Laser Cooling and Optical Detection of Excitations in a *LC* Electrical Circuit

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We explore a method for laser cooling and optical detection of excitations in a room temperature *LC* electrical circuit. Our approach uses a nanomechanical oscillator as a transducer between optical and electronic excitations. An experimentally feasible system with the oscillator capacitively coupled to the *LC* and at the same time interacting with light via an optomechanical force is shown to provide strong electromechanical coupling. Conditions for improved sensitivity and quantum limited readout of electrical signals with such an "optical loud speaker" are outlined.

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Cooling plays an essential role in most areas of physics, in part because it reduces detrimental thermal fluctuations. For sensing application, where thermal fluctuations may hide the small signals one is trying to measure, strong coupling of mechanical and electrical oscillators to systems in a pure quantum state, such as light or polarized atomic ensembles, opens up new possibilities for quantum sensing of fields and forces [1]. This in principle allows for enhanced sensitivity of the oscillators, where readout of their state is limited only by quantum fluctuations. In recent years, dramatic advances in optomechanical coupling and cooling of high quality-factor (Q) mechanical systems have been made [2–7]. Similarly, cooling of electrical circuits have also been considered [8] using an electro-optical coupling [9].

In this Letter, we propose to extend laser cooling of mechanical objects to electrical circuits. By strongly coupling a high-Q inductor-capacitor resonator (LC) to a near-resonant nanomechanical membrane [10,11] in the radio frequency (rf) domain, the electrical circuit can be effectively cooled by the cold mechanical system. Since such electrical circuits are used in a wide variety of settings, the reduction of thermal fluctuations in these systems will likely find numerous applications. We show that the cooling techniques explored here allow for optical readout of electrical signals in the circuit. Since light fields are routinely measured with quantum limited precision, this allows for high sensitivity broadband detection of weak electric signals. We analyze the possible gain in sensitivity and bandwidth obtainable using this method. Moreover, light and atomic ensembles can behave as oscillators with a negative mass and thus provide the means to measure fields and forces beyond the standard quantum limit using the power of entanglement [8,12–14] which could improve the sensitivity even further. Importantly, the strong coupling considered here can be implemented with room temperature setups, which is a major asset of our work. It can, however, be extended to cryogenic setups where it would provide a versatile interface for quantum information, allowing for the reliable transfer of quantum states from rf to optical domains and back [15,16]. The key idea in this work is to achieve strong coupling between a nanomechanical membrane and a high-Q LC circuit, and then observe the electrical excitations via optomechanical coupling between the membrane and a high-Q optical cavity. Our suggested method is insertion of the membrane into the fringing field of a capacitor [17], as shown in Fig. 1, such that the capacitance depends upon the displacement of the membrane. We demonstrate that for reasonable component parameters, when combined with a voltage bias and an inductive component to make a resonant electrical circuit near the frequency of a mechanical resonance ω_m, the coupling g between rf photons and the membrane phonons can become sufficiently large to induce normal mode splitting [18], where the resonant response of the system comprises combined electromechancial excitations.

A model Hamiltonian describing the coupled electromechanical system (Fig. 1) in the limit of well separated, high-Q resonant electrical and mechanical modes is

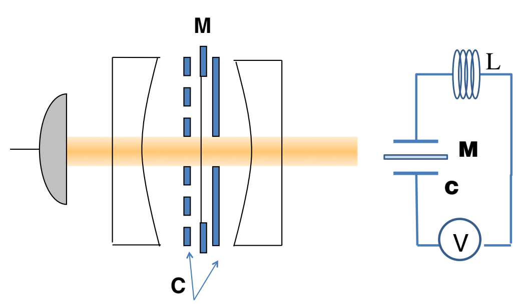

FIG. 1 (color online). (Left) Schematic of a Fabry-Perot cavity with a nanomechanical membrane inserted in the waist of the cavity; the membrane, in turn, is part of a parametric capacitor. (Right) Equivalent circuit with a dc voltage bias describing the coupled electromechanical system.

$$H = \frac{\phi^2}{2L} + \frac{q^2}{2C} + \frac{p_m^2}{2m} + \frac{m\omega_m^2 x_m^2}{2} + \frac{g}{q_0 x_0}(q x_m). \quad (1)$$

Here $\hbar = 1$, $q_0 = 1/\sqrt{2L\omega_0}$, $x_0 = 1/\sqrt{2m\omega_m}$, and $\omega_0 = \sqrt{1/LC}$. The flux $\phi = Ldq/dt$ and the membrane momentum $p_m = mdx_m/dt$ are the canonical momenta conjugate to the charge q and position x_m. When the two modes are brought into resonance $\omega_m = \omega_0 = \omega$, the natural canonical variables become normal mode solutions $Y_\pm, P_\pm$ with $x_m = (Y_+ + Y_-)/\sqrt{2m}$; $q = (Y_+ - Y_-)/\sqrt{2L}$; $p_m = (P_+ + P_-)\sqrt{m/2}$; $\phi = (P_+ - P_-)\sqrt{L/2}$, with frequencies $\omega_\pm = \omega\sqrt{1 \pm g/\omega}$ for $g < \omega$ as considered below.

This coupled-mode system could be inserted into an optical cavity (Fig. 1), whose mode couples to the membrane position x_m via radiation pressure [10]. Both normal modes interact with the cavity mode as $x_m \propto Y_+ + Y_-$; thus, radiation pressure-based detection can be applied independently to each normal mode when the cavity linewidth κ is narrow enough to resolve the normal mode splitting $\omega_+ - \omega_-$. Alternatively, both normal modes can be observed simultaneously when $|\omega_-| > \kappa > \omega_+ - \omega_-$. In both cases the optical system can also cool the combined electromechanical system. In essence this cooling is achieved because the membrane-cavity system acts as a transducer up-converting excitations in the LC circuits to optical frequencies. This means that the LC circuit will equilibrate with the optical modes which are in the quantum mechanical ground state even at room temperature. At the same time the interface between rf excitations in the LC circuits and optical photons also allows for detection of rf-electric signals by optical measurement. Since optical measurement can be quantum noise limited, this opens up new possibilities for detection of weak electrical signals, as we outline below.

Capacitor.—We provide a specific design which admits a strong coupling of a nanomechanical resonator to a LC circuit and simultaneously admits optical coupling (Fig. 1). Specifically, we envision a parallel plate capacitor of area A where we replace one of the plates by a set of wires of thickness t, width r, and separated by a distance d; see Fig. 2(a). In order to have an interaction with light we introduce a hole in the capacitor to allow for laser beams to get through (not shown). Below we are mainly interested in relative changes of the capacitance, and this hole is of minor importance. A dielectric membrane of thickness h is inserted into the capacitor. We assume the membrane to be much larger than the capacitor such that the position of the membrane is given a single number x_m describing the distance from the top of the wires. The replacement of one of the capacitor plates by the set of wires creates a spatial inhomogeneity of the electric field, which attracts the membrane towards the wires when the capacitor is charged. At the same time this inhomogeneity also means that the capacitance $C(x_m)$ will depend on x_m. Expanding the capacitance around the equilibrium position gives rise

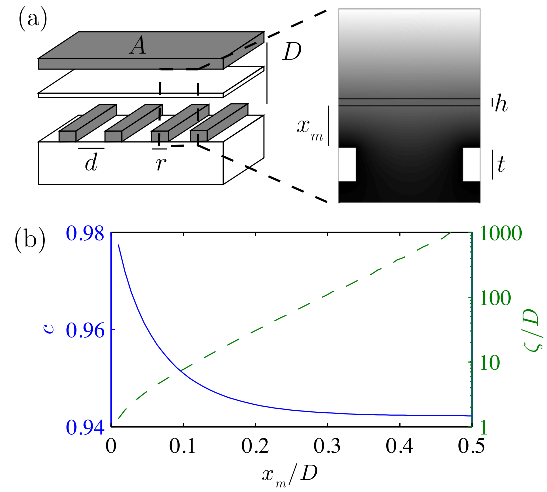

FIG. 2 (color online). Coupling of capacitor and membrane. (a) Schematic of the considered setup (left). A capacitor is made of a plate of area A separated from a set of wires by a distance D. The wires have thickness t and width r and are separated by a distance d. A membrane of thickness h is separated from the wires by a distance x_m. To find the capacitance we perform finite element method simulations over a cross section of the capacitor (right). The gray scale indicates the simulated electric potential with $r = D/4$, $d = 3D/4$, $t = D/4$, $h = D/20$, and $\epsilon = 7.6$. (b) Ratio of capacitance with the membrane to a parallel plate geometry without membrane, c (full curve, left axis), and characteristic length $-\zeta = C/(dC/dx)$ (dashed curve, right axis) for the same parameters as in (a).

to a LC-membrane coupling $\propto x_m q^2$. This is analogous to the radiation pressure coupling that occurs in the optical domain, i.e., a $\chi^{(2)}$-type nonlinearity. As such, we can enhance the coupling strength by providing a classical displacement of the LC circuit's charge, with either a dc or an ac voltage bias V providing an offset charge Q_0. For simplicity we restrict ourselves to the case of a dc voltage, though generalization to the ac case is a simple extension of these ideas and allows us to frequency match the LC and mechanical systems. The coupling between the membrane position and the charge fluctuations around the equilibrium $\hat{q} = q - Q_0$ is then $\propto x_m Q_0 \hat{q}$ and is enhanced by the large charge Q_0 induced on the capacitor, in direct analogy to the similar effect for cavity optomechanics. The full Hamiltonian including electrical and mechanical contributions is

$$H = \frac{\phi^2}{2L} + \frac{p_m^2}{2m} + \frac{m\omega_m^2(x_m - x_e)^2}{2} + \frac{q^2}{2C(x_m)} - qV. \quad (2)$$

Here x_e is the equilibrium membrane position at $V = 0$. The fixed point of the classical charge Q_0 at a given bias voltage and the equilibrium displacement X_0 are then found from $\partial_q H|_{Q_0,X_0} = \partial_{x_m} H|_{Q_0,X_0} = 0$, which yields

$$Q_0 = C(X_0)V, \qquad X_0 = x_e - \frac{Q_0^2}{C(X_0)}\frac{1}{\omega_m}\frac{x_0^2}{\zeta}.$$

Here we have introduced a characteristic length scale ζ defined by $\zeta^{-1} = -C^{-1}\partial C/\partial x_m|_{X_0}$, which describes the relative change in the capacitance at the new equilibrium position X_0.

Around these classical values, we consider the remaining quantum fluctuations $\hat{q}, \hat{x}$. We change to annihilation and creation operators $\hat{a}(\hat{b})$ for the membrane (LC), and find a Hamiltonian

$$H = \omega_m \hat{a}^\dagger \hat{a} + \omega_0 \hat{b}^\dagger \hat{b} + \frac{g}{2}(\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger), \quad (3)$$

with $g = \sqrt{\omega_m \omega_0}\sqrt{(x_e - X_0)/2\zeta}$. This corresponds to the model Hamiltonian examined in the beginning. Assuming a constant value of $\partial C/\partial x_m$ from x_e to X_0, the coupling constant can be expressed in a more intuitive form $g = \sqrt{\omega_m \omega_0}\sqrt{\frac{\Delta C}{2C}}$, i.e., through the capacitance change ΔC caused by the displacement of the membrane due to the applied voltage V. The solution is a stable point under the condition $g < \sqrt{\omega_0 \omega_m}$. We have here neglected several higher order nonlinear corrections arising from the position dependence of the capacitance, which are negligible for the parameters considered below. We have also ignored a small shift in the resonance frequency, which is of minor interest here.

To obtain quantitative estimates of the feasible coupling constant, we assume the plate electrode to be much larger than the separation of the plate and transverse dimensions of the wires $\sqrt{A} \gg d, D, r$. We can then find the capacitance for a given position of the membrane by solving for the potential using the finite element method. We express the capacitance as $C(x_m) = \frac{\epsilon_0 A}{D} c(x_m)$, where $c(x_m)$ is a dimensionless number of order unity, which describes the deviation from a standard parallel plate capacitor.

As an example, we take a SiN membrane of dielectric constant $\epsilon = 7.6$ and thickness $h = 100$ nm inserted into a capacitor with a separation $D = 2\ \mu$m and dimensions $r = D/4$, $d = 3D/4$, and $t = D/4$. A simulation with these values is shown in Fig. 2(b). From this simulation we extract the values of $\zeta \approx 30D = 60\ \mu$m for a distance of $X_0 \approx 0.2D = 0.4\ \mu$m. Hence, if the applied voltage shifts the equilibrium position by $x_e - X_0 \approx 10$ nm around $X_0 \approx 0.2D = 0.4\ \mu$m, the coupling constant is $g/\omega \approx 0.01$ and the system is in the strong-coupling regime if the Q values of the LC circuit and membrane exceed 100. Assuming an operating frequency of $\omega = (2\pi)1$ MHz, an oscillator length $x_0 = 3$ fm, and a capacitor area of $A = 0.1\ \text{mm}^2$ (consistent with a 1 mm^2 membrane), this displacement only requires an applied voltage on the order of 25 V. If a larger initial separation is desirable, a similar coupling ($g/\omega \approx 0.02$) could be achieved if the equilibrium distance is shifted from $x_e \approx 0.6D = 1.2\ \mu$m to $X_0 \approx 0.4D = 0.8\ \mu$m ($\zeta \approx 400D = 800\ \mu$m for $X_0 \approx 0.4D = 0.8\ \mu$m) with an applied voltage of 450 V.

Cooling the LC circuit.—The membrane may be efficiently cooled via optomechanical coupling between the radiation pressure force of a cavity field and the position of the central area of the membrane. The details of this process have been analyzed by a wide variety of groups [7]. In essence the effect on the membrane degree of freedom $\hat{a}$ is to induce a damping Γ_m, which is much greater than the intrinsic damping rate of the membrane $\Gamma_m \gg \gamma_m$, but is limited by the cavity decay rate $\Gamma_m \lesssim \kappa$. This additional damping only produces moderate additional quantum fluctuations associated with the vacuum noise of the light field. We are mainly interested in room temperature applications where the obtainable Q values are a few hundreds. Assuming the resolved sideband limit $\kappa \ll \omega$, the thermal load is then mainly limited by the heating of the LC circuit and we shall ignore these optical heating effects. Working with the LC circuit with damping γ, resonant with the membrane ($\omega_0 = \omega_m = \omega$), we may expect an efficient coupling between the optomechanical system and the LC circuit, provided that $g > \gamma$ such that we can get excitations out of the system faster than they leak in.

We use the input-output formalism in the rotating wave approximation (valid for $g \ll \omega$) to describe this combined mode cooling. The Heisenberg-Langevin equations for this situation are

$$\dot{\hat{a}} = -\Gamma_m \hat{a} + \sqrt{2\gamma_m}\hat{a}_{\text{in}} - i\frac{g}{2}\hat{b},$$
$$\dot{\hat{b}} = -\gamma \hat{b} + \sqrt{2\gamma}\hat{b}_{\text{in}} - i\frac{g}{2}\hat{a}.$$

In the strong damping limit ($\Gamma_m > g$), we can treat the coupled LC resonator as a perturbation and arrive at

$$\dot{\hat{b}} \approx -(\gamma + \Gamma)\hat{b} + \sqrt{2\gamma}\hat{b}_{\text{in}} - i\frac{g}{2\Gamma_m}\sqrt{2\gamma_m}\hat{a}_{\text{in}}.$$

This equation describes the cooling of $\hat{b}$ through the membrane-light system with a rate $\Gamma = g^2/4\Gamma_m$. In the continuous cooling limit, we expect to achieve a thermal population in $\hat{b}$ given by

$$\langle \hat{b}^\dagger \hat{b} \rangle \approx \frac{\gamma}{\Gamma + \gamma} n_b + \frac{2\gamma_m}{g} n_a,$$

where n_a, n_b are the original thermal occupation of modes $\hat{a}$ and $\hat{b}$. Typically this population will be dominated by the heating of the LC circuit (the first term) since the membrane can have a very large $Q \sim 10^6$ [11].

In the mode-resolved, strong-coupling limit, with $\omega_0 = \omega_m = \omega$ and $\gamma, \Gamma_m < g$, each of the two normal modes $\hat{a} + \hat{b}$ and $\hat{a} - \hat{b}$ have frequencies $\omega \pm g/2$ and a damping rate given by the average of the two damping rates $(\gamma + \Gamma_m)/2$. The optomechanical coupling then works independently on each of the two modes. Assuming again the Q of the membrane to be much higher than the Q of the LC, a standard argument for optomechanical cooling [19] leads to a thermal occupation number of $\gamma n_b/\Gamma_m$.

Comparing the two limits derived above we see that the minimal thermal occupation is achieved at a cooling laser power and detuning such that $\Gamma_m \approx g$, where we obtain a population $\sim\gamma n_b/g$. The cooling of the membrane is, however, limited by the cavity decay rate $\Gamma_m \lesssim \kappa$. Hence the cooling limit is the larger of $\gamma n_b/g$ and $\gamma n_b/\kappa$.

Sensitivity of optical readout of LC circuit.—The cooling identified above realizes an interface between optical fields and rf excitations of the LC circuits at the single photon level, and we now turn to a possible application of this interface. Often LC circuits are used in sensitive detectors to pick up very small signals [20]. As we will now show, the sensitivity in such experiments can be improved by detecting the cooling light leaving the cavity. This takes advantage of the fact that the homodyne detection of laser light can be quantum noise limited with near-unit quantum efficiency, thus avoiding many of the noise sources present for low frequency signals. To show this we will work in the strong damping limit identified above $\Gamma_m > g$ with the LC circuit tuned into resonance with the membrane ($\omega_0 = \omega_m = \omega$). Again we also assume the damping of the mechanical motion of the membrane to be negligible $\gamma_m \ll g$. In this limit the membrane and the cavity mediate an effective interaction between the LC mode $\hat{b}$ and the optical cavity input or output modes $\hat{d}_{\text{in(out)}}$ with the effective cooling rate Γ introduced above. In the rotating wave approximation this situation is described by the generic equations

$$\dot{\hat{b}} = -(\gamma + \Gamma)\hat{b} + if(t) + \sqrt{2\gamma}\hat{b}_{\text{in}} - \sqrt{2\Gamma}\hat{d}_{\text{in}}, \quad (4)$$

$$\hat{d}_{\text{out}} = \hat{d}_{\text{in}} + \sqrt{2\Gamma}b. \quad (5)$$

Here, we have introduced an incoming signal to be measured, which is described by $f(t)$; e.g., if the signal is from a voltage V applied to the system, $f(t) = -(C/4\hbar^2 L)^{1/4}V(t)$.

Suppose that we want to measure the Fourier components of the incoming signals $f(\nu)$ detuned by a frequency ν with respect to the resonance frequency of the LC circuit ω_0 within a certain bandwidth $|\nu| \lesssim \delta\omega_0$. This can be done by splitting the outgoing signal on a beam splitter [$d_\pm = (d \pm d_b)/\sqrt{2}$, where d_b is the annihilation operator for the other mode incident on the beam splitter] and inferring the two quadratures $x_f(\nu) = [f(\nu) + f^*(\nu)]/\sqrt{2}$ and $p_f = [f(\nu) - f^*(\nu)]/i\sqrt{2}$ from a homodyne detection of the $x_+ = (d_+ + d_+^\dagger)/\sqrt{2}$ [$p_- = (d_- - d_-^\dagger)/i\sqrt{2}$] quadrature of the d_+ (d_-) mode. The signal-to-noise ratio for, e.g., a measurement of the amplitude can be defined by $S = [\langle x_+(\nu)^2\rangle + \langle p_-(\nu)^2\rangle]/2N$. Here N describes the noise $\langle x_+(\nu)x_+(\nu')\rangle = \langle p_-(\nu)p_-(\nu')\rangle = N\delta(\nu - \nu')$ in the absence of any signal. From the equations of motion, we find

$$S = \frac{2\Gamma|\langle f(\nu)\rangle|^2}{2\gamma\Gamma(2n_b + 1) + [\gamma^2 + \Gamma^2 + \nu^2](2n_d + 1)}. \quad (6)$$

Here n_d describes the number of thermal excitations in the field used to probe the circuit, and we assume that the fields incident on the beam splitter are of the same type such that $\langle d^\dagger d\rangle = \langle d_b^\dagger d_b\rangle$.

Let us compare our approach to the case where the LC circuit is readout by homodyne detection with a rf amplifier assumed to have a similar number of thermal excitations as the system being measured $n_d = n_b \gg 1$. Disregarding any additional noise added during the amplification, S is optimized for $\Gamma = \gamma$ and is limited to $S = |\langle f(\nu)\rangle|^2/4\gamma n_b$ with a detection bandwidth $\delta\omega = 2\gamma$. In contrast, with the optical readout, the incoming laser fields can be quantum noise limited with $n_d = 0$ if light is in a coherent state. In this case we obtain twice the signal-to-noise ratio S for $\gamma \leq \Gamma \lesssim \gamma n_b$. The optimal sensitivity is thus better with optical detection, even if we assume ideal detection of the fields in both cases. Such an ideal detection is routinely achieved by homodyne detection of optical fields with near unity quantum efficiency, whereas it is hard to achieve for rf fields. For realistic limited detector efficiencies of rf fields, the sensitivity may thus be significantly improved using optical readout. Furthermore, the high sensitivity with laser cooling is obtained over a much larger bandwidth which is determined by $\delta\omega = 2\sqrt{2\Gamma\gamma n_b}$. In other words, if, prior to laser cooling, the LC circuit had a high Q and a narrow bandwidth γ that is less than the bandwidth $\delta\omega_0$ required for a particular application, laser cooling allows an increase of the bandwidth with a limited decrease in the sensitivity (< 3 dB) if $\delta\omega_0 \lesssim \delta\omega$. Using regular rf techniques, an alternative approach would be to increase the bandwidth by increasing the damping of the circuit, but this would result in a decrease of the sensitivity by a factor of $\sqrt{\delta\omega_0/\gamma}$. Crucially, since optical fields are shot-noise limited even at room temperature, this measurement setup does not involve cryogenics.

The potential benefits of this approach—high quantum efficiency conversion from rf to optical photons, and the corresponding potential for low temperature detection of rf signals—are limited by the finite Q values for room temperature inductors. Appropriate replacements may be considered, such as crystal resonators or cryogenic superconducting resonators. An additional benefit of a cryogenic setup is the possibility to enter the quantum strong-coupling limit, $g \gtrsim \gamma n_{\text{thermal}}$, at which point the conversion from rf to optical domain can be used as a quantum interface. However, understanding of these features and improvements requires further investigation.

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