State-Independent Proof of Kochen-Specker Theorem with 13 Rays

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Quantum contextuality, as proved by Kochen and Specker, and also by Bell, should manifest itself in any state in any system with more than two distinguishable states and recently has been experimentally verified. However, for the simplest system capable of exhibiting contextuality, a qutrit, the quantum contextuality is verified only state dependently in experiment because too many (at least 31) observables are involved in all the known state-independent tests. Here we report an experimentally testable inequality involving only 13 observables that is satisfied by all noncontextual realistic models while being violated by all qutrit states. Thus our inequality facilitates a state-independent test of the quantum contextuality for an indivisible quantum system. We also provide a record-breaking state-independent proof of the Kochen-Specker theorem with 13 directions determined by 26 points on the surface of a magic cube.

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It is believed, almost religiously, that every effect has its own cause and the same cause shall lead to the same effect. The predictions of quantum mechanics (QM) are however probabilistic and the effect that different outcomes appear in different runs of a measurement seems to have no definite cause. Einstein, Podolsky, and Rosen [1] initiated a long-lasting quest for a quantum reality by questioning the completeness of quantum mechanics. Hidden variable (HV) models are introduced in order to explain why a certain outcome appears in each run of a measurement, attempting to make QM complete. Years later Kochen and Specker (KS) [2] and Bell [3] discovered that quantum mechanics can be completed only by a hidden variable model that is *contextual*: the outcome of a measurement depends on which compatible observable might be measured alongside. Simply put, the Kochen-Specker theorem states that noncontextual HV models cannot reproduce all the predictions of QM, or quantum mechanics is contextual.

In any noncontextual HV model all observables have definite values determined only by some HVs λ that are distributed according to a given probability distribution ϱ_λ with normalization $\int d\lambda \varrho_\lambda = 1$. Two observables are compatible if they can be measured in a single experimental setup and a maximal set of mutually compatible observables defines a context. Noncontextuality is a typical classical property: the value of an observable revealed by a measurement is predetermined by HVs λ only regardless of which compatible observable might be measured alongside. Local realism is a form of noncontextuality enforced by the locality, and thus Bell's inequalities [4] are a special form of KS inequalities [5–9], experimentally testable inequalities that are satisfied by all noncontextual HV models, some of which have been tested in recent experiments [10–18]. In general KS inequalities reveal the nonclassical nature of single systems demanding neither spacelike separation nor entanglement, i.e., independent of state.

However, for the simplest system capable of exhibiting the quantum contextuality, a qutrit, only a state-dependent verification has been made in a recent experiment [18]. This is because all the known state-independent KS inequalities for a qutrit [9] are based on the proofs of the KS theorem and involve too many observables to be tested practically. For example, the original KS proof involves 117 rays [2] and the number of rays is reduced to 33 by Peres [19] and Schuttle as reported by Svozil in 1994 and pointed out by Bub [20]. The best KS proof known so far, due to Conway and Kochen [21], still involves 31 rays. Also there are many state-dependent KS proofs among which a 5-ray proof [7] has been used in the recent state-dependent experimental verification of quantum contextuality for a qutrit. In this Letter we report a state-independent proof of the KS theorem for a qutrit with only 13 directions. Based on this proof we propose an experimental testable inequality that involves only 13 observables and two observable correlations and is satisfied by all noncontextual HV models while being violate by all qutrit states. Thus our inequality will make a state-independent test of quantum contextuality for an indivisible system practical.

How can we exclude noncontextual HV models for QM or prove the quantum contextuality? The answer depends on what kinds of quantum mechanical predictions we want the HV model to reproduce. For example, if we only want the predictions on nonsequential measurements to be reproduced, then a noncontextual HV model does exist according to Kochen and Specker [2]. Because of this toy model Kochen and Specker imposed a rather strong constraint on the HV models as a way out [2]: the algebraic structure of compatible observables must be preserved. That is to say the value assigned to the product or the sum of two compatible observables must be equal to the product or the sum of the values assigned to these two compatible observables, which will be referred to as the product rule and the sum rule, respectively. As we shall see later this constraint can be lifted if we consider sequential measurements.

In what follows we shall slightly abuse the notation of a ray to represent both a vector in the projective Hilbert space and a normalized rank-1 projector on the ray since they are in a one-to-one correspondence. As a result of the product rule the value assigned to the product of two orthogonal rays, which are compatible, must be zero. As a result of the sum rule there is one and only one ray that is assigned to value 1 among all the rays in a complete orthonormal basis since the identity is always assigned to value 1. Thus in every noncontextual HV model preserving the partial algebraic structure of compatible observables there exists a KS value assignment to all rays in the corresponding Hilbert space satisfying the following: (1) The value $\{0, 1\}$ assigned to a ray is independent of which bases it finds itself in, and (2) one and only one ray is assigned to value 1 among all the rays in a complete orthonormal basis. The first condition reflects the noncontextuality and the second condition arises from the requirement that the algebraic structure of compatible observables be preserved. For a Hilbert space of a dimension greater than 2 there always exists a finite set of rays to which the KS value assignment is impossible. In addition to various KS proofs for qutrits [2,19,20], the 18-ray proof in four dimensions due to Cabello, Estebaranz, and García-Alcaine [22] is the smallest state-independent KS proof known so far.

First let us present a state-independent proof of the KS theorem for a qutrit using only 13 rays. In a given basis $\{|0\rangle, |1\rangle, |2\rangle\}$ each ray $r = (a, b, c)$ is in a one-to-one correspondence to a normalized rank-1 qutrit projector $\hat{r} = |r\rangle\langle r|/\langle r|r\rangle$ in which $|r\rangle = a|0\rangle + b|1\rangle + c|2\rangle$. Consider the following 13 rays

$$\begin{aligned}
&y_1^- = (0, 1, -1) \quad && h_1 = (-1, 1, 1) \quad && z_1 = (1, 0, 0)\\
&y_2^- = (-1, 0, 1) && h_2 = (1, -1, 1) && z_2 = (0, 1, 0)\\
&y_3^- = (1, -1, 0) && h_3 = (1, 1, -1) && z_3 = (0, 0, 1)\\
&y_1^+ = (0, 1, 1) && h_0 = (1, 1, 1) &&\\
&y_2^+ = (1, 0, 1) && &&\\
&y_3^+ = (1, 1, 0) && &&
\end{aligned} \tag{1}$$

that are determined by 26 points on the surface of a 3×3 magic cube as illustrated in Fig. 1. If we regard those 13 rays as 13 vertices and link two vertices if and only if the corresponding rays are orthogonal, then we obtain the orthogonality graph Δ_{13} as shown in Fig. 2. Obviously a given set of rays determines uniquely the orthogonality graph and usually not vice versa. However, those 13 rays are determined uniquely by the orthogonality relationships specified by the graph Δ_{13} up to a global unitary transformation.

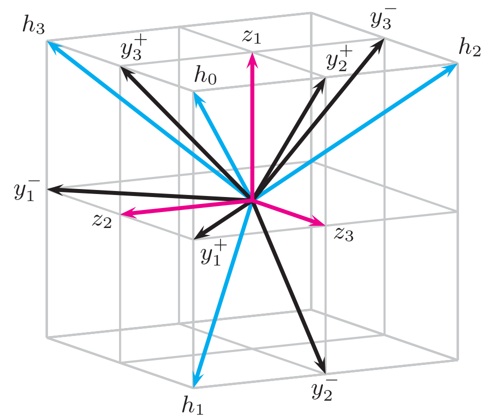

FIG. 1 (color online). Illustration of 13 directions that are determined by 26 points on the surface of a 3×3 magic cube.

In fact, without loss of generality we can choose z_k as in Eq. (1) since they form a basis. Because $\{z_k, y_k^\pm\}$ are mutually orthogonal for each $k = 1, 2, 3$ there exist nonzero t_1, t_2, t_3 such that $y_1^+ = (0, t_1, 1)$ and $y_1^- = (0, -1, t_1^*)$, $y_2^+ = (1, 0, t_2)$ and $y_2^- = (t_2^*, 0, -1)$, $y_3^+ = (t_3, 1, 0)$ and $y_3^- = (-1, t_3^*, 0)$. As a result we have $h_1 = (-t_2^*, t_1, 1)$, $h_2 = (1, -t_3^*, t_2)$, and $h_3 = (t_3, 1, -t_1^*)$. Since h_k is orthogonal to y_{k-1}^+ for $k = 1, 2, 3$, we have $t_1^* = t_2 t_3$, $t_2^* = t_1 t_3$, and $t_3^* = t_1 t_2$ from which it follows that $|t_k| = 1$ and $t_1 t_2 t_3 = 1$, i.e., $t_k = e^{i(\theta_{k+1} - \theta_{k+2})}$ for some real θ_k. Finally we obtain $h_0 = (e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3})$ which is orthogonal to $y_{1,2,3}^-$. The diagonal unitary transformation taking h_0 to $(1, 1, 1)$ leaves z_k unchanged so that the standard form of 13 rays in Eq. (1) is obtained.

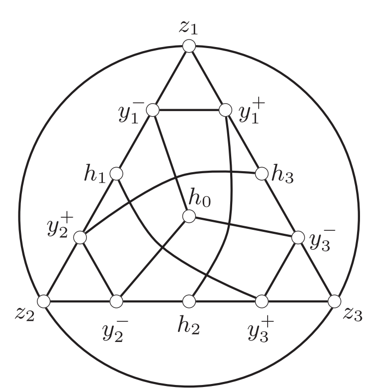

FIG. 2. The orthogonality relationships among the 13 rays in Eq. (1) determine a graph Δ_{13} with 13 vertices (hollow dots) representing the 13 rays and edges, straight or curved, linking two rays that are orthogonal.

The KS value assignments to the 13-ray set are possible; i.e., no logical contradiction can be extracted by considering conditions 1 and 2 only. However, in any possible KS value assignment there is at most one ray among $\{\hat{h}_\alpha|\alpha = 0, 1, 2, 3\}$ that can be assigned to value 1. Suppose that this is not true; i.e., there are two or more $\hat{h}_\alpha$ that are assigned to value 1. Because of the symmetry of the graph Δ_{13} as shown in Fig. 2, we need only to consider the following two cases: (i) If $\hat{h}_0$ and $\hat{h}_1$ are assigned to value 1 then $\hat{y}_2^\pm$ and $\hat{y}_3^\pm$ must be assigned to 0 so that both $\hat{z}_2$ and $\hat{z}_3$ must be assigned to value 1 which is impossible, and (ii) if $\hat{h}_1$ and $\hat{h}_2$ are assigned to 1 then $\hat{y}_1^\pm$ and $\hat{y}_2^\pm$ must be zero so that both $\hat{z}_1$ and $\hat{z}_2$ must be assigned to value 1, also a contradiction. In the reasonings above we have taken into account condition 2 which demands that linked rays not be assigned simultaneously to value 1 and in a triangle one and only one ray be assigned to value 1. A set of eight rays in each case considered above, e.g., $\hat{h}_{0,1}$, $z_{2,3}$, and $y_{2,3}^\pm$, constitutes in fact a 3-box paradox [23] which also appears in Cliffton's state-dependent proof of the KS theorem [24] and a recent proposal to close the compatibility loopholes [25], in addition to its first use in the 117-ray proof [2].

Denoting by $h_\alpha^\lambda \in \{0, 1\}$ the value assigned to $\hat{h}_\alpha$ for given λ, we have $\sum_\alpha h_\alpha^\lambda \leq 1$ from the above arguments. As a result the following inequality

$$\sum_{\alpha=0}^{3} \langle \hat{h}_\alpha \rangle_c := \sum_{\alpha=0}^{3} \int d\lambda \varrho_\lambda h_\alpha^\lambda \leq 1 \qquad (2)$$

must be satisfied by all noncontextual HV models that admit a KS value assignment. However, the quantum mechanics predicts $\sum_{\alpha=0}^{3} \langle \hat{h}_\alpha \rangle_q = 4/3$ due to the identity $\sum_{\alpha=0}^{3} \hat{h}_\alpha = \frac{4}{3} I$ with I being the identity operator of qutrit (see the Appendix). This proves the original KS theorem: the noncontextual HV model satisfying conditions 1 and 2 cannot reproduce all the predictions of QM.

Usually the KS theorem is proved by finding a set of rays to which the KS value assignment does not exist so that we need not check other predictions of QM. Our proof here is a set of 13 rays, to which all possible KS value assignments, which do exist, fail to reproduce a certain prediction of QM. Notably the inequality Eq. (2), involving only 4 observables explicitly, provides a state-independent test for the noncontextual HV models that preserve the algebraic structure of compatible observables, in the spirit of Kochen and Specker. In an experimental test of the inequality Eq. (2) each average must be measured on different subensembles prepared in the same state, similar to a standard test of a Bell inequality as pointed out by Cabello [8].

It turns out that the requirement of preserving the algebraic structure, i.e., the product and sum rules, of compatible observables is too strong and unnecessary. Peres noticed that the sum rule can be abandoned and kept the product rule [26] as in the KS proofs via the Mermin-Peres square [19,27] for 2 qubits and Mermin's pentagram [27] for 3 qubits. However, the product rule is not necessary either. Instead we only need to require that the quantum correlations of compatible observables, the quantum mechanical predictions on sequential measurements, be reproduced. This imposes no additional constraints on the noncontextual HV models since they must reproduce all the predictions of QM in the first place. As noticed earlier, the toy model by Kochen and Specker did not take into account the quantum correlations of compatible observables.

Indeed every KS proof mentioned previously can be turned into a state-independent KS inequality [9] that is obeyed by all noncontextual HV models, no matter whether the algebraic structure of compatible observables is preserved or not. All known state-independent KS inequalities involve the correlations of at least three compatible observables; i.e., predictions on three or more sequential measurements must be reproduced by the HV models. In the case of a qutrit the resulting KS inequalities involve too many observables, e.g., at least 31, to be tested experimentally. Recently Klyachko *et al.* [7] proposed a simple KS inequality, called a pentagram inequality since it is based on the graph of a pentagon, to test noncontextual HVs with only 5 dichotomic observables and correlations of two compatible observables. However, this pentagram inequality, which is based on the assumption of noncontextuality only and whose violation has been verified in a recent experiment [18], has a drawback of being state dependent.

Our state-independent inequality is based on the orthogonality graph Δ_{13} as shown in Fig. 2. We denote by $V = \{y_k^\sigma, h_\alpha, z_k | k = 1, 2, 3; \sigma = \pm; \alpha = 0, 1, 2, 3\}$ its vertex set and by Γ its adjacency matrix which is a 13×13 symmetric matrix with vanishing diagonal elements. And $\Gamma_{uv} = 1$ if two vertices $u, v \in V$ are neighbors and $\Gamma_{uv} = 0$ otherwise. For arbitrary 13 variables $a_v = \pm 1$ with $v \in V$ it holds

$$\sum_{v \in V} a_v - \frac{1}{4} \sum_{u,v \in V} \Gamma_{uv} a_u a_v \leq 8, \qquad (3)$$

which can be easily verified with the help of a computer by exhausting all 2^{13} possibilities and an analytic proof is provided in the Appendix. Let $\{A_v | v \in V\}$ be a set of 13 dichotomic observables taking values $a_v^\lambda = \pm 1$ for given λ. Then from inequality Eq. (3) we obtain our magic-cube inequality

$$\sum_{v \in V} \langle A_v \rangle_c - \frac{1}{4} \sum_{u,v \in V} \Gamma_{uv} \langle A_u A_v \rangle_c \leq 8, \qquad (4)$$

where we have denoted $\langle A_v \rangle_c := \int d\lambda \varrho_\lambda a_v^\lambda$ and $\langle A_u A_v \rangle_c := \int d\lambda \varrho_\lambda a_u^\lambda a_v^\lambda$. Though the correlation of two observables is always well defined in a noncontextual HV model regardless of whether they are compatible or not, in order to compare with the predictions of QM those observables labeled with linked vertices in the orthogonality graph Δ_{13} should be compatible (commuting) so that their correlation is also well defined in QM.

Fortunately if we define 13 observables $\hat{A}_v = I - 2\hat{r}_v$ from 13 rays $\hat{r}_v \in \{\hat{y}_k^\sigma, \hat{h}_\alpha, \hat{z}_k\}$, then $\hat{A}_u$ and $\hat{A}_v$ are commuting, i.e., compatible, whenever two vertices $u, v \in V$ are linked, i.e., $\Gamma_{uv} = 1$. Therefore all the expectation values appear on the left-hand side of Eq. (4) are also well defined in QM and therefore should be reproduced. Because of the linearity of QM the left-hand side of Eq. (4) must reproduce the expectation value of

$$\hat{L} = \sum_{v\in V} \hat{A}_v - \frac{1}{4}\sum_{u,v\in V} \Gamma_{uv}\hat{A}_u\hat{A}_v. \tag{5}$$

Simple calculations yield $\hat{L} = \frac{25}{3}I$ and therefore $\langle \hat{L}\rangle_q = 25/3 > 8$ for any qutrit state, meaning that the magic-cube inequality Eq. (4) is violated by all qutrit states. Thus we have proved in a state-independent fashion that no noncontextual HV model, no matter whether the algebraic structure of compatible observables is preserved or not, can reproduce all the predictions of QM, especially those quantum correlations of two compatible observables, by using only 13 observables.

To summarize, our magic-cube inequality Eq. (4) provides the simplest state-independent proof of quantum contextuality. Firstly, it is a test for the contextuality for an indivisible system capable of exhibiting the quantum contextuality, i.e., smallest system. Secondly, it involves the smallest number of observables so far and we conjecture that the KS theorem does not have a state-independent proof with less than 13 rays. Thirdly, only correlations of at most two, the smallest number, of compatible observables are involved. In comparison correlations of at least three compatible observables are involved in all the state-independent tests of contextuality known so far. If we impose a minimal requirement on the noncontextual HV models, i.e., only the correlations of at most two compatible observables are required to be reproduced, then our magic-cube inequality will be the first state-independent proof of the KS theorem in this case. We believe that there are also proofs of the KS theorem for higher dimensional systems that involve only two observable correlations. Finally, we have proved the KS theorem by finding a 13-ray set, which is the smallest state-independent proof so far, for which all possible KS value assignments, which do exist, fail to reproduce a certain prediction of QM. Since only a relatively small number of observables and correlations of only two compatible observables are involved, an experimental test of our inequality is well within the reach of current technologies. We believe that the nitrogen-vacancy center in diamond [28] should be a promising choice to test our inequality.

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Appendix.—

Explicit normalized rank-1 projectors corresponding to 13 rays in Eq. (1): By denoting $\bar{1} = -1$ we have

$$\hat{y}_1^\pm = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \pm 1 \\ 0 & \pm 1 & 1 \end{pmatrix}, \qquad \hat{y}_2^\pm = \frac{1}{2}\begin{pmatrix} 1 & 0 & \pm 1 \\ 0 & 0 & 0 \\ \pm 1 & 0 & 1 \end{pmatrix}, \qquad \hat{y}_3^\pm = \frac{1}{2}\begin{pmatrix} 1 & \pm 1 & 0 \\ \pm 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\hat{h}_0 = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad \hat{h}_1 = \frac{1}{3}\begin{pmatrix} 1 & \bar{1} & \bar{1} \\ \bar{1} & 1 & 1 \\ \bar{1} & 1 & 1 \end{pmatrix}, \qquad \hat{h}_2 = \frac{1}{3}\begin{pmatrix} 1 & \bar{1} & 1 \\ \bar{1} & 1 & \bar{1} \\ 1 & \bar{1} & 1 \end{pmatrix}, \qquad \hat{h}_3 = \frac{1}{3}\begin{pmatrix} 1 & 1 & \bar{1} \\ 1 & 1 & \bar{1} \\ \bar{1} & \bar{1} & 1 \end{pmatrix},$$

$$\hat{z}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \hat{z}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \hat{z}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Thus we have identities

$$\hat{y} := \sum_{k=1}^{3}\sum_{\sigma=\pm} \hat{y}_k^\sigma = 2I, \qquad \hat{h} := \sum_{\alpha=0}^{3} \hat{h}_\alpha = \frac{4}{3}I,$$
$$\hat{z} := \sum_{k=1}^{3} \hat{z}_k = I. \tag{A1}$$

Let $\hat{A}_v = I - 2\hat{r}_v$ with $\hat{r}_v \in \{\hat{y}_k^\sigma, \hat{h}_\alpha, \hat{z}_k\}$. Since $24 = \sum_{u,v\in V}\Gamma_{uv}/2$ is the number of the edges in Δ_{13}, $\sum_{u\in V}\Gamma_{uv}$ is the degree of the vertex v, and $\hat{r}_v\hat{r}_u = 0$ if $\Gamma_{uv} = 1$, it holds

$$\begin{aligned}\hat{L} &= \sum_{v\in V}\hat{A}_v - \frac{1}{4}\sum_{u,v\in V}\Gamma_{uv}\hat{A}_u\hat{A}_v \\ &= 13I - 2(\hat{y} + \hat{h} + \hat{z}) - \frac{I}{4}\sum_{uv}\Gamma_{uv} + \sum_{uv}\Gamma_{uv}\hat{r}_v \\ &\quad - \sum_{u,v\in V}\Gamma_{uv}\hat{r}_u\hat{r}_v \\ &= 13I - 2(\hat{y} + \hat{h} + \hat{z}) - 12I + 4(\hat{z} + \hat{y}) + 3\hat{h} \\ &= I + 2(\hat{z} + \hat{y}) + \hat{h} = \frac{25}{3}I.\end{aligned} \tag{A2}$$

Proof of Eq. (4).—There are 9 vertices $\{z_k, y_k^\sigma\}$ of degree 4 and 4 vertices $\{h_\mu\}$ of degree 3 in Δ_{13}, where the degree of a vertex denotes the number its neighbors. Let t be the total number of a_v that take value -1, f the number of a_v that take value -1 with v being of degree 4, and l the number of pairs of vertices u, v such that $a_u = a_v = -1$ and $\Gamma_{uv} = 1$. We then have $L = 1 + t + f - 2l$. Since the quadratic term in L is unchanged under replacements $a_v \mapsto -a_v$, we need only to consider $t \le 6$. If $t \le 3$, since $l \ge 0$ and $f \le t$, then we have $L \le 7$. If $t = 4$, then from $f - 2l \le 3$ since it is impossible to have $f = 4$ and $l = 0$, i.e., among any 4 vertices of degree 4 there is at least a connected pair, it follows $L \le 8$, which is attained when $f = 3$ and $l = 0$, e.g., z_1, y_2^-, y_3^+, and h_3. In the case of $t = 5$, if $f = 5$ then $l \ge 2$ so that $L \le 7$. If $f \le 4$ then $l \ge 1$ so that $L \le 8$, which is attained by, e.g., z_1, y_2^-, y_3^+, y_1^-, and h_3. In the case of $t = 6$, if $f = 6$ then $l \ge 3$ so that $L \le 7$; if $2 \le f \le 5$ then we have again $L \le 8$ because $l \ge 2$; if $f = 1$ then $L \le 7$ since $l \ge 0$.

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[2] S. Kochen and E. P. Specker, J. Math. Mech. **17**, 59 (1967).
[3] J. S. Bell, Rev. Mod. Phys. **38**, 447 (1966).
[4] J. S. Bell, Physics (Long Island City, N.Y.) **1**, 195 (1964).
[5] C. Simon, Č Brukner, and A. Zeilinger, Phys. Rev. Lett. **86**, 4427 (2001).
[6] J. Å. Larsson, Europhys. Lett. **58**, 799 (2002).
[7] A. A. Klyachko, M. A. Can, S. Binicioglu, and A. S. Shumovsky, Phys. Rev. Lett. **101**, 020403 (2008).
[8] A. Cabello, Phys. Rev. Lett. **101**, 210401 (2008).
[9] P. Badziąg, I. Bengtsson, A. Cabello, and I. Pitowsky, Phys. Rev. Lett. **103**, 050401 (2009).
[10] M. Michler, H. Weinfurter, and M. Zukowski, Phys. Rev. Lett. **84**, 5457 (2000).
[11] Y. F. Huang, C. F. Li, Y. S. Zhang, J. W. Pan, and G. C. Guo, Phys. Rev. Lett. **90**, 250401 (2003).
[12] Y. Hasegawa, R. Loidl, G. Badurek, M. Baron, and H. Rauch, Phys. Rev. Lett. **97**, 230401 (2006).
[13] H. Bartosik *et al.*, Phys. Rev. Lett. **103**, 040403 (2009).
[14] B. H. Liu *et al.*, Phys. Rev. A **80**, 044101 (2009).
[15] G. Kirchmair *et al.*, Nature (London) **460**, 494 (2009).
[16] E. Amselem, M. Rådmark, M. Bourennane, and A. Cabello, Phys. Rev. Lett. **103**, 160405 (2009).
[17] O. Moussa, C. A. Ryan, D. G. Cory, and R. Laflamme, Phys. Rev. Lett. **104**, 160501 (2010).
[18] R. Lapkiewicz *et al.*, Nature (London) **474**, 490 (2011).
[19] A. Peres, J. Phys. A **24**, L175 (1991).
[20] J. Bub, Found. Phys. **26**, 787 (1996).
[21] J. H. Conway and S. Kochen (unpublished); A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer, Dordrecht, 1993), p. 114; see also J. H. Conway and S. Kochen, in *Quantum [Un]speakables: From Bell to Quantum Information*, edited by R. A. Bertlmann and A. Zeilinger (Springer-Verlag, Berlin, 2002), p. 257.
[22] A. Cabello, J. M. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).
[23] D. Z. Albert, Y. Aharonov, and S. D'Amato, Phys. Rev. Lett. **54**, 5 (1985).
[24] R. Clifton, Am. J. Phys. **61**, 443 (1993).
[25] A. Cabello and M. T. Cunha, Phys. Rev. Lett. **106**, 190401 (2011).
[26] A. Peres, Phys. Lett. A **151**, 107 (1990).
[27] N. D. Mermin, Phys. Rev. Lett. **65**, 3373 (1990).
[28] J. R. Maze *et al.*, New J. Phys. **13**, 025025 (2011).