlations was not supported by the original earthquake data from southern California [32]. It was shown, in particular, that catalog incompleteness can lead to the spurious detection of magnitude correlations. Almost all earthquake catalogs suffer from this effect—even if one constrains the observation to larger earthquakes—due to the presence of short-term aftershock incompleteness [17,33–36]. It arises mainly because seismicity directly after a large earthquake can be masked by overlapping arrivals of waves from different events. Thus, it is conceivable that magnitude correlations do exist over short space and time scales but they are typically hidden due to catalog incompleteness.

Here, we address exactly this point by studying seismicity, for which the influence of catalog incompleteness is minimal. Specifically, we focus on an aftershock sequence of nano- and picoseismicity—corresponding to earthquakes with moment magnitudes M_W in the range $[-4, 0]$ as discussed in [37]—observed at the Mponeng mine, South Africa, with a magnitude of completeness of $M_C = -4.3$ [38,39]. We find that there is no significant evidence for magnitude correlations. This is even true if one considers events that are close in space and/or time, which were speculated to exhibit particularly strong magnitude correlations.

The data we analyze are recorded as part of the JAGUARS (Japanese-German underground acoustic emission research in South Africa) project [40]. The project aimed to close the gap between the laboratory research on rock samples and seismicity measured *in situ*. For that purpose the high-frequency JAGUARS network was installed at a depth of 3550 m in the Mponeng deep gold mine in South Africa. The network was composed of a 3-component (3C) accelerometer and 8 acoustic emission sensors. The sensors were sensitive in a broad frequency range (50 Hz—200 kHz) and allowed us to record extremely small seismic events (M_W between -5.0 and -0.8) [38,41]. The recorded seismicity (except for man-made sources) falls into two major groups: (1) postblasting seismic activity related to production blastings, and (2) aftershock sequences following seismic events not directly provoked by blastings (for details, see [39]).

Here, we analyze part of the aftershock sequence of a large $M_W = 1.9$ seismic event that occurred on 27 December, 2007, 30 m from the center of JAGUARS network (see Refs. [39,41–43] for details). Because of the Christmas vacation period, the data set does not contain working noises and postblasting activity [38,44]. The initial data set contained more than 20 000 events detected in the *whole* area over a period of 6 days [38,43]. From these, more than 11 500 seismic events were located automatically. The remaining events were not located due to an insufficient number of picks (less than 6 P and 1 S arrival) resulting from a low signal-to-noise ratio. This is *typically* related to an extremely low magnitude of the event and/or large source-receiver distances; i.e., the event occurred far away from the network. The automatically located events were manually reviewed in order to confirm P and S picks and hypocenter locations [38]. As the data set does not contain man-made noises, practically all located events rejected by this manual review were due to wrongly placed P and/or S onsets. These inappropriate picks were caused by low signal-to-noise ratio, phases following direct arrivals or multiple (sometimes overlapping) events in a single time window.

The moment magnitudes M_W were calculated using the acoustic emission sensors calibrated with the 3C accelerometer (see [38] for details of the calibration procedure). For 64% of all events, M_W was calculated using the maximum number of five sensors available, as the remaining three sensors displayed problems related to coupling and were not used. The uncertainty in the estimation of the moment magnitude was within $M_W \pm 0.46$, corresponding to 3 standard deviations [38].

The manually reviewed data set contained 9444 events with M_W ranging from -5.0 to -0.8. For the purpose of this study we analyzed a subset of aftershocks located in the vicinity of the fault plane (see Fig. 4, area F and Table I of [38] for details). This subset contained 7107 high-quality seismic events located very close to the network, where location precision and detection threshold are highest [39]. The subset follows the GR scaling relation [38]. The magnitude of completeness was estimated using the methodology presented in [45]. We defined M_C as the point where the GR power law explains 90% or more of the frequency-magnitude distribution. As a result, the final subset contained $N = 4242$ seismic events with magnitudes above $M_C = -4.3$. To test for any potential bias due to events that were not automatically located, we considered an extended catalog consisting of $N = 4674$ events above $M_C = -4.3$. In addition to the located events it included nonlocated aftershocks that occurred closer than 80 m from the sensors as estimated using S-P times. We assumed these events occurred in the vicinity of the fault plane. Whenever possible, we calculated magnitudes using the same methodology as for the located earthquakes. We found that none of the results presented in the following changes significantly if the extended catalog is considered instead of the original one with $N = 4242$ events.

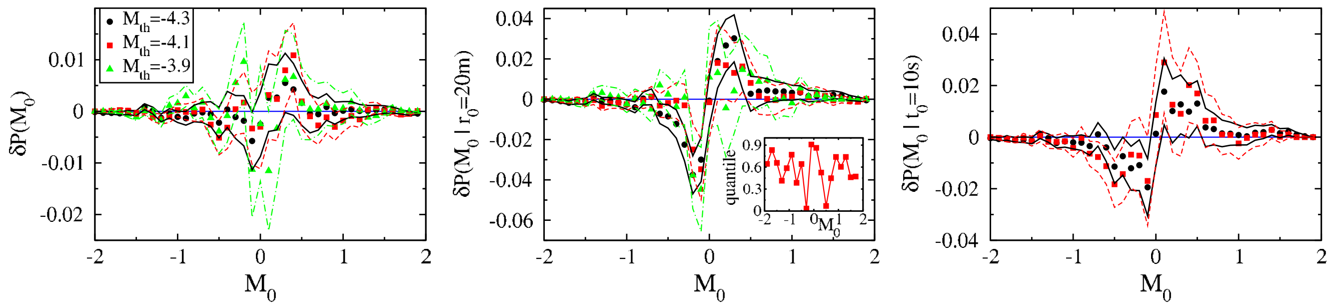
TABLE I. Number of events N in the respective subcatalogs of the aftershock sequence studied here.

M_{th}	r_0	t_0	N
-4.3	∞	∞	4242
-4.3	20 m	∞	998
-4.3	∞	10 s	1071
-4.1	∞	∞	2549
-4.1	20 m	∞	531
-4.1	∞	10 s	456
-3.9	∞	∞	1352
-3.9	20 m	∞	261

To test for the presence of magnitude correlations, we study the magnitude difference $\Delta M_i = M_{i+1} - M_i$ between subsequent events above a given magnitude threshold M_{th} as in Ref. [32]. If subsequent magnitudes were correlated, the statistical distribution of ΔM should deviate from the distribution of magnitude differences between randomly chosen earthquakes above the same M_{th}. The latter distribution can be obtained by defining the magnitude differences as $\Delta M_i^* = M_{i^*} - M_i$ where each i^* is a random number drawn with equal probability from the set $\{1, \ldots, N\}$ and N is the number of earthquakes considered. Averaging over many different realizations of the series ΔM_i^*, we can not only estimate the distribution of random magnitude differences but also the expected deviations from it for a given N [46]. Specifically, we consider here the differences in the (cumulative) distributions $\delta P(M_0) = P(\Delta M < M_0) - P(\Delta M^* < M_0)$ for different magnitude thresholds M_{th}. In the absence of correlations, $\delta P(M_0)$ should not significantly deviate from 0 for all M_0.

$\delta P(M_0)$ is shown in Fig. 1(a) for different magnitude thresholds M_{th} that are greater than or equal to the established magnitude of completeness $M_C = -4.3$. The vast majority of data points are within 1 standard deviation, σ, of zero. None of them is further away than two σ—independent of the magnitude threshold M_{th}. Thus, there is no significant evidence for the presence of magnitude correlations [47]. Moreover, no systematic dependence on the magnitude threshold is visible—in contrast to what one would expect for catalog incompleteness as discussed in Ref. [32]. This provides additional support for the high level of completeness of the considered catalog.

To further take into account the effect of spatial and/or temporal proximity, we also consider the differences in the *conditional* distributions $\delta P(M_0|r_0) = P(\Delta M < M_0|\Delta r < r_0) - P(\Delta M^* < M_0|\Delta r < r_0)$, where $\Delta M_i^* = M_{i^*} - M_i$ as before and $\Delta r_i = |\vec{r}_{i+1} - \vec{r}_i|$ is the distance between the



(a) (b) (c)

FIG. 1 (color online). (a) Differences in the probability to observe a magnitude difference $M_{i+1} - M_i < M_0$ between the original catalog with magnitude threshold M_{th} and randomized versions, which do not exhibit any magnitude correlations—see text for a detailed discussion of the method. Magnitude correlations correspond to significant deviations from zero. The solid, dashed, and dash-dotted curves correspond to one σ error bars for $M_{\text{th}} = -4.3$, -4.1, and -3.9, respectively. The different numbers of events are summarized in Table I. (b) Similar to Fig. 1(a) but only considering those pairs of consecutive earthquakes that are separated by less than 20 m. Inset: quantile of the distribution of the values of $P(M_0 - 0.1 < \Delta M^* < M_0 + 0.1|20\text{ m})$ that corresponds to the value of $P(M_0 - 0.1 < \Delta M < M_0 + 0.1|20\text{ m})$ for $M_{\text{th}} = -4.1$. (c) Similar to Fig. 1(a) but only considering those pairs of consecutive earthquakes that are separated by less than 10 s.

hypocenters of subsequent earthquakes. The high spatial resolution of our catalog allows us to study hypocenters in contrast to studies of standard seismicity that have focused on epicenters [31,32]. Note that each M_{i^*} is now a magnitude randomly drawn from the reduced set $\{M_i|\Delta r_i < r_0\}$. Analogously, we define $\delta P(M_0|t_0)$.

Figure 1(b) shows $\delta P(M_0|r_0)$ for $r_0 = 20$ m. Overall, it allows us to draw conclusions similar to the unconditional case. Namely, significant magnitude correlations independent of M_{th} are absent. Only for $M_0 = -0.2$ and $M_0 = -0.1$, there is an indication of a systematic deviation of $\delta P \approx -0.034$, which is present for all M_{th}. The significance lies between two and four σ depending (non-monotonically) on the magnitude threshold and on M_0. To investigate this further, we compare $P(M_0 - \epsilon < \Delta M < M_0 + \epsilon|r_0)$ for fixed ϵ to the distribution of the values of $P(M_0 - \epsilon < \Delta M^* < M_0 + \epsilon|r_0)$ obtained from the different random realizations of the series ΔM_i^*. The inset of Fig. 1(b) shows the quantile of the latter distribution that corresponds to the value of $P(M_0 - 0.1 < \Delta M < M_0 + 0.1|20\text{ m})$, as a function of M_0. For $M_0 = -0.3$, $P(M_0 - 0.1 < \Delta M < M_0 + 0.1|20\text{ m})$ corresponds to the 3.4% quantile. This indicates that a magnitude difference in the range $[-0.4, -0.2]$ is less likely in the original catalog compared to what is typically expected for the random case. It is, however, not significant at the 93.2% confidence level—assuming a symmetric distribution. Similarly, this is true for the range [0.4, 0.6] at the 86.2% confidence level. For $M_0 = -0.1$ and $M_0 = 0.1$, $P(M_0 - 0.1 < \Delta M < M_0 + 0.1|20\text{ m})$ corresponds to the 90.9% and 86.3% quantile, respectively. Thus, a magnitude difference in the range $[-0.2, 0.2]$ is more likely in the original catalog compared to what is typically expected for the random case. This is similar to the effect of catalog incompleteness [32]. Yet, it is not significant at the 81.8% confidence level. Since the remaining 15 (independent) quantiles in the inset of Fig. 1(b) exhibit variations within the 70% confidence levels, our findings are not sufficient to reject our null hypothesis that the magnitudes are independent of one another. These findings are not specific to the choice of r_0 and ϵ (not shown).

Figure 1(c) provides further evidence for the absence of magnitude correlations even if subsequent events are close in space and/or time. Specifically, $\delta P(M_0|t_0 = 10\text{ s})$ does not show any significant deviation from zero at the two σ level, independent of M_{th}. This includes the cases $M_0 = -0.2$ and $M_0 = -0.1$. Again, all these observations are not specific to the choice of t_0 (not shown). Thus, even if one considers subsequent events that are close in time, our analysis does not show any significant deviation from the null hypothesis of independent magnitudes.

To summarize, our analysis of magnitude correlations in an aftershock sequence of nano- and picoseismicity has not provided any significant evidence for such correlations. This implies that the assumption of independent earthquake magnitudes often used for forecasting seismicity rates and hazard assessment is indeed justified. We would like to point out that the question of magnitude correlations has *per se* nothing to do with the self-similar distribution of magnitudes as described by the GR law. Since the level of catalog completeness is much higher than for typical earthquake catalogs, our findings strongly suggest that earlier claims of the existence of magnitude correlations were purely based on artifacts related to catalog incompleteness, thereby confirming the results presented in Ref. [32].

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[47] We find that this is also true if we consider $\Delta_l M_i = M_{i+l} - M_i$ for $l > 1$.