Observation of Peregrine Solitons in a Multicomponent Plasma with Negative Ions

H. Bailung,[1] S. K. Sharma,[1] and Y. Nakamura[1,2]

[1]Plasma Physics Laboratory, Physical Sciences Division, Institute of Advanced Study in Science and Technology, Paschim Boragaon, Guwahati-35, India

[2]On leave from Yokohama National University, Yokohama, Japan

(Received 29 July 2011; published 16 December 2011)

The experimental observation of Peregrine solitons in a multicomponent plasma with the critical concentration of negative ions is reported. A slowly amplitude modulated perturbation undergoes self-modulation and gives rise to a high amplitude localized pulse. The measured amplitude of the Peregrine soliton is 3 times the nearby carrier wave amplitude, which agrees with the theory. The numerical solution of the nonlinear Schrödinger equation is compared with the experimental results.

DOI: 10.1103/PhysRevLett.107.255005 PACS numbers: 52.35.Fp, 52.27.Cm

Korteweg–de Vries (KdV) solitons were theoretically predicted for ion-acoustic waves by Washimi and Taniuti [1]. The existence of ion-acoustic solitons was experimentally confirmed by Ikezi *et al.* using a novel double-plasma device [2]. For ion-acoustic waves, the coefficient of the nonlinear term in the KdV equation is positive so that only solitons with a positive potential are possible. However, when a certain density of negative ions that is above a critical value is introduced into the plasma previously consisting of positive ions and electrons, the coefficient becomes negative. Then, instead of positive potential solitons, negative potential solitons can propagate. Negative potential solitons (rarefaction of positive ions) have been experimentally observed in a negative ion plasma which contains F^- ions [3].

A positive potential pulse steepens due to the fluid velocity of positive ions. Electrons follow the steepening since their mass is much lighter than ions. On the other hand, negative ions pull back the steepening due to their large inertia. At a certain density of negative ions, which is called the critical density, the steepening does not occur, which corresponds to the vanishing of the nonlinear term in the KdV equation. In this case, the second order nonlinear term that is proportional to the square of the wave potential must be considered and the resulting equation is called the modified KdV equation (mKdV). Since the coefficient of the second order nonlinear term is positive, the mKdV equation permits the existence of both positive and negative potential solitons. The coexistence of both the positive and negative potential solitons was confirmed experimentally when the negative ion density was equal to the critical density [4].

Saito *et al.* have shown that the nonlinear Schrödinger equation (NLSE) is equivalent to the mKdV equation [5]. It is well known that the NLSE possesses envelope soliton solutions. Envelope solitons of ion-acoustic waves were observed experimentally when the negative ion density was equal to the critical value in a double-plasma device [6]. This experimental result confirms that the NLSE is applicable in describing ion-acoustic waves in the negative ion plasma when the density of negative ions is equal to the critical density.

Peregrine analyzed the NLSE and predicted that there are additional localized solutions to the NLSE [7]. It has been suggested that rogue waves in the ocean are related to what are now called Peregrine solitons. Peregrine solitons have been observed in nonlinear fiber optics experiments [8]. They have also recently been observed in deep water wave experiments performed in a water tank [9].

In this Letter, we report the first observation of Peregrine solitons of ion-acoustic waves in a multicomponent plasma with negative ions when the density of negative ions is equal to the critical value.

In a multicomponent plasma with the critical density of negative ions, the NLSE describing the evolution of ion-acoustic wave with weak nonlinearity is given by [5,6]

$$i\frac{\partial \psi}{\partial \tau} + p\frac{\partial^2 \psi}{\partial \xi^2} + \frac{q}{4}|\psi|^2\psi = 0, \tag{1}$$

where ψ represents the wave amplitude normalized by the electron temperature (T_e), the time τ and distance ξ in the wave frame are normalized, respectively, by the ion plasma period ${\omega_{\rm pi}}^{-1} = (\varepsilon_0 m_i/ne^2)^{1/2}$ and the electron Debye length $\lambda_D = (\varepsilon_0 \kappa T_e/ne^2)^{1/2}$, where m_i is the positive ion mass and n is the unperturbed electron density. The group velocity V_g of the propagating wave normalized by $(\kappa T_e/m_i)^{1/2}$ is given by

$$V_g = \frac{\omega^3}{k^3}\left(\frac{1-r}{1+r/\mu}\right) \equiv \frac{d\omega}{dk}.$$

Here, r and μ represent the density ratio and the mass ratio, respectively, of negative ions to that of the positive ions. The angular frequency ω and the wave number k are related with the following dispersion relation:

$$\omega^2 = \frac{k^2}{1+k^2}\left(\frac{1+r/\mu}{1-r}\right).$$

The dispersion coefficient p in Eq. (1) is given by

$$p = -\frac{3}{2}\frac{\omega^5}{k^4}\left(\frac{1-r}{1+r/\mu}\right)^2 \equiv \frac{1}{2}\frac{d^2\omega}{dk^2},$$

with the nonlinear coefficient $q = -\frac{d\omega}{d|\psi|^2}$.

As the dispersion coefficient p is always negative, a finite amplitude sinusoidal wave is modulationally unstable for $q < 0$ [10]. The NLSE (1) has a rational solution of the form [7,11]

$$\psi(\xi, \tau) = \frac{2}{\sqrt{q}}\left[\frac{4(1+i\tau)}{1+4\tau^2+2\xi^2/p} - 1\right]\exp(i\tau). \qquad (2)$$

The development of the initial amplitude modulated wave packet is then given by

$$\eta(x,t) = \mathrm{Re}\{\psi(x,t)\exp[i(kx-\omega t)]\}, \qquad (3)$$

where $\psi(x,t)$ is the dimensional form of Eq. (2) which can be obtained by using the transformation $\xi \rightarrow a_0 k_D(x - V_g t)$ and $\tau \rightarrow a_0^2 \omega_{\mathrm{pi}} t$, where a_0 represents the initial wave amplitude of the background carrier wave.

The evolution of the wave with increasing value of a_0 is shown in Fig. 1 by numerically solving Eq. (3). Here we consider $\omega = 0.7\omega_{\mathrm{pi}}$, $k = 0.74k_D$, which are the experimentally measured values for the ion-acoustic wave in the multicomponent plasma. The space coordinate is fixed at $x = 13.2$ cm. The values for the dispersion coefficient (p) and the nonlinear coefficient ($q/4$) are taken to be -0.5 and -0.4, respectively, for the present plasma condition [6]. For smaller value of a_0 the perturbation evolves as a wave envelope which undergoes amplitude and phase modulation leading to the formation of the solitary wave with increasing carrier wave amplitude. The wave amplitude is nearly 2.5 times the carrier amplitude for $a_0 = 0.035$. The special feature of Eq. (3) is that the solution is localized both in space and time as already shown [8,9].

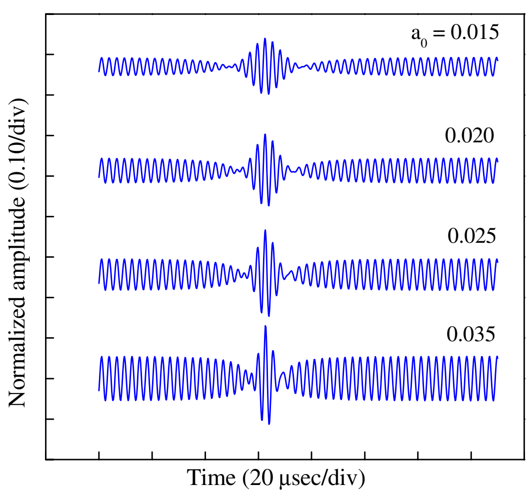

FIG. 1 (color online). Numerical result of Eq. (3) for different values of a_0. Parameters used are $\omega = 0.7\omega_{\mathrm{pi}}$, ($\omega_{\mathrm{pi}} = 492$ kHz), $k = 0.74k_D$, $k_D = 1/\lambda_D = 20.0\ \mathrm{cm}^{-1}$, $x = 13.0$ cm.

The experiment has been carried out in a multidipole double-plasma machine [12]. The diameter of the device is 30 cm and its total length is 120 cm. The device is separated into a source and a target section with a floating grid. The grid consists of a stainless steel mesh 50 lines/inch with 83% transparency. The cathodes consist of 0.1 mm diameter tungsten filaments and are placed 6 cm from the surface of the anode. Each section has five filaments with a length of 6 cm. The chamber is evacuated down to 2.0×10^{-4} Pa with an oil diffusion pump. Argon and sulfur hexafluoride are introduced independently into the chamber under continuous pumping. The pressure of Ar is 5.7×10^{-2} Pa and the pressure of SF_6 is varied from 0 to 1×10^{-3} Pa. The discharge voltage is 70 V and the discharge currents of the two sections are 10–50 mA. Plasma parameters as measured with a Langmuir probe of 6 mm diameter are: the electron temperature is approximately 1.1 eV and the electron density is approximately $3.8 \times 10^8\ \mathrm{cm}^{-3}$. Wave signals are detected with the axially movable Langmuir probe which is biased positively with respect to the plasma potential (≈ 1.5 V) to collect the electron saturation current and is therefore sensitive to the perturbed electron density.

First, we start the experiment with a positive ion plasma, i.e., without introducing SF_6 gas into the Ar discharge. A positive sinusoidal pulse is applied to the anode of the source. The pulse excites ion-acoustic solitons (KdV solitons) [2,13]. Next SF_6 gas is introduced into the chamber to a critical partial pressure. At the critical partial pressure, the effective density of F^- ions divided by Ar^+ ions is 0.1 [4]. At this pressure, the KdV solitons disappear and the modified KdV solitons (both compressive and rarefactive solitons are excited simultaneously) are seen to propagate [4,13,14].

Then instead of the positive pulse, a slowly varying amplitude modulated continuous sinusoidal signal is applied to the source anode and ion-acoustic perturbations are excited. Examples of observed signals at different distances from the separation grid for fixed carrier amplitude (5.4 V peak to peak) are shown in Fig. 2. The carrier frequency is 350 kHz and modulation frequency is 31 kHz. Close to the grid the observed signals resemble the applied signal. With increasing distance the compression of the wave packet sets in and the first Peregrine soliton emerges at 10.5 cm. The signals recorded at 10.5, 12.5, and 14.5 cm are shown with different amplitude scale for better resolution. The perturbation grows in amplitude (which is measured as the ratio of the soliton amplitude to that of the nearby carrier wave) and reaches a maximum at 12.5 cm. The observed maximum amplitude is 2.5 times the nearby carrier wave. The width of the perturbation becomes narrower during growing stage. At further distances ($\geq$14.5 cm) the perturbation decays. This observation

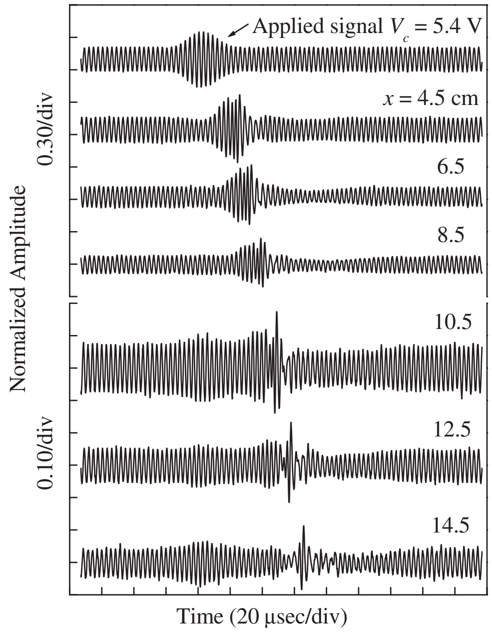

FIG. 2. Observed signals of the electron density perturbation at different probe positions from the separation grid. The top trace is the applied signal with carrier and modulation frequencies 350 and 31 kHz, respectively. Peak to peak amplitude of the applied carrier wave (V_c) is fixed at 5.4 V. Signals observed at 10.5 to 14.5 cm are shown with different amplitude scale (0.10/div) for better resolution.

indicates one important characteristic of Peregrine soliton that is the spatiotemporal focusing effect which distinguishes it from other solitary waves. The perturbation moves with group velocity and measured to be 2.1×10^5 cm s^{-1} (Mach number ~ 1.17).

In order to compare the observed Peregrine soliton with the numerical results we excite the wave with different excitation voltages (V_c) of the carrier wave at a fixed probe position ($x = 13.6$ cm). The observed signals are shown in Fig. 3. At lower excitation amplitude the perturbation shape is of the envelope type which is similar to the numerical result (Fig. 1) for smaller a_0 value. The Peregrine soliton is created when the excitation amplitude is ≥ 4.7 V. The amplitude of the highest Peregrine soliton (corresponding to $V_c = 6.0$ V) observed is approximately 3.5 times that of the nearby carrier wave which agrees with the theoretical prediction [7]. The same effect is seen in the numerical results when the Peregrine soliton is formed for higher values of a_0. A direct comparison of the observed time series of the Peregrine soliton with numerical results is shown in Fig. 4. The shape of the observed soliton reasonably agrees with the numerical results. The peak to peak amplitude of the observed soliton is ~2.5 times the carrier wave amplitude and the breather maximum is flanked by deep trough on the either side which is an important criterion to be identified as a Peregrine soliton. This observation is consistent with the previous experiment

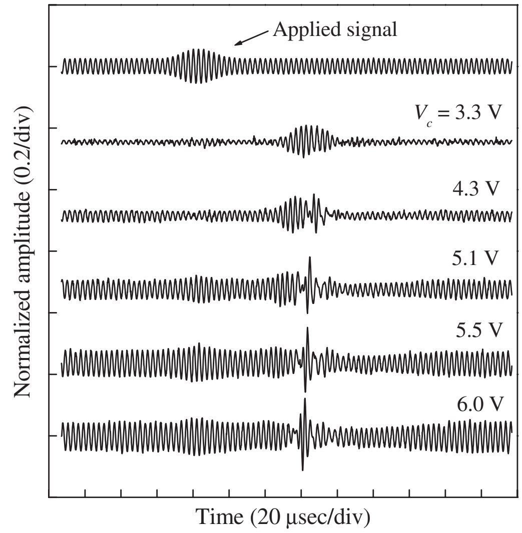

FIG. 3. Signals recorded for different excitation amplitudes of the carrier wave. The probe is fixed at 13.6 cm from the separation grid. Top trace represents the applied signal with carrier and modulation frequencies 350 and 31 kHz, respectively.

[9]. The slight shift in the phase of the carrier part with theory is probably due to the presence of pseudowave in front of the solitons [15]. However, detailed investigation is necessary for confirmation. We analyzed the wave signals shown in Fig. 3 using the fast Fourier transform (FFT). The obtained frequency spectrum is triangular of which the peak is at the carrier frequency. This result is the same as the one obtained in the experiments using fiber optics [8] and deep water waves [9].

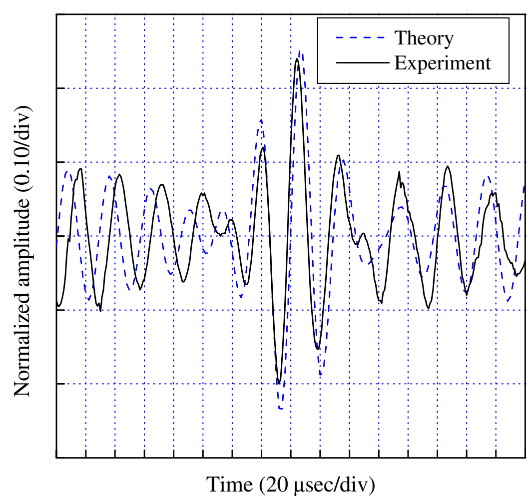

FIG. 4 (color online). Comparison of the time series signal (solid line) observed at 13.6 cm with the theoretical Peregrine soliton (dashed line) obtained by using Eq. (3). The applied carrier and modulation frequencies are 350 and 31 kHz, respectively. $V_c = 5.9$ V. The parameters used for numerical calculations are $\omega = 0.7\omega_{\rm pi}$, ($\omega_{\rm pi} = 492$ kHz), $k = 0.74k_D$, $k_D = 1/\lambda_D = 20.0$ cm^{-1}.

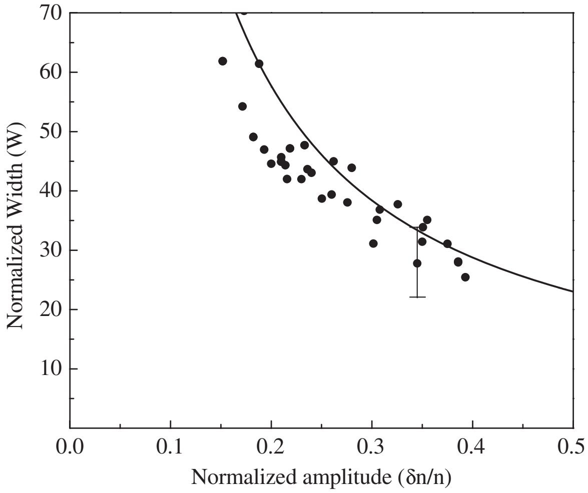

FIG. 5. Measured widths W normalized with the electron Debye length λ_D as a function of amplitude δn (peak to peak)/n, where δn is the perturbed electron density. The solid curve is the theoretical width estimated from the wave signals as shown in Fig. 1 [$W \times (\delta n/n) = 11.5$].

The width W of the observed Peregrine solitons is measured and the results are shown in Fig. 5 as a function of the normalized amplitude. The result shows that $W \times (\delta n/n)$ is almost constant, which agrees with the theoretical result obtained from the numerical calculation of Eq. (2). This characteristic of the Peregrine soliton, i.e., the width $\times$ amplitude is constant, is the same as the modified KdV soliton [4] and the envelope soliton [6]. Equation (7) of Ref. [9] also predicts a similar result.

In summary, Peregrine solitons are observed in the multicomponent plasma with negative ions when the density of the negative ions is equal to the critical value. Considering the previous experiment on mKdV solitons and envelope solitons, Peregrine solitons have also been found to propagate in the negative ion plasma.

The authors would like to thank Dr. K. E. Lonngren for suggesting the present experiment.

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