# Quantum Nonlocality in Weak-Thermal-Light Interferometry

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In astronomy, interferometry of light collected by separate telescopes is often performed by physically bringing the optical paths together in the form of Young's double-slit experiment. Optical loss severely limits the efficiency of this so-called direct detection method, motivating the fundamental question of whether one can achieve a comparable performance using separate optical measurements at the two telescopes before combining the measurement results. Using quantum mechanics and estimation theory, here I show that any such spatially local measurement scheme, such as heterodyne detection, is fundamentally inferior to coherently nonlocal measurements, such as direct detection, for estimating the mutual coherence of bipartite thermal light when the average photon flux is low. This surprising result reveals an overlooked signature of quantum nonlocality in a classic optics experiment.

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The basic goal of stellar interferometry is to retrieve astronomical information from the mutual coherence between optical modes collected by telescopes [1–3]. The imaging resolution increases with the distance between the collected optical modes called the baseline, motivating the development of long-baseline stellar interferometry using light collected from a telescope array [2,3]. The standard method of stellar interferometry in the optical regime is called direct detection, which coherently combines the optical paths in the form of the classic Young's double-slit experiment, but its efficiency suffers from decoherence in the form of accumulating optical loss along the paths as the baseline is increased. To avoid optical loss, an alternative method is to perform separate heterodyne detection at the two telescopes, before combining the measurement results via classical communication and data processing [2,3]. In quantum information theory, direct detection can be classified as a nonlocal measurement scheme, which requires joint quantum operations on the two optical modes, while heterodyne detection is a local measurement scheme, which does not require quantum coherence between the separate detectors [4,5]. Townes has previously analyzed the quantum noises in direct and heterodyne detection and concluded that direct detection is superior at high optical frequencies and heterodyne detection is superior at low frequencies [3,6]. Heterodyne detection is, however, only one example of local measurements, and it remains a fundamental and important question whether any other local measurement can perform as well as nonlocal measurements while not suffering from decoherence.

The main purpose of this Letter is to prove that, in the case of weak thermal light, *any* local measurement scheme must be significantly inferior to a nonlocal one for the estimation of the mutual coherence according to quantum mechanics. This is a surprising result in quantum metrology, since the disadvantage of local measurements does not otherwise occur for coherent states at any strength, a well-studied case in quantum metrology [7], strong thermal light, in which case there is little difference between direct and heterodyne detection [8], or even the single-photon state assumed by Gottesman, Jennewein, and Croke in their proposal of shared-entanglement stellar interferometry [9]. This quantum measurement nonlocality can be regarded as a dual of Einstein-Podolsky-Rosen entanglement [4,10]: Despite the fact that bipartite thermal light has a well-defined classical description and possesses no quantum entanglement, nonlocal quantum measurements are necessary to extract the most information from the light. For optical interferometry and imaging applications in general, the result demonstrates the fundamental advantage of nonlocal measurements for weak thermal light and motivates the development of coherent optical measurement techniques, such as integrated optical information processing [2,11,12] and entanglement sharing [9].

Consider the estimation of first-order spatial coherence ($g^{(1)}$) between two distant optical modes. In quantum optics, bipartite thermal light is described by the density operator

$$\rho = \int d^2\alpha d^2\beta \Phi(\alpha, \beta)|\alpha, \beta\rangle\langle\alpha, \beta|, \tag{1}$$

where $|\alpha, \beta\rangle$ is a coherent state with amplitudes $\alpha$ and $\beta$ in the two modes and $\Phi(\alpha, \beta)$ is the Sudarshan-Glauber representation [1], given by

$$\Phi(\alpha, \beta) = \frac{1}{\pi^2 \det\Gamma} \exp\left[-\begin{pmatrix}\alpha^* & \beta^*\end{pmatrix}\Gamma^{-1}\begin{pmatrix}\alpha \\ \beta\end{pmatrix}\right]. \tag{2}$$

$\Gamma$ is the mutual coherence matrix:

 

$$\Gamma \equiv \begin{pmatrix} \Gamma_{aa} & \Gamma_{ab} \\ \Gamma_{ba} & \Gamma_{bb} \end{pmatrix} = \begin{pmatrix} \langle a^\dagger a\rangle & \langle b^\dagger a\rangle \\ \langle a^\dagger b\rangle & \langle b^\dagger b\rangle \end{pmatrix}, \tag{3}$$

and $a$ and $b$ are annihilation operators of the optical modes. The zero-mean Gaussian statistics are a standard assumption for astronomical sources in theoretical optics [1,3]. The positive $\Phi$ function indicates that the two modes are classically correlated only and possess no quantum entanglement [13].

Let $\langle a^\dagger a\rangle = \langle b^\dagger b\rangle = \epsilon/2$ for simplicity. For an incoming light with photon-flux spectral density $S(\nu)$ and a relatively narrow detector bandwidth $\Delta\nu$ around a center frequency $\nu_0$, the filtered photon flux is $S(\nu_0)\Delta\nu$. Over the duration of the effective temporal mode $\Delta t \sim 1/\Delta\nu$, $\epsilon = S(\nu_0)\Delta\nu\Delta t \sim S(\nu_0)$ turns out to be independent of the detector bandwidth and a function of the source and the telescope efficiency only. Considering the case $\epsilon \ll 1$, as is common for interferometry with high optical $\nu_0$, the density operator can be approximated in the photon-number basis as

$$\rho = (1-\epsilon)|0,0\rangle\langle 0,0| + \frac{\epsilon}{2}[|0,1\rangle\langle 0,1| + |1,0\rangle\langle 1,0| + g^*|0,1\rangle\langle 1,0| + g|1,0\rangle\langle 0,1|] + O(\epsilon^2), \tag{4}$$

where I have defined $\epsilon g/2 \equiv \Gamma_{ab} = \Gamma_{ba}^*$ and $g \equiv g_1 + ig_2$ as the complex degree of coherence with $|g| \le 1$ [1]. In the following, I neglect the small $O(\epsilon^2)$ terms, assume that $\epsilon$ is known, and $g_1$ and $g_2$ are the unknown parameters to be estimated. The assumption of a known $\epsilon$ should be reasonable, as other noninterferometric imaging methods can be used to estimate the average photon flux and are usually much less sensitive to noise [2]. Otherwise, $\epsilon$ should also be regarded as an unknown parameter to be estimated by the interferometer, a complication outside the scope of this Letter.

Any measurement in quantum mechanics can be modeled by a positive operator-valued measure (POVM) $E(y)$ [4,14], which determines the probability of the observation $y$:

$$P(y|g) = \mathrm{tr}[E(y)\rho]. \tag{5}$$

For example, in the direct detection scheme [Fig. 1(a)], the two optical modes are brought to interfere at a 50-50 beam splitter and the photons at the two output ports are counted. It can be shown by standard quantum optics calculations [8] that the POVM $E(n,m)$ for photon counts $n$ and $m$ are

$$E(0,0) = |0,0\rangle\langle 0,0|, \tag{6}$$

$$E(1,0) = \tfrac{1}{2}(|1,0\rangle + e^{-i\delta}|0,1\rangle)(\langle 1,0| + e^{i\delta}\langle 0,1|), \tag{7}$$

$$E(0,1) = \tfrac{1}{2}(|1,0\rangle - e^{-i\delta}|0,1\rangle)(\langle 1,0| - e^{i\delta}\langle 0,1|), \tag{8}$$

where $\delta$ is an adjustable phase shift on the $b$ mode. The observation probabilities become

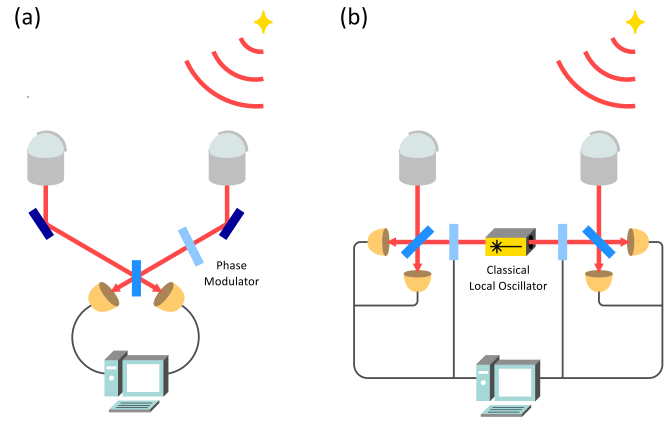


FIG. 1 (color online). Schematics of (a) the direct detection scheme, an example of nonlocal quantum measurement, and (b) a local measurement scheme, which performs spatially separate measurements and permits only classical communication and control between the two sites. Examples of the latter include heterodyne and homodyne detection.

$$P(0,0|g) = 1-\epsilon, \tag{9}$$

$$P(1,0|g) = \frac{\epsilon}{2}[1 + \mathrm{Re}(ge^{-i\delta})], \tag{10}$$

$$P(0,1|g) = \frac{\epsilon}{2}[1 - \mathrm{Re}(ge^{-i\delta})]. \tag{11}$$

To evaluate the parameter-estimation capability of a measurement scheme, consider the Fisher-information matrix, defined as [15]

$$F \equiv \sum_y \frac{1}{P(y|g)} D(y|g), \tag{12}$$

$$D(y|g) \equiv \begin{pmatrix} \left[\frac{\partial P(y|g)}{\partial g_1}\right]^2 & \frac{\partial P(y|g)}{\partial g_1}\frac{\partial P(y|g)}{\partial g_2} \\ \frac{\partial P(y|g)}{\partial g_2}\frac{\partial P(y|g)}{\partial g_1} & \left[\frac{\partial P(y|g)}{\partial g_2}\right]^2 \end{pmatrix}. \tag{13}$$

The inverse of the Fisher-information matrix provides a lower Cramér-Rao bound to the mean-square estimation error covariance matrix $\Sigma$ for any unbiased estimate in the form of $\Sigma \ge F^{-1}$. The eigenvalues of $F$, which must be nonnegative as $F \ge 0$, hence quantify the amounts of independent information obtainable from the measurement. In a total observation time interval $T$ over which the model parameters can be approximated as time constant, $M \sim T/\Delta t \sim T\Delta\nu$ measurements can be performed, and the total Fisher information is $F^{(M)} = MF \sim T\Delta\nu F$. In the limit of large $M$, the Cramér-Rao bound is asymptotically achievable by maximum-likelihood estimation. This makes the Fisher information a rigorous metric for comparing the inherent capabilities of different measurement schemes for parameter estimation.

The Fisher information for direct detection is

$$F = \frac{\epsilon}{1 - \mathrm{Re}(ge^{-i\delta})^2}\begin{pmatrix} \cos^2\delta & \sin\delta\cos\delta \\ \sin\delta\cos\delta & \sin^2\delta \end{pmatrix}, \tag{14}$$

and the eigenvalues of $F$ are

$$\lambda_1 = 0, \qquad \lambda_2 = \frac{\epsilon}{1 - \mathrm{Re}(ge^{-i\delta})^2}. \tag{15}$$

The zero eigenvalue corresponds to the absence of information about the unobservable quadrature $\mathrm{Im}(ge^{-i\delta})$. In practice, $\delta$ is varied over measurements to retrieve information about both quadratures of $g$. The important point to note here is that $||F|| = \lambda_1 + \lambda_2 \geq \epsilon$ if we take the trace norm. The norm of the total Fisher information for $M$ measurements becomes $||F^{(M)}|| = M||F|| \geq M\epsilon$, which scales linearly with the average photon number $M\epsilon$, thereby achieving the optimal "shot-noise" scaling for parameter estimation using classical states [16]. Similarly, it is shown in [8] that the Fisher information for the shared-entanglement scheme proposed by Gottesman, Jennewein, and Croke [9] has in theory the same expression but reduced by a factor of 2.

Both of the aforementioned schemes can be considered as nonlocal quantum measurements, which require bringing the two modes together physically or sharing entanglement between the two sites. The physical nonlocality makes such schemes increasingly challenging to implement technically as the distance between the two modes increases, primarily due to accumulating decoherence in the form of optical loss along the paths [2]. Local measurement schemes, on the other hand, measure the two modes separately before combining the results via classical communication [Fig. 1(b)], and can therefore be implemented over a much greater distance in principle. To investigate the general performance of any local measurement, let us write the observation probability distribution for POVM $E(y)$ explicitly as

$$P(y|g) = (1 - \epsilon)E_{00,00}(y) + \frac{\epsilon}{2}[E_{01,01}(y) + E_{10,10}(y) + 2|E_{10,01}(y)|\mathrm{Re}(ge^{-i\delta})], \tag{16}$$

where

$$E_{nm,n'm'}(y) \equiv \langle n, m|E(y)|n', m'\rangle \tag{17}$$

and $\delta$ is the phase of $E_{10,01}$. To put a bound on the Fisher information given by Eq. (12), note that

$$P(y|g) \geq (1 - \epsilon)E_{00,00}(y), \tag{18}$$

and the positive-semidefinite matrix

$$D = \epsilon^2|E_{10,01}(y)|^2\begin{pmatrix} \cos^2\delta & \sin\delta\cos\delta \\ \sin\delta\cos\delta & \sin^2\delta \end{pmatrix} \tag{19}$$

defined in Eq. (13) has a trace norm given by $\epsilon^2|E_{10,01}(y)|^2$. Applying the subadditivity property of matrix norms to Eq. (12) results in an upper bound on $||F||$:

$$||F|| \leq \frac{\epsilon^2}{1 - \epsilon}\sum_y \frac{|E_{10,01}(y)|^2}{E_{00,00}(y)}. \tag{20}$$

For generality, I define local measurements as the ones performed using local operations with classical communication (LOCC), which permits the measurement at one site to be conditioned upon the observation at the other site. A necessary condition for a spatial-LOCC POVM is the positive-partial-transpose condition $E^{\mathcal{T}_a}(y) \geq 0$ [17]. By the Cauchy-Schwarz inequality, $|\langle 1, 0|E|0, 1\rangle|^2 = |\langle 0, 0|E^{\mathcal{T}_a}|1, 1\rangle|^2 = |\langle 0, 0|\sqrt{E^{\mathcal{T}_a}}\sqrt{E^{\mathcal{T}_a}}|1, 1\rangle|^2 \leq \langle 0, 0|E^{\mathcal{T}_a} \times |0, 0\rangle\langle 1, 1|E^{\mathcal{T}_a}|1, 1\rangle = \langle 0, 0|E|0, 0\rangle\langle 1, 1|E|1, 1\rangle$, or

$$|E_{10,01}(y)|^2 \leq E_{00,00}(y)E_{11,11}(y). \tag{21}$$

Combining Eqs. (20) and (21), I obtain an $O(\epsilon^2)$ upper bound on $||F||$:

$$||F|| \leq \frac{\epsilon^2}{1 - \epsilon}\sum_y E_{11,11}(y) = \frac{\epsilon^2}{1 - \epsilon}, \tag{22}$$

where $\sum_y E_{11,11}(y) = 1$ comes from the completeness property of a POVM. The neglected $O(\epsilon^2)$ term in the density operator in Eq. (4) contributes an additional $O(\epsilon^2)$ term to $P$ and an $O(\epsilon^3)$ term to $D$, so the Fisher information would be modified by an $O(\epsilon^3)$ term and the upper bound in Eq. (22) should be rewritten as

$$||F|| \leq \epsilon^2 + O(\epsilon^3). \tag{23}$$

For $M$ measurements, the bound can be generalized to allow for adaptive measurements conditioned upon previous observations, as shown in Ref. [8]:

$$||F^{(M)}|| \leq M[\epsilon^2 + O(\epsilon^3)]. \tag{24}$$

This upper bound shows that the best Fisher information any spatiotemporal-LOCC measurement can achieve is still substantially worse than that of the spatially nonlocal methods ($||F^{(M)}|| \sim M\epsilon$) when $\epsilon \ll 1$. In other words, spatially local measurements are fundamentally much less efficient than nonlocal methods in extracting coherence information from weak-thermal-light interferometry. This general proof is supported by explicit Fisher-information calculations for heterodyne and homodyne detection [8], signal-to-noise-ratio calculations for direct and heterodyne detection of the full thermal state given by Eq. (1) [8], and the known fact in astronomy that direct detection performs better than heterodyne detection for high optical $\nu_0$ [3,6]. Reference [8] also includes a discussion of the quantum origin of the nonlocality in terms of the semiclassical photodetection picture.

Note that the advantage of nonlocal measurements is lost for coherent states, strong thermal light with $\epsilon \gg 1$ [8], or even the nonclassical single-photon state studied in Ref. [9]. For coherent states, $|g| = 1$ and the unknown parameters are the phases of the two optical modes in a product of coherent states, in which case it can easily be shown that nonlocal measurements are not necessary, analogous to the case of single-parameter phase estimation with a product state [18]. For strong thermal light with $\epsilon \gg 1$, calculations in Ref. [8] show that the performances of direct detection and heterodyne detection converge and suggest that the noise in this regime is dominated by the thermal statistics of the source rather than the detection statistics. The single-photon state studied in Ref. [9] can also be analyzed using the formalism here by omitting $O(\epsilon^2)$ terms and then putting $\epsilon = 1$, resulting in comparable performances for local and nonlocal measurements.

The peculiar existence of quantum nonlocality for weak thermal light, as a property of bipartite measurements applied to certain separable states, can be regarded as a dual of Einstein-Podolsky-Rosen entanglement [4,10], a property of bipartite states that can produce higher correlations in certain separable measurements. In the context of quantum communication theory, it is well known that nonlocal measurements can extract more information from states with no entanglement [4,5,19]; the result here provides a striking example in which the same type of quantum nonlocality readily exists for observers extracting information from nature.

For practical applications, the result here demonstrates the fundamental advantage of nonlocal quantum measurements for weak-thermal-light interferometry and may have further implications for optical imaging systems, such as compound-eye imaging and fluorescence microscopy [11]. The shared-entanglement proposal in Ref. [9] requires a path-entangled single-photon source and quantum repeaters, both of which are unlikely to become feasible in the near future, but standard linear optics can also perform nonlocal measurements by coherently processing multiple optical modes before detection, provided that optical loss can be minimized. In the short term, the result here thus motivates the development of low-loss coherent optical information devices, such as photonic crystal fibers and integrated photonics, for thermal-light interferometry and imaging [2,11,12].

Accurate coherence information can be obtained only in the limit of many collected photons. This corresponds to measurements of many copies of the quantum state. A more general quantum measurement strategy than the ones considered here involves joint quantum operations on the multiple copies before measurements. This kind of temporal nonlocality is not needed for parameter estimation when spatially nonlocal measurements can be performed [16]. It remains an interesting open question whether coherent temporally nonlocal strategies can offer any significant advantage when one is restricted to spatially local measurements. Other potential generalizations include time-varying parameters and the estimation of temporal coherence for spectroscopy in addition to spatial coherence. One must then take into account the dynamics of the source and colored noise, which can be analyzed using the quantum waveform estimation framework developed in Refs. [20].

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