Do Baryons Trace Dark Matter in the Early Universe?

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Baryon-density perturbations of large amplitude may exist if they are compensated by dark-matter perturbations such that the total density is unchanged. Primordial abundances and galaxy clusters allow these compensated isocurvature perturbations (CIPs) to have amplitudes as large as $\sim 10\%$. CIPs will modulate the power spectrum of cosmic microwave background (CMB) fluctuations—those due to the usual adiabatic perturbations—as a function of position on the sky. This leads to correlations between different spherical-harmonic coefficients of the temperature and/or polarization maps, and induces polarization B modes. Here, the magnitude of these effects is calculated and techniques to measure them are introduced. While a CIP of this amplitude can be probed on large scales with existing data, forthcoming CMB experiments should improve the sensitivity to CIPs by at least an order of magnitude.

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We have been conditioned to believe that the $\sim 10^{-5}$ variations in the cosmic microwave background (CMB) temperature [1,2] imply that the matter in the early Universe was distributed with similarly small variations. This is certainly true if primordial perturbations are adiabatic, i.e., if there are perturbations only to the *total* matter content, with the fractional contributions of baryons, dark matter, photon, and neutrinos the same everywhere. It is also true for many isocurvature models [3], where the total density is fixed.

It is therefore a surprise that perturbations in the baryon density can be almost arbitrarily large—far larger than 10^{-5}—as long as they are compensated by dark-matter perturbations in such a way that the total matter density remains unchanged [4,5]. These compensated isocurvature perturbations (CIPs) induce no gravitational fields, as the total matter density in this mode is spatially homogeneous. Baryon-pressure gradients induce motions at the baryon sound speed which, at the time of primary CMB decoupling ($z \sim 1091$, *decoupling* hereafter), is $(v/c) \sim (T/m_p)^{1/2} \sim (\mathrm{eV/GeV})^{1/2} \sim 10^{-4.5}$. These motions affect the photon temperature only on distances $\lesssim 10^{-4.5}$ times the horizon at decoupling, that is, CMB multipole moments with $l \gtrsim 10^6$ [4], far larger than those ($l \lesssim 10^4$) probed by CMB experiments. Thus, while the CMB power spectrum currently constrains the *mean* baryon–to–dark-matter ratio precisely, it tells us nothing about spatial variations in this ratio.

Big-bang nucleosynthesis (BBN) and galaxy-cluster baryon fractions constrain the CIP amplitude to be less than $\sim 10\%$ [5]. Consequences of CIPs for galaxy surveys are small [4]. Measurements of 21-cm radiation from the dark ages would be sensitive to CIPs [4,6,7], but such measurements are still off in the future.

Here we show that the primordial relative distribution of baryons and dark matter can be determined with the CMB. Our principle motivation is curiosity—is the common assumption that baryons trace dark matter in the early Universe justified empirically? However, a search for CIPs is also motivated by the curvaton models [8] that predict their existence [4,9] and perhaps by recent ideas linking baryon and dark-matter densities [10]. Moreover, if the Planck satellite finds evidence for primordial isocurvature perturbations, the CIP measurements we describe below will be essential to determine how that perturbation is distributed between baryons and dark matter.

CIPs modulate the baryon and dark-matter densities at decoupling, where $\sim 90\%$ of photons last scatter, and at reionization, where $\sim 10\%$ of CMB photons last scatter. There will thus be a modulation of the small-scale temperature and polarization power spectra from one patch of sky to another. This modifies the power spectrum obtained by averaging over the entire sky, induces polarization B modes, and causes correlations between different spherical-harmonic coefficients of the temperature and/or polarization maps. The effects on the CMB are analogous to those of gravitational lensing [11]. The B-mode power spectrum induced by CIPs through the modulation of the reionization optical depth has already been calculated [5].

We show, however, that the CMB effects induced by modulation of the baryon density at *decoupling* are considerably larger than those induced at reionization. Our calculation follows Ref. [12], where the CMB effects of a spatially varying cosmological parameter (there the fine-structure constant) were considered. This variation induces a spatially varying power spectrum. We have extended the formalism of Ref. [12] to calculate the effect of CIPs on top of the usual adiabatic initial conditions, extending the flat-sky formalism developed there to the full sky, and generalizing the calculation to scales of smaller width than the decoupling surface. Since the technical details are complicated, we present them elsewhere [13] and focus here on the principal science results.

The CIP involves baryon and cold-dark-matter densities $\rho_b(\mathbf{x}) = \bar{\rho}_b[1 + \Delta(\mathbf{x})]$ and $\rho_c(\mathbf{x}) = \bar{\rho}_c - \rho_b\Delta(\mathbf{x})$, written as functions of position $\mathbf{x}$ in terms of a fractional baryon-density perturbation $\Delta(\mathbf{x})$. Note that the total matter density $\rho_b(\mathbf{x}) + \rho_c(\mathbf{x})$ associated with the CIP does not vary with $\mathbf{x}$. We assume that $\Delta(\mathbf{x})$ is a random field with a scale-invariant power spectrum $P_\Delta(k) = Ak^{-3}$, as may be expected if CIPs arise somehow from inflation, and A is a dimensionless amplitude. The rms variation $\Delta_{\rm cl}$ in the baryon–to–dark-matter ratio between galaxy clusters obeys the constraint $\Delta_{\rm cl} \lesssim 0.08$.

When the three-dimensional field is projected onto a narrow spherical surface, the resulting angular power spectrum for Δ will be $C_L^\Delta \simeq A/(\pi L^2)$ for mulipole moments $L \lesssim (\eta_0 - \eta_{\rm ls})/\sigma_\eta$, where $\eta_{\rm ls}$ and η_0 are the conformal time at last scatter and today, respectively, and σ_η is the rms conformal-time width of the last-scattering surface. At smaller angular scales (larger L), the variation in Δ is suppressed by the finite width of the scattering surface. The angular power spectrum for Δ can then be approximated by $C_L^\Delta \simeq A(\eta_0 - \eta_{\rm ls})/(2\sqrt{\pi}L^3\sigma_\eta)$ for $L \gtrsim (\eta_0 - \eta_{\rm ls})/\sigma_\eta$ [5,13]. The rms variation $\Delta_{\rm cl}$ in the baryon–to–dark-matter ratio on galaxy cluster scales is

$$\Delta_{\rm cl}^2 = \frac{1}{2\pi^2}\int k^2 dk[3j_1(kR)/(kR)]^2 P_\Delta(k), \qquad (1)$$

where R is the mean separation between galaxy clusters. The integral has a formal logarithmic divergence at low k which is cut off, however, by the horizon $k_{\rm min} \simeq (10\ {\rm Gpc})^{-1}$. Taking $R \simeq 10$ Mpc, we find $\Delta_{\rm cl}^2 \simeq A\ln(1000)/2\pi^2$. Thus, $\Delta_{\rm cl} \lesssim 0.08$ implies $A \lesssim 0.017$. A weaker bound ($A \lesssim 0.046$) comes from BBN.

Now consider the CMB fluctuations produced at decoupling. The CIP-induced variation of the baryon and dark-matter densities across the sky modulates the small-scale power spectrum, and this modulation induces off-diagonal correlations in the CMB [12,13].

Moreover, B modes are induced in the CMB polarization [12]. The induced spherical-harmonic coefficients are

$$a_{lm}^B = -i\sum_{L+l+l'\,{\rm odd}} \xi_{lml'm'}^{LM}\begin{pmatrix} l & L & l' \\ 2 & 0 & -2 \end{pmatrix}\Delta_{LM}\frac{da_{l'm'}^E}{d\Delta}, \qquad (2)$$

where Δ_{LM} are the spherical-harmonic coefficients for $\Delta(\hat{n})$; $da_{l'm'}^E/d\Delta$ is the derivative of the usual E-mode spherical-harmonic coefficient with respect to Δ (computed using the CAMB code [13]) and

$$\xi_{lml'm'}^{LM} = \begin{pmatrix} l & L & l' \\ 0 & 0 & 0 \end{pmatrix}^{-1}\int d\hat{n}Y_{lm}^*(\hat{n})Y_{LM}(\hat{n})Y_{l'm'}(\hat{n}). \quad (3)$$

This induced B mode arises by modulating the first-order adiabatic perturbation to first order in the CIP. This is because the sound speed, photon diffusion length, and visibility function, assumed spatially homogeneous in the standard treatment, all depend on the local baryon density. In contrast, when the CMB is gravitationally lensed, $da_{l'm'}^E/d\Delta$ is replaced by a function encoding a deflection.

Figure 1 shows the results of our calculations for the B-mode power spectrum C_l^{BB} induced by a scale-invariant spectrum of CIPs with the largest amplitude ($A \simeq 0.017$) consistent with galaxy-cluster baryon–to–dark-matter ratios. CIPs modulate the reionization optical depth, and as noted in Ref. [5], this also generates B modes, through patchy screening [14] and scattering [15] of primordial CMB fluctuations. We plot these reionization contributions in Fig. 1 for the same CIP amplitude. We see that the B mode power spectrum induced at decoupling is larger (by up to 3 orders of magnitude) than that induced at reionization, for $l \gtrsim 50$. The decoupling-induced B modes are larger because (a) they involve ~90% of the photons, rather than ~10%, and (b) the finite width of the reionization rescattering surface smooths the angular Δ fluctuations to larger angular scales (lower L) than it does for decoupling. Reconstruction of Δ_{LM} depends primarily on higher-l modes, and so the baryon-density modulation at decoupling is more important in probing CIPs than that at reionization.

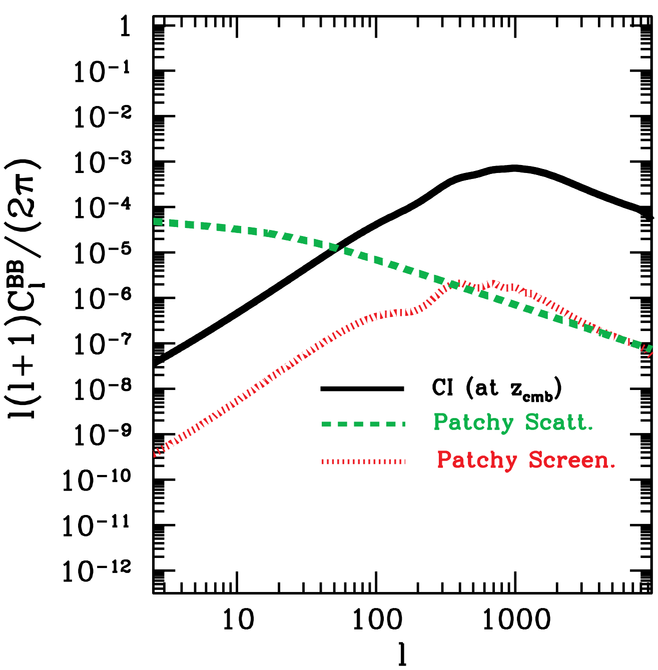

FIG. 1 (color online). The B-mode power spectrum (in μK^2) for the CIP-induced contribution from decoupling (solid black curve), contrasted with contributions from patchy scattering (short-dashed green curve), and patchy screening (dotted red curve) at reionization. We use a scale-invariant spectrum of CIPs with the amplitude $A \simeq 0.017$ that saturates the galaxy-cluster bound.

We now turn to the reconstruction of CIPs from CMB maps. In the absence of CIPs, the multipole moments a^X_{lm} obey the relation $\langle a^X_{lm} a^{X'}_{l'm'} \rangle = C^{XX'}_l \delta_{ll'} \delta_{mm'}$ ($X \in \{T, E, B\}$); i.e., spherical-harmonic coefficients with $(lm) \neq (l'm')$ are statistically independent. However, if there is spatial modulation of the power spectrum, then there will be off-diagonal ($l \neq l'$, $m \neq m'$) correlations,

$$\langle a^X_{lm} a^{X'}_{l'm'} \rangle = C^{XX'}_l \delta_{ll'} \delta_{mm'} + \sum_{LM} D^{LM,XX'}_{ll'} \xi^{LM}_{lml'm'}, \qquad (4)$$

where $D^{LM,XX'}_{ll'} = \Delta_{LM} S^{L,XX'}_{ll'}$ are bipolar spherical harmonics [16], and $S^{L,XX'}_{ll'}$ are coupling coefficients that are calculated in Ref. [13].

Construction of minimum-variance estimators for the Δ_{LM} and their associated errors is straightforward [17]. For example, for the EB correlation, the estimator is

$$\hat{\Delta}_{LM} = \sigma^2_{\Delta_L} \sum_{l' \geq l} \frac{S^{L,EB}_{ll'} \hat{D}^{LM,EB}_{ll'}}{C^{BB,\mathrm{map}}_l C^{EE,\mathrm{map}}_{l'}} + \{E \leftrightarrow B\}, \qquad (5)$$

and it has a variance

$$\sigma^{-2}_{\Delta_L} = \sum_{l' \geq l} \frac{(2l+1)(2l'+1)(S^{L,EB}_{ll'})^2}{4\pi C^{BB,\mathrm{map}}_l C^{EE,\mathrm{map}}_{l'}} + \{E \leftrightarrow B\}, \qquad (6)$$

where $C^{XX'}_l$ are power spectra including noise, and $\hat{D}^{LM,EB}_{ll'}$ is the minimum-variance estimator for $D^{LM,EB}_{ll'}$ [17]. From these one can estimate C^{Δ}_L.

Figure 2 shows the predicted errors in the CIP power spectrum reconstruction from the TT, EE, TE, TB, and EB estimators for the Wilkinson Microwave Anisotropy Probe (WMAP) and the proposed Experimental Probe of Inflationary Cosmology (EPIC). These are $\delta C^{XX'}_L \equiv [(2L+1)]^{-1/2} (\sigma^{XX'}_{\Delta_L})^2 / f_{\mathrm{sky}}$, where the $(2L+1)^{-1/2}$ factor results from the multiple modes available at each L.

Instrumental parameters for WMAP are a beamwidth of $21'$ (full width at half-maximum), noise-equivalent temperature (NET) of 1200 $\mu\mathrm{K}\sqrt{\mathrm{s}}$, fraction of sky analyzed $f_{\mathrm{sky}} = 0.65$, and observation time $t_{\mathrm{obs}} = 7$ years. For EPIC (150 GHz channel) we assume a beamwidth of 5 arcmin, NET of 2.0 $\mu\mathrm{K}\sqrt{\mathrm{s}}$, and observation time $t_{\mathrm{obs}} = 4$ years, also with $f_{\mathrm{sky}} = 0.65$.

For WMAP, the best sensitivity comes from TT. For EPIC, the sensitivity at $L \gtrsim 100$ comes primarily from the TB estimator. We have checked that the best sensitivity for Planck comes from TT, while some ground-based experiments (e.g., SPTPol) benefit from polarization.

The left panel of Fig. 3 shows the errors in the CIP power spectrum reconstruction obtained by combining the TT, TE, EE, TB, and EB estimators for a variety of CMB experiments. The signal-to-noise ratio is given by $S/N = \{(f_{\mathrm{sky}}/2) \sum_{L > f_{\mathrm{sky}}^{-1/2}} (2L+1)[(C^{\Delta}_L / \sigma^2_{\Delta_L})]^2\}^{1/2}$. The right panel shows the S/N for detection of a scale-invariant spectrum of CIPs as a function of Δ_{cl}. With WMAP, a CIP saturating the cluster bound is marginally accessible on the largest scales. Planck should be able to probe rms CIP amplitudes of 3×10^{-2} and higher. Significant improvements in sensitivity should be obtained with upcoming experiments like Polarbear, SPTPol, and ACTPol. We see that S/N values $\gtrsim 3$ may be possible with EPIC for an rms CIP amplitude of 4×10^{-3}, a factor of $\sim$20 lower than the current limit.

The tools for these measurements should be generalizations of those used for weak lensing of the CMB [18], which also produces off-diagonal correlations. CIPs should be distinguishable from lensing, since these physical effects are distinct, as evidenced by differing forms for the

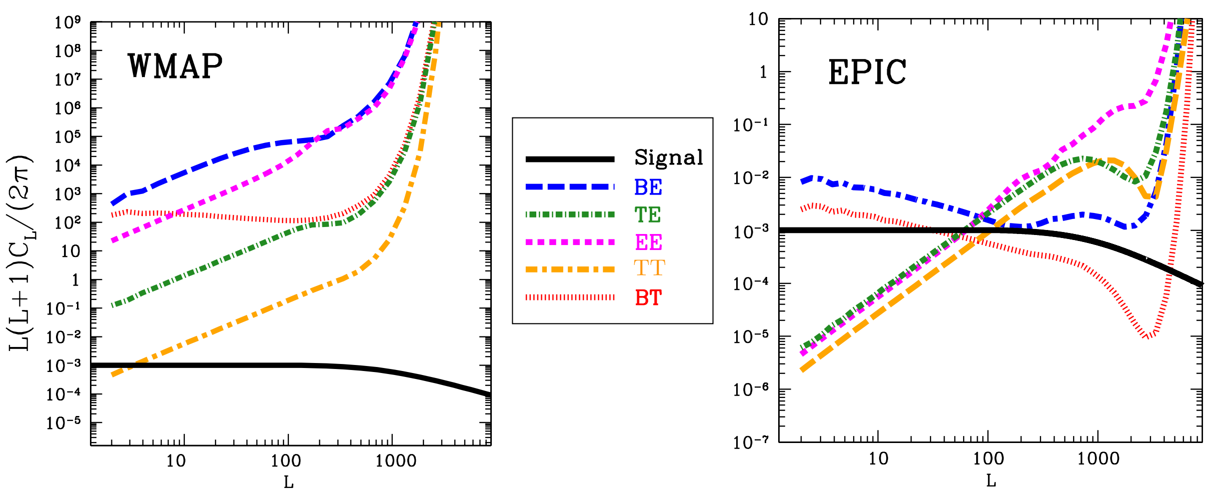

FIG. 2 (color online). Shown are the errors in C^{Δ}_L from the TT, TE, EE, TB, and EB estimators for the CIP perturbation Δ at the surface of last scatter for (a) WMAP and (b) a CMB polarization satellite, with the specifications spelled out in the EPIC mission concept study. Also shown (signal) is the power spectrum C^{Δ}_L for a scale-invariant spectrum of CIPs with the maximum amplitude allowed by galaxy clusters.

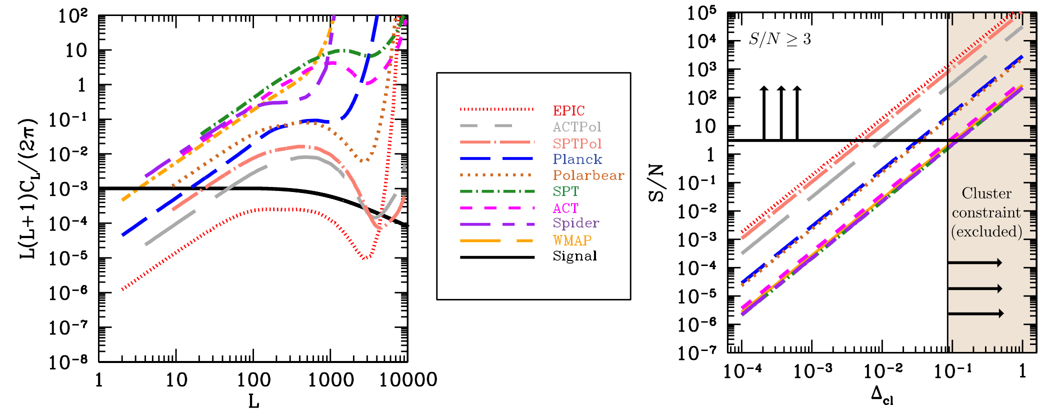

FIG. 3 (color online). (a) Shown are the total expected errors in C_L^Δ from the combined TT, TE, EE, TB, and EB estimators for the CIP perturbation Δ at the surface of last scatter for several current and forthcoming CMB experiments. Also shown (signal) is the power spectrum C_L^Δ for a scale-invariant spectrum of CIPs with the maximum amplitude allowed by galaxy clusters. (b) The signal-to-noise as a function of the rms fluctuation in the galaxy cluster baryon–to–dark-matter ratio along different lines of sight at $z \simeq 1091$. The vertical line shows the upper limit to the rms amplitude from galaxy clusters (excluded region is shaded).

coupling coefficients $S_{ll'}^{L,XX'}$. In Ref. [19], it is shown that for the analogous case of patchy reionization, optical depth fluctuations may be separated from lensing without significant loss in S/N, and we expect that this is also true for CIPs. We leave for future work the development of tools to distinguish CIPs from weak lensing and contaminants like Galactic foregrounds.

CIPs are an intriguing possibility and a prediction of some inflationary models. With the measurements we have described here, we may soon know empirically how closely dark matter and baryons trace each other in the early Universe.

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