## Understanding Jet Scaling and Jet Vetos in Higgs Searches

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Jet counting and jet vetos are crucial analysis tools for many LHC searches. We can understand their properties from the distribution of the exclusive number of jets. LHC processes tend to show either a distinct staircase scaling or a Poisson scaling, depending on kinematic cuts. We illustrate our approach in a detailed study of jets in weak boson fusion Higgs production.

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Higgs searches at hadron colliders are typically plagued by large backgrounds. To extract Higgs signals we develop strategies to suppress backgrounds where the structure of QCD effects plays an important role.

Searching for a Higgs boson produced we first reject backgrounds by reconstructing the invariant or transverse mass of the Higgs decay products. Based on the gluonfusion production mechanism, an additional discriminating feature turns out to be the boost of the Higgs boson, i.e., its recoil against a number of quarks or gluons. If the Higgs boson is usually produced with small transverse momentum, we require a large opening angle of its decay products and veto hard jet activity.

Combined Higgs search results based on exactly zero jets and one jet have recently been shown for the LHC [\[1,](#page-3-0)[2](#page-3-1)]. For each case the kinematic extraction cuts can be optimized once we understand how signal and backgrounds fall into these jet bins.

Extending these to two jets will include Higgs bosons produced in weak boson fusion (WBF), i.e., in association with two so-called tagging jets [[3](#page-3-2)]. For this process the color structure of the signal together with the two forward jets forbids gluon exchange between the incoming quarks. Correspondingly, jet radiation into the central detector is suppressed. Requiring exactly two jets is equivalent to a central jet veto as a means to suppress backgrounds [[4\]](#page-3-3).

As another example, the production of a Higgs boson in association with a  $W$  or  $Z$  boson [\[5](#page-3-4)] is mediated by an electroweak process. We therefore do not expect significant jet activity in the signal, in contrast to  $W/Z$  + jets backgrounds. A jet veto makes use of this feature.

In all these cases we need to compute and measure the numbers of jets in signal and background channels. From the theory perspective the requirement of observing exactly  $n_{\text{jets}}$  is problematic [\[6\]](#page-3-5). Any computation based on parton densities which obey the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation is per se jet inclusive; i.e., we always compute cross sections including an unspecified number of collinear jets. In contrast, a jet veto corresponds to the probability

$$
1 - P_{\text{veto}} = \frac{\sigma_0}{\hat{\sigma}_{\text{tot}}} = \frac{\hat{\sigma}_{\text{tot}} - \hat{\sigma}_1}{\hat{\sigma}_{\text{tot}}},\tag{1}
$$

<span id="page-0-0"></span>in terms of the inclusive  $(\hat{\sigma}_n)$  and exclusive  $(\sigma_n)$  n-jetassociated cross sections and  $\hat{\sigma}_0 \equiv \hat{\sigma}_{\text{tot}}$ .<br>The problem with Eq. (1) is that we can

The problem with Eq. [\(1\)](#page-0-0) is that we cannot really include a sensitive detector region for the number of (visible) jets. The collinear limit of the DGLAP equation is not a useful approximation because we only observe jets with finite transverse momentum. In an improved description the initial state parton shower unfolds the parton splittings and produces more or less collinear initial state radiation. The approximate simulation of initial state radiation over the entire phase space critically limits our understanding of the *exclusive*  $n_{\text{iets}}$  distribution and associated veto survival probabilities.

Matrix element and parton shower matching is the key to simulating jet radiation at the LHC [[7](#page-3-6)[–9\]](#page-3-7). It correctly describes the radiation of any number of jets over the entire radiation phase space, for example, for pure QCD jet events [\[10\]](#page-3-8), W, Z or photon production in association with jets [\[10–](#page-3-8)[13\]](#page-3-9), or Higgs production in gluon fusion [\[14\]](#page-3-10) as well as in weak boson fusion.

We propose to study exclusive  $n_{\text{iets}}$  distributions. A jet veto then becomes nothing but a cut on another well understood distribution. It will turn out that jet radiation is usually governed by one of two patterns, staircase scaling and Poisson scaling. We have access to both of them with the current LHC sample sizes. Based on such scaling studies we can for the first time count exclusive jets in a completely testable framework.

<span id="page-0-1"></span>Staircase scaling is defined as constant ratios of successive  $n_{\text{lets}}$  rates, for example, for  $W + \text{jets}$  or pure QCD jets production,

$$
R_{(n+1)/n} = \frac{\sigma_{n+1}}{\sigma_n} \equiv R.
$$
 (2)

The scaling parameter  *depends on the core process and* on the requirements on the jets, but not on  $n$ . This feature has been observed at UA2, Tevatron, and the LHC [\[11,](#page-3-11)[12\]](#page-3-12). Historically, Eq.  $(2)$  $(2)$  is defined inclusively as  $\ddot{R}$ . However, exclusive and inclusive staircase scaling is equivalent with  $K \equiv$ exch  $R \equiv \hat{R}$  [\[10\]](#page-3-8). Following the above argument, we rely on the exclusive formulation to study the  $n_{\text{jets}}$  distribution. From<br>the constant jet ratio  $R$  we can derive the normalized the constant jet ratio  $R$  we can derive the normalized distribution of the exclusive number of jets

$$
\sigma_n = \sigma_0 e^{-bn} \quad \text{with} \quad R \equiv e^{-b}.
$$
 (3)

At the LHC we can study staircase scaling, for example, in  $W$  + jets and  $Z$  + jets production [[10](#page-3-8)[–12](#page-3-12)[,15\]](#page-3-13), in pure QCD jet production [\[10\]](#page-3-8), and in  $\gamma$  + jets production [[13\]](#page-3-9).<br>OCD production of Z + jets is defined as Z radiation off QCD production of  $Z$  + jets is defined as Z radiation off strong jet production  $|\mathcal{M}|^2 \propto \alpha \alpha_s^n$ .<br>To observe staircase scaling it is a

To observe staircase scaling it is crucial to consider total cross sections with as few kinematic cuts as possible. While the reason for this scaling behavior from first principles is not entirely clear, we know that it is closely linked with the non-Abelian nature of QCD. In our simulations we see that the majority of jets arise through initial state radiation (ISR) off the parton entering the hard process. This radiation mediates between the virtuality scales of a parton inside the proton and the parton entering the hard process. The large jet multiplicities constituting staircase scaling are then driven by further splitting of very few ISR quarks or gluons; i.e., we are sensitive to final state radiation patterns starting from hard ISR. This splitting ISR feature is driven by multiple gluon splitting; i.e., it only occurs for non-Abelian massless gauge bosons with a selfcoupling.

In Fig. [1](#page-1-0) we see that with an approximately constant  $R$ , SHERPA [[16](#page-4-0)] based on CKKW matching [\[8\]](#page-3-14) with at least four hard jets correctly reproduces experimental observations. Note that the quoted values for  $(n + 1)/n$  are counted in addition to the two tagging jets which define our core process.

<span id="page-1-0"></span>

FIG. 1 (color online). Simulated  $R_{(n+1)/n}$  distributions for Z + jets and  $H +$  jets production via the effective g-g-H coupling. We only apply the basic cuts  $p_{Ti} > 20(30)$  GeV and  $|y_i| < 4.5$ for the solid (dotted) entries. The quoted  $n$  values are in addition to the two leading jets.

A proper analysis of the theoretical uncertainties shows that this scaling behavior is not affected [\[10\]](#page-3-8). Varying  $\alpha_s(m_Z) = 0.114 - 0.122$  [\[17\]](#page-4-1) merely shifts R. A consistent variation of all renormalization, factorization, and shower scales also leaves the scaling feature untouched, but with a large enough shift in  $R$  that such a shift should be viewed as a tuning parameter for jet merging [[10\]](#page-3-8). Fixed-order QCD corrections leave the scaling untouched [\[18\]](#page-4-2). The only complication we observe in Fig. [1](#page-1-0) is the variation of the first entry  $R_{1/0}$  which corresponds to a need to properly define the hard process. Counting only jets above 30 GeV instead of 20 GeV alleviates this problem.

In Fig. [2](#page-1-1) we show the  $R_{(n+1)/n}$  distribution for the WBF Higgs signal as well as the electroweak  $Z +$  jets channel at order  $|\mathcal{M}|^2 \propto \alpha^3 \alpha_s^n$ . From signal and background studies for WBF Higgs production [3.4] we know that these profor WBF Higgs production [[3](#page-3-2),[4\]](#page-3-3) we know that these processes have different jet radiation patterns. However, for the total cross section with minimal cuts we still observe an approximate staircase scaling  $R_{(n+1)/n} = \text{const.}$  The slight drop in  $R_{(n+1)/n}$  is due to different classes of Feynman diagrams contributing to the electroweak process at different jet multiplicities, including WBF topologies, Z bremsstrahlung, and  $WW/WZ$  pair production.

If we want to use jet scaling for Higgs searches, we need to consider background suppression cuts. As an illustrative example we study WBF Higgs production  $qq \rightarrow qqH$  with a decay  $H \to \tau \tau$  or  $H \to WW$  [[3\]](#page-3-2). Fixed-order corrections to the inclusive rate are known to be moderate [\[19\]](#page-4-3). We are only interested in the production process so neglect the Higgs decay products entirely. Starting from exactly two tagging jets defined as the hardest jets with

<span id="page-1-2"></span>
$$
p_{T,j} > 20 \text{ GeV}, \t |y_j| < 4.5, \t y_1 y_2 < 0,
$$
  

$$
|y_1 - y_2| > 4.4, \t m_{jj} > 600 \text{ GeV},
$$
 (4)

<span id="page-1-3"></span>we compute the jet activity in the to-be-vetoed region

 $p_T^{\text{veto}} > 20 \text{ GeV},$  $\min y_1$ ,  $y \leq y^{\text{veto}} \leq \max y_1$ , (5)

<span id="page-1-1"></span>

FIG. 2 (color online).  $R_{(n+1)/n}$  distributions for electroweak<br>(FW)  $Z +$  jets production and WBF Higgs production. Again (EW)  $Z + \text{jets}$  production and WBF Higgs production. Again,<br>we only apply  $n_{\text{max}} > 20 \text{ GeV}$  and  $|y_1| < 4.5$  and count n in we only apply  $p_{Ti} > 20$  GeV and  $|y_i| < 4.5$  and count *n* in addition to the two leading jets.

Instead of the two hardest jets fulfilling Eq. ([4\)](#page-1-2), we can also use the most forward jets without our conclusions being affected. In Fig. [3](#page-2-0) we first show the signal-like processes, now only counting jets inside the veto region Eq. ([4\)](#page-1-2). These cuts reduce backgrounds while capturing the signal properties, most notably the large invariant mass of the tagging jets. Therefore, they do not have a major effect on weak boson fusion and electroweak Z production, and the approximate staircase scaling persists. The suppression of the first additional jets merely reflects the suppression due to the color structure combined with the strictly enforced tagging jet structure. Slight deviations from a perfect staircase scaling are also expected because the WBF cuts in Eq. ([4](#page-1-2)) are given by the LHC analyses and not optimized to test scaling features.

<span id="page-2-2"></span>Poisson scaling is a different scaling property which we observe, for example, in QCD  $Z$  + jets production after cuts. For a limited number of emissions up to  $n^{\text{crit}}$ , it reads

$$
R_{(n+1)/n} = \frac{\sigma_{n+1}}{\sigma_n} = \frac{\bar{n}}{n+1} \quad \text{for } n < n^{\text{crit}}, \qquad (6)
$$

in terms of  $\bar{n}$ , the number of jets expected. Statistically, a Poisson distribution represents the number of positive outcomes of independent trials. We observe it in multiple soft photon radiation, for example, off an electron [\[20\]](#page-4-4). Its theoretical derivation rests on two features: first, one splitting dominates, e.g., successive photon radiation off an electron or gluon radiation off a quark; second, soft radiation is automatically ordered by the radiation angle, which means there is no combinatorial factor.

From the previous discussion we know that such a radiation pattern does not occur for total jet rates in proton-proton collisions. The dominant splitting ISR via the gluon self-coupling instead leads to staircase scaling.

This changes once we apply cuts forcing our events into a specific kinematic configuration. The form of the Poisson distribution is reminiscent of Sudakov factors, a solution to the DGLAP equation on which the parton shower approach

<span id="page-2-0"></span>

FIG. 3 (color online).  $R_{(n+1)/n}$  distributions for electroweak  $Z +$  jets production and Higgs production in weak boson fusion. Unlike in Fig. [2](#page-1-1) we now count only additional jets in the veto region defined by Eq. ([5](#page-1-3)).

is based. Such Sudakov factors model the leading nonsplitting probabilities, for example, of partons entering the hard process coming from the proton. The approximation underlying the DGLAP equation is collinearity and a sizable difference in virtualities between partons inside the proton and the hard process. It then corresponds to resumming the collinear logarithm arising from successive ISR.

A similar kinematic situation we enforce through the WBF cuts Eq.  $(4)$  $(4)$  $(4)$ : the partons have to generate the large invariant mass  $m_{ij} > 600$  GeV which favors quarks in the initial state. These quarks can most efficiently evolve via successive soft-collinear gluon emission; i.e., they show the same radiation pattern as soft photons being radiated off a hard electron. The moment such a large (collinear) logarithm enhances jet radiation we observe Poisson scaling of the jet ratios in QCD Z + jets and  $H$  + jets, Fig. [4.](#page-2-1) Values of  $R_{1/0} \gg 1$  reflect a much higher probability to observe jet radiation.

Because Poisson scaling only enhances radiation as long as the large logarithm dominates the kinematics, for large  $n_{\text{jets}}$  we do not approach zero, but the staircase limit. Fitting Poisson curves to the first three bins in Fig. [4](#page-2-1) gives us  $\bar{n}$  = 1.4 for QCD Z production and  $\bar{n} = 1.8$  for Higgs production in gluon fusion. The last two bins follow a staircase pattern. A comparison with Fig. [2](#page-1-1) confirms that we see consistent  *values in the staircase setup and the high*multiplicity Poisson regime.

Comparing the QCD and electroweak  $Z +$  jets production processes, we observe a very clear difference. While the staircase scaling property is indeed slightly sculpted due to the different subprocesses, the Poisson shape for the QCD process significantly enhances the first two jet radiations.

Among other observables, Table [I](#page-3-15) tells us how to compute a jet veto survival probability from the exclusive jet scaling. Based on our numerical results we cannot expect to simply extract the two scaling parameters  $b$  and  $\bar{n}$  and

<span id="page-2-1"></span>

FIG. 4 (color online).  $R_{(n+1)/n}$  distributions for Z + jets production and Higgs production via the effective gluon-gluon-Higgs coupling. Unlike in Fig. [1](#page-1-0) we now count only additional jets in the veto region defined by Eq. ([5\)](#page-1-3). The curves are fits to Eq. [\(6\)](#page-2-2).

<span id="page-3-15"></span>TABLE I. Observables for staircase and Poisson scaling. The ratios R and  $\hat{R}$  correspond to the exclusive and inclusive  $n_{\text{jets}}$ distribution, respectively.  $\Gamma(n, \bar{n})$  is the upper incomplete gamma<br>function. In the last line we show the jet veto survival probability function. In the last line we show the jet veto survival probability  $P_{\text{veto}} \equiv \sigma_0/\hat{\sigma}_{\text{tot}}.$ 

	Staircase scaling	Poisson scaling
$\sigma_n$ $R_{(n+1)/n}$	$\sigma_0 e^{-bn}$	$\hat{\sigma}_0 \frac{e^{-\bar{n}} \bar{n}^n}{\frac{\bar{n}}{n+1}}$
	$\rho^{-b}$	$(\frac{(n+1)e^{-\bar n} \bar n^{-(n+1)}}{\Gamma(n+1)-n \Gamma(n,\bar n)}+1)^{-1}$
$\hat{R}_{(n+1)/n}$ $\langle n_{\text{jets}} \rangle$	$\overline{2} \cosh b - 1$	
$P_{\text{veto}}$	$1 - e^{-b}$	$e^{-\bar{n}}$

insert them into the resummed form for  $P_{\text{veto}}$ . On the other hand, the distinct patterns shown in Table [I](#page-3-15) allow us to study the  $n<sub>jets</sub>$  distributions before and after cuts, validate the appropriate simulation, and quantify the scaling ratios  $R_{(n+1)/n}$  including their uncertainties. For WBF signal-like processes, where a measurement is difficult, it is crucial that we can understand and simulate the staircase scaling pattern which is typical for inclusive LHC processes.

Outlook.—In this Letter we have laid out a strategy to understand jet counting and the associated veto survival probabilities. The tool behind this study is the exclusive  $n<sub>jets</sub>$  distribution, as currently measured in several processes at the LHC.

There exist two scaling types for exclusive jet rates at the LHC. First, staircase scaling for inclusive as well as for exclusive jet rates is defined by a constant ratio  $R =$  $\sigma_{n+1}/\sigma_n$ . It occurs for total cross sections, for example, in  $W/Z/\gamma$  + jets and pure QCD jet production [[10](#page-3-8)[,12,](#page-3-12)[13\]](#page-3-9) and is due to non-Abelian gluon splitting. In Higgs proand is due to non-Abelian gluon splitting. In Higgs production staircase scaling is realized by all signal and background inclusive rates and by the signal and electroweak Z production after tagging jet cuts.

Second, Poisson scaling appears for processes in which jet radiation is enhanced by a large logarithm. It corresponds to successive gauge boson radiation. In the past, this scaling has been used to describe all signal and back-ground processes after tagging jet cuts [\[3](#page-3-2),[4](#page-3-3)]. For QCD  $Z +$ jets and gluon-fusion Higgs production we confirm this result after requiring two tagging jets.

The main effect of these two scaling patterns is visible in the first emission. While for electroweak processes we find  $R_{1/0}=0.15$ –0.3, typical QCD processes after WBF cuts range around  $R_{1/0} \sim 1.5$ –2.5. An exclusive jet requirement or central jet veto is a powerful tool to suppress backgrounds, provided that kinematic cuts drive the backgrounds into a Poisson regime and leave the signal of staircase type.

An understood  $n_{\text{jets}}$  distribution with and without Poisson-inducing cuts allows us to carefully study all aspects of jet counting, including experimental and theoretical uncertainties. It should enable LHC searches which otherwise are plagued by severe theoretical uncertainties.

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- <span id="page-3-1"></span><span id="page-3-0"></span>[1] ATLAS Collaboration, Report No. ATLAS-CONF-2011- 111 (to be published).
- <span id="page-3-2"></span>[2] CMS Collaboration, Report No. CMS-HIG-11-003 (to be published).
- <span id="page-3-3"></span>[3] D. L. Rainwater, D. Zeppenfeld, and K. Hagiwara, [Phys.](http://dx.doi.org/10.1103/PhysRevD.59.014037) Rev. D 59[, 014037 \(1998\);](http://dx.doi.org/10.1103/PhysRevD.59.014037) T. Plehn, D. L. Rainwater, and D. Zeppenfeld, Phys. Rev. D 61[, 093005 \(2000\).](http://dx.doi.org/10.1103/PhysRevD.61.093005)
- [4] U. Baur and E.W.N. Glover, [Phys. Lett. B](http://dx.doi.org/10.1016/0370-2693(90)90506-2) 252, 683 [\(1990\)](http://dx.doi.org/10.1016/0370-2693(90)90506-2); V. D. Barger, K. Cheung, T. Han, and D. Zeppenfeld, Phys. Rev. D 44[, 2701 \(1991\)](http://dx.doi.org/10.1103/PhysRevD.44.2701); V. D. Barger, R. J. N. Phillips, and D. Zeppenfeld, [Phys. Lett. B](http://dx.doi.org/10.1016/0370-2693(95)00008-9) 346, 106 [\(1995\)](http://dx.doi.org/10.1016/0370-2693(95)00008-9); D. L. Rainwater, R. Szalapski, and D. Zeppenfeld, Phys. Rev. D 54[, 6680 \(1996\);](http://dx.doi.org/10.1103/PhysRevD.54.6680) D. L. Rainwater, D. Summers, and D. Zeppenfeld, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.55.5681) 55, 5681 [\(1997\)](http://dx.doi.org/10.1103/PhysRevD.55.5681); T. Figy, V. Hankele, and D. Zeppenfeld, [J. High](http://dx.doi.org/10.1088/1126-6708/2008/02/076) [Energy Phys. 02 \(2008\) 076](http://dx.doi.org/10.1088/1126-6708/2008/02/076);B. E. Cox, J. R. Forshaw, and A. D. Pilkington, [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2010.12.011) 696, 87 (2011).
- <span id="page-3-5"></span><span id="page-3-4"></span>[5] J. M. Butterworth, A. R. Davison, M. Rubin, and G. P. Salam, Phys. Rev. Lett. 100[, 242001 \(2008\).](http://dx.doi.org/10.1103/PhysRevLett.100.242001)
- <span id="page-3-6"></span>[6] For recent theory developments, see I.W. Stewart and F.J. Tackmann, [arXiv:1107.2117.](http://arXiv.org/abs/1107.2117)
- [7] For an early correct description of one jet over its entire  $p_T$ range, see M. Bengtsson and T. Sjostrand, [Phys. Lett. B](http://dx.doi.org/10.1016/0370-2693(87)91031-8) 185[, 435 \(1987\);](http://dx.doi.org/10.1016/0370-2693(87)91031-8) G. Miu and T. Sjostrand, [Phys. Lett. B](http://dx.doi.org/10.1016/S0370-2693(99)00068-4) 449[, 313 \(1999\)](http://dx.doi.org/10.1016/S0370-2693(99)00068-4); M. H. Seymour, [Nucl. Phys.](http://dx.doi.org/10.1016/0550-3213(94)00554-R) B436, 443 [\(1995\)](http://dx.doi.org/10.1016/0550-3213(94)00554-R); G. Corcella and M. H. Seymour, [Nucl. Phys.](http://dx.doi.org/10.1016/S0550-3213(99)00672-0) B565[, 227 \(2000\)](http://dx.doi.org/10.1016/S0550-3213(99)00672-0).
- <span id="page-3-14"></span><span id="page-3-7"></span>[8] S. Catani, F. Krauss, R. Kuhn, and B. R. Webber, [J. High](http://dx.doi.org/10.1088/1126-6708/2001/11/063) [Energy Phys. 11 \(2001\) 063](http://dx.doi.org/10.1088/1126-6708/2001/11/063); S. Höche, F. Krauss, S. Schumann, and F. Siegert, [J. High Energy Phys. 05](http://dx.doi.org/10.1088/1126-6708/2009/05/053) [\(2009\) 053.](http://dx.doi.org/10.1088/1126-6708/2009/05/053)
- <span id="page-3-8"></span>[9] M. L. Mangano, M. Moretti, and R. Pittau, [Nucl. Phys.](http://dx.doi.org/10.1016/S0550-3213(02)00249-3) B632[, 343 \(2002\)](http://dx.doi.org/10.1016/S0550-3213(02)00249-3).
- <span id="page-3-11"></span>[10] C. Englert, T. Plehn, P. Schichtel, and S. Schumann, [Phys.](http://dx.doi.org/10.1103/PhysRevD.83.095009) Rev. D 83[, 095009 \(2011\)](http://dx.doi.org/10.1103/PhysRevD.83.095009).
- [11] G. Aad et al. (ATLAS Collaboration), [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2011.03.012) 698, [325 \(2011\);](http://dx.doi.org/10.1016/j.physletb.2011.03.012) [arXiv:1106.2061](http://arXiv.org/abs/1106.2061); [J. High Energy Phys. 09](http://dx.doi.org/10.1007/JHEP09(2011)053) [\(2011\) 053](http://dx.doi.org/10.1007/JHEP09(2011)053); [Eur. Phys. J. C](http://dx.doi.org/10.1140/epjc/s10052-011-1763-6) 71, 1763 (2011); CMS Collaboration, [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2011.07.067) 702, 336 (2011); V. M. Abazov et al. (D0 Collaboration), [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2009.11.012) 682, [370 \(2010\)](http://dx.doi.org/10.1016/j.physletb.2009.11.012); T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 100[, 102001 \(2008\).](http://dx.doi.org/10.1103/PhysRevLett.100.102001)
- <span id="page-3-12"></span>[12] S. D. Ellis, R. Kleiss, and W. J. Stirling, [Phys. Lett. B](http://dx.doi.org/10.1016/0370-2693(85)90425-3) 154, [435 \(1985\)](http://dx.doi.org/10.1016/0370-2693(85)90425-3); F. A. Berends, W. T. Giele, H. Kuijf, R. Kleiss, and W. J. Stirling, [Phys. Lett. B](http://dx.doi.org/10.1016/0370-2693(89)91081-2) 224, 237 (1989); F. A. Berends, H. Kuijf, B. Tausk, and W. T. Giele, [Nucl. Phys.](http://dx.doi.org/10.1016/0550-3213(91)90458-A) B357[, 32 \(1991\).](http://dx.doi.org/10.1016/0550-3213(91)90458-A)
- <span id="page-3-10"></span><span id="page-3-9"></span>[13] S. Höche, S. Schumann, and F. Siegert, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.81.034026) 81, [034026 \(2010\);](http://dx.doi.org/10.1103/PhysRevD.81.034026) C. Englert, T. Plehn, P. Schichtel, and S. Schumann (to be published).
- <span id="page-3-13"></span>[14] J.M. Butterworth *et al.*, [arXiv:1003.1643.](http://arXiv.org/abs/1003.1643)
- [15] V. M. Abazov et al. (D0 Collaboration), [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2007.10.046) 658, [112 \(2008\);](http://dx.doi.org/10.1016/j.physletb.2007.10.046) T. Aaltonen et al. (CDF-Run II Collaboration), Phys. Rev. Lett. 100[, 102001 \(2008\)](http://dx.doi.org/10.1103/PhysRevLett.100.102001).
- <span id="page-4-0"></span>[16] T. Gleisberg, S. Höche, F. Krauss, M. Schönherr, S. Schumann, F. Siegert, and J. Winter, [J. High Energy](http://dx.doi.org/10.1088/1126-6708/2009/02/007) [Phys. 02 \(2009\) 007.](http://dx.doi.org/10.1088/1126-6708/2009/02/007)
- <span id="page-4-1"></span>[17] H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. M. Nadolsky, J. Pumplin, and C.-P. Yuan, Phys. Rev. D 82[, 074024 \(2010\).](http://dx.doi.org/10.1103/PhysRevD.82.074024)
- <span id="page-4-2"></span>[18] C. F. Berger et al., Phys. Rev. Lett. **106**[, 092001 \(2011\)](http://dx.doi.org/10.1103/PhysRevLett.106.092001).
- <span id="page-4-3"></span>[19] T. Han, G. Valencia, and S. Willenbrock, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.69.3274) 69[, 3274 \(1992\)](http://dx.doi.org/10.1103/PhysRevLett.69.3274); T. Figy, D. Zeppenfeld, and C. Oleari,

Phys. Rev. D 68[, 073005 \(2003\);](http://dx.doi.org/10.1103/PhysRevD.68.073005) M. Ciccolini, A. Denner, and S. Dittmaier, Phys. Rev. Lett. 99[, 161803 \(2007\)](http://dx.doi.org/10.1103/PhysRevLett.99.161803); Phys. Rev. D 77[, 013002 \(2008\);](http://dx.doi.org/10.1103/PhysRevD.77.013002) P. Bolzoni, F. Maltoni, S. O. Moch, and M. Zaro, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.105.011801) 105, 011801 [\(2010\)](http://dx.doi.org/10.1103/PhysRevLett.105.011801).

<span id="page-4-4"></span>[20] M. E. Peskin and D. V. Schroeder, An Introduction to Quantum Field Theory (Addison-Wesley, Reading, MA, 1995), p. 842.