Superluminal Neutrinos at OPERA Confront Pion Decay Kinematics

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Violation of Lorentz invariance (VLI) has been suggested as an explanation of the superluminal velocities of muon neutrinos reported by OPERA. In this Letter, we show that the amount of VLI required to explain this result poses severe difficulties with the kinematics of the pion decay, extending its lifetime and reducing the momentum carried away by the neutrinos. We show that the OPERA experiment limits $\alpha = (v_\nu - c)/c < 4 \times 10^{-6}$. We then take recourse to cosmic-ray data on the spectrum of muons and neutrinos generated in Earth's atmosphere to provide a stronger bound on VLI: $(v - c)/c < 10^{-12}$.

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The recent OPERA report [1] of superluminal velocities for the muon neutrinos, $v(\nu_\mu)/c = 1 + \alpha$, $\alpha = 2.5 \times 10^{-5}$, has evoked much interest. Indeed present information on neutrino oscillations suggests much stronger bounds on putative superluminal anomalies for neutrinos [2,3]. Still this recent experiment and previous measurements at Fermilab [4] and MINOS [5] supporting this result prompted many theoretical and phenomenological comments. These possibilities include speculations of segregating the effect only into the ν_μ sector [6–8]. In this Letter, we study the implications of the superluminal velocities of the neutrinos on the kinematics of pion decay and show that superluminal velocities for ν_μ are severely constrained by these considerations. The constraints derived here are not restricted to any specific model but merely probe into consequences of superluminal motion of ν_μ from pion, kaon, and other decays.

Most of the attempts to understand the OPERA result consider violation of Lorentz invariance (VLI) at the phenomenological level [2,3,9–12]. There are also theoretical motivations stemming from string theory and from models with extra dimensions. In these models VLI increases with energy as a power law and have the general characteristic of modifying the maximum attainable velocity of the particles.

The phenomenology of these models has been extensively studied [3,10–12], and important constraints on the level of VLI have been established. Of particular interest is the work of Cohen and Glashow [13], who discuss the possibility of $\nu_\mu \rightarrow \nu_\mu + e^- + e^+$ or $\nu_\mu \rightarrow \nu_\mu + \nu_e + \bar{\nu}_e$ and derive strong constraints on VLI. Other ideas of emission of gravitational radiation have also been discussed [14]. Keeping these in mind, additional assumptions are required to accommodate the large superluminal velocities reported by the OPERA collaboration. The very severe constraints come from the neutrino sector: neutrino-oscillation experiments suggest that the amount of Lorentz noninvariant contribution for all the three neutrinos to be the same ($\alpha_{\nu_e} = \alpha_{\nu_\mu} = \alpha_{\nu_\tau} = \alpha_\nu$), as noted by Coleman and Glashow [2,3]. The observations of neutrinos from SN1987A [15] require that $|\alpha_\nu| < 10^{-9}$. The recent OPERA claim of $\alpha_{\nu_\mu} = 2.5 \times 10^{-5}$, together with SN1987A constraints seems to indicate that the VLI parameter grows rapidly with energy, as suggested by some models.

In this Letter, we note that such a large value of α_{ν_μ}, whether energy-independent or energy-dependent, will have many other phenomenological manifestations. Specifically, they would affect the kinematics and the rate of $\pi \rightarrow \mu + \nu_\mu$ decay, for high energy pions in ways that many experiments (OPERA included) would have detected. Moreover, the change in the rate of pion decay would affect the flux of the cosmic-ray muons and muon neutrinos significantly, in conflict with observations which extend up to $\sim 10^4$ and $\sim 10^5$ GeV, respectively. In the present analysis we assume that the neutrinos detected at Gran Sasso arise exclusively from pion decay, even though there could be some contributions from kaons. As a justification of this assumption we note that the charged kaon multiplicity at these energies is low ~ 0.3, much smaller than the pion multiplicity of ~ 6, and that the transverse momenta of kaons are larger than that of the pions and the transverse momenta of neutrinos arising in K decay are larger than those from π decay, so that the probability of $K \rightarrow \mu + \nu_\mu$ contributing to the detector at Gran Sasso 730 kms away is reduced. Furthermore, the considerations presented here are equally applicable (with some numerical modifications) to K decay as well. A more detailed analysis of the kaon contributions is certainly warranted both in the context of OPERA results and for the analysis of cosmic-ray data, but will not change the conclusions of this Letter significantly.

In the formalism for VLI, given by Coleman and Glashow [2], different particles achieve different terminal velocities, and accordingly, for the discussion of π decay, we make the minimal assumption that muon neutrinos have superluminal motion and the $\pi^{\pm}$, $\mu^{\pm}$, being charged particles, have terminal velocities equal to the velocity of light to avoid Cherenkov radiation in vacuum. Thus, unlike the analysis reviewed in the introduction, our analysis presented here does not directly apply to ν_e and ν_τ, except indirectly because of neutrino oscillations. However, the two-body kinematics presented here, with appropriately chosen α, is valid in all cases where one of the emergent particles has a superluminal terminal velocity. In models, where α increases with energy, the constraints derived in this Letter become much more stringent. We make the following standard assumptions. (A1) Energy-momentum conservation. (A2) The relation $\partial E/\partial p = v$ between the velocity of a particle and the change of its energy with momentum. This classical relation applies also to the group velocity of waves, $v_{\text{group}} = \partial\omega/\partial k$, and extends it to wave mechanics as well. (A3) The positivity of energy for free particles (by which we exclude Tachyons). The assumed $E - p$ relations for different fields are variants of deformed forward light cones or mass hyperboloids. These criteria are applicable to most existing VLI models.

The assumption A2 for the muon neutrinos implies

$$\int dE = \int v(p)dp, \tag{1}$$

where $v(p) > 1$ beyond some small value of momentum $p_{\min}$ that is much larger than the tiny sub-eV mass of the neutrinos, m_ν, which we neglect. Defining, in general,

$$\left\langle\frac{\partial E}{\partial p}\right\rangle = 1 + \alpha, \tag{2}$$

as the effective average over the neutrino momenta detected in the OPERA experiment, centered around 17 GeV, we write the energy-momentum relation at high energies as

$$E_\nu = p_\nu(1 + \alpha), \tag{3}$$

where α corresponds to VLI, required to understand the OPERA anomaly. The kinematic analysis begins with the standard mass-energy relation for π, μ:

$$E_i = (p_i^2 + m_i^2)^{1/2}. \tag{4}$$

We then express the four-vector of the decaying pion as

$$(E_\pi, p_\pi, 0, 0) \tag{5}$$

and those of the final neutrino and muon as

$$(E_\nu, p_{\nu\ell}, p_{\nu t}, 0) \quad \text{and} \quad (E_\mu, p_{\mu\ell}, p_{\mu t}, 0), \tag{6}$$

respectively, where the longitudinal components of momenta are taken to be $p_{\nu\ell} = \eta p_\pi$ and $p_{\mu\ell} = (1 - \eta)p_\pi$, and the transverse components as $p_{\nu t} = -p_{\mu t} = p_t$. With this choice the conservation of all the spatial components of momenta is evident.

We still need to satisfy the energy conservation:

$$E_\pi = E_\nu + E_\mu, \tag{7}$$

with:

$$\begin{aligned} E_\pi &= [p_\pi^2 + m_\pi^2]^{1/2}; \\ E_\nu &= [p_\pi^2\eta^2 + p_t^2]^{1/2}(1 + \alpha) \quad \text{and} \\ E_\mu &= [p_\pi^2(1 - \eta)^2 + p_t^2 + m_\mu^2]^{1/2}. \end{aligned} \tag{8}$$

Keeping in mind that in accelerator experiments including OPERA, m_π/p_π, m_μ/p_π, and p_t/p_π are very small, we expand the square root and keep only the leading term to get

$$\frac{m_\pi^2}{2p_\pi} = \frac{p_t^2 + m_\mu^2}{2p_\pi(1 - \eta)} + \eta\alpha p_\pi + \frac{p_t^2(1 + \alpha)}{2p_\pi\eta}. \tag{9}$$

Rearranging we can write

$$\frac{m_\pi^2}{2p_\pi^2} - \frac{m_\mu^2 + p_t^2}{2p_\pi^2(1 - \eta)} - \frac{p_t^2}{2p_\pi^2\eta} = \alpha\left(\eta + \frac{p_t^2}{2p_\pi^2\eta}\right) \tag{10}$$

or

$$\alpha = \frac{1}{2p_\pi^2\eta^2 + p_t^2}\left[m_\pi^2\eta - m_\mu^2\frac{\eta}{(1 - \eta)} - p_t^2\frac{1}{(1 - \eta)}\right]. \tag{11}$$

Since p_t^2 is positive, this yields a constraint:

$$\alpha\eta \le \frac{1}{2p_\pi^2}\left[m_\pi^2 - \frac{m_\mu^2}{(1 - \eta)}\right]. \tag{12}$$

In the OPERA experiment the typical energy of the neutrinos is ~17 GeV that arise from the decay of pions with a mean energy of ~60 GeV, so that the typical value of $\langle\eta\rangle \approx 0.3$. Inserting this value of η into Eq. (12) we obtain the bound:

$$\alpha_{\text{OPERA}} \le \frac{1}{0.6p_\pi^2}\left[m_\pi^2 - \frac{m_\mu^2}{0.7}\right] \sim 4 \times 10^{-6}. \tag{13}$$

Note that this bound on the superluminal parameter, α, is significantly smaller than 2.5×10^{-5} estimated from the time profiles of the events and the GPS timing in their experiment.

Next, we address the question, whether η could indeed be smaller than the assumed value of ~0.3 which would allow the value of α, estimated in our analysis, to be consistent with the OPERA result. For this, special selection effects should conspire to push η down to ~0.05. We note that this hypothesis would imply significant enhancement of the lifetime of the pions. To see this, note that within this standard kinematics of pion decay, the value of the η parameter is uniformly distributed in the range

$$0 \le \eta \le 1 - \frac{m_\mu^2}{m_\pi^2}, \tag{14}$$

i.e., in the range $\sim 0 - 0.5$. The phase space for the pion decay is directly proportional to this range and any reduction in this range will have a corresponding reduction in the rate of decay. It is straightforward to perform the Lorentz noninvariant phase space integral after modifying the δ functions representing mass shell conditions according to Eqs. (3)–(8). Such a calculation yields an integral directly proportional to $\eta_{\max}$. Thus with the reduction of $\langle\eta\rangle$ to 0.05, the pion lifetime will be extended by a factor of 6 or more, which is excluded by various accelerator experiments, including OPERA.

As seen clearly from Eqs. (10)–(12), the bounds get stronger in proportion to p_π^2, or even with higher powers in models with α increasing with energy invoked recently for explaining how the OPERA results need not be flavor specific and still be consistent with the small α inferred from SN1987A neutrinos. Accordingly, the bounds on VLI become extremely stringent for the ultrahigh energy muons and neutrinos observed in deep underground experiments at Kolar Gold Fields, Kamiokande, Baksan, IceCube, and other experiments [16–18].

Before we discuss these cosmic-ray observations, we note that the fraction of the momentum carried away by the muon in the standard decay kinematics of the pion, $(1 - \eta)$ is in the range of $\sim$0.5–1. The spectrum of muons generated in the Earth's atmosphere is well measured up to energies of $\sim 10^5$ GeV and we confine our analysis to the spectrum up to $\sim 4 \times 10^4$ GeV where the muons arise mainly from the decay of pions and kaons and the contributions of muons generated by neutrino interactions in rock to the depth intensity curve could be neglected. The observed differential energy spectrum is well represented by the theoretical estimate [19]:

$$f_\mu(E) \cong \left[A_\pi\langle 1 - \eta\rangle_\pi^{\beta-1}\left\{\frac{\langle 1-\eta\rangle_\pi \mathcal{E}_\pi}{E + \langle 1-\eta\rangle_\pi \mathcal{E}_\pi}\right\} + A_K\langle 1 - \eta\rangle_K^{\beta-1}\left\{\frac{\langle 1-\eta\rangle_K \mathcal{E}_K}{E + \langle 1-\eta\rangle_K \mathcal{E}_K}\right\}\right]E^{-\beta}, \quad (15)$$

where: β = spectral index of the cosmic-ray spectrum $\sim$2.65; $\mathcal{E}_\pi = h_0(\theta)m_\pi/c\tau_\pi$; $\mathcal{E}_K = h_0(\theta)m_K/c\tau_K$; $h_0(\theta) = 7 \times 10^5 \sec\theta$ cm, the scale height of Earth's atmosphere at a zenith angle θ; $\tau_{\pi/K}$ = decay lifetimes of pions/kaons at rest; $\langle 1 - \eta\rangle_{\pi/K}$ = the fractional momenta carried away by the muons in pion/kaon decay averaged over the spectrum of cosmic rays, around the energy band of interest; $A_{\pi/K}$ = Constants.

These constants are estimated from the inclusive cross sections for the production of pions and kaons at high energies and indicate that the net contribution of K decay is $\sim$10% for the muons and about $\sim$70% of the total flux of neutrinos at the highest energies. A similar expression for the flux of neutrinos generated in the atmosphere results when we replace $\langle 1 - \eta\rangle$ by η in Eq. (15). Notice that at very high energies $\gtrsim 10^3$ GeV, with $E \gg \mathcal{E}_{\pi/K}$, the spectra of muons and neutrinos become steeper with a spectral index $\sim(\beta + 1)$. Furthermore, the spectral intensities became proportional to $\langle 1 - \eta\rangle^\beta$ or $\langle\eta\rangle^\beta$ as the case might be.

Now the spectrum of muons presented by Novoseltsev *et al.* [17], fits well with Eq. (15), that assumes that $\tau_{\pi/K}$ are constants. Thus $\langle\eta\rangle$ has to be constant up to energies of $\sim 4 \times 10^4$ GeV. Note that Eq. (15) is sensitive to change in $\langle\eta\rangle$ in two ways—first through the change $\langle 1 - \eta\rangle^\beta$ and more importantly through its effect on extending the lifetime of pions and kaons. Thus the muon data imply

$$\alpha\eta < 10^{-11}. \quad (16)$$

Much more extensive data of the atmospheric muons (2×10^{10} events) and upward neutrinos (2×10^4 events) of energies in the range of 1–400 TeV, generated by energetic cosmic rays from the other side of the Earth have been provided recently by the south pole IceCube experiment [16,18], which shows a good fit with an index $\sim(\beta + 1) \sim 3.65$ at energies $E \gg \mathcal{E}_{\pi/K}$. Thus their observations imply a constraint

$$\alpha\eta < 10^{-13}. \quad (17)$$

Keeping in mind that we can not allow significant changes in η as they will affect the spectral slope and spectral intensities of the muons and neutrinos, the limits derived here represent bounds on the superluminal parameter α, which may be stated conservatively as

$$\alpha < 10^{-12}, \quad (18)$$

allowing nearly a factor of 10 variance for any contributions of the uncertainties in the cosmic-ray observations and the approximations used in our analysis. It should be noted that since spectra of both the muons and the neutrinos very well fit the estimates, which assume τ_π and η to be constants, the bound on the VLI parameter α follows exclusively from kinematic considerations. Indeed accurate spectra of atmospheric muons and/or neutrinos even at lower energies of several TeV can be used to improve the limits presented here. Our limit on α_{ν_μ} is far more stringent than that derived from the observations of the neutrinos emitted in SN1987A. It may be appropriate to note here that the observation of even a single event initiated by the neutrinos generated through the Greisen-Zatsepin-Kuzmin process in detectors like ANITA [20] would improve the bound on α to $\sim 10^{-20}$. To assess the impact of these results on specific models, we note that in general the matrix element in VLI theories may also have a novel energy dependence; however, they are unlikely to exactly cancel the above purely kinematic effects derived in this Letter.

We would like to draw attention to an independent analysis of the IceCube data by Bi *et al* [21], who assume that the superluminal ν_μ may be treated as having an effective mass, $m_{\rm eff} = [m_\nu^2 + 2\alpha p_\nu^2]^{1/2}$, so that the decay mode $\pi \rightarrow \mu + \nu_\mu$ becomes forbidden beyond a threshold energy for the neutrinos. This analysis yields results similar to our results, which we have derived showing the progressive kinematic restriction of the phase space available for π decay, leading to a monotonic increase of pion lifetime with energy.

In summary, we presented here a strong constraint $(v - c)/c < 10^{-12}$ on the amount of violation of Lorentz invariance from pion decay kinematics and cosmic-ray data. Careful observations of the fluxes of very high energy muons and neutrinos at accelerators and in cosmic rays, and their comparison with the expected fluxes will constrain any possible variation of the decay lifetime of the pion, which in turn, will lead to better bounds than those reported here.

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