

## Three-Body Correlation Functions and Recombination Rates for Bosons in Three Dimensions and One Dimension

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We investigate local three-body correlations for bosonic particles in three dimensions and one dimension as a function of the interaction strength. The three-body correlation function  $g^{(3)}$  is determined by measuring the three-body recombination rate in an ultracold gas of Cs atoms. In three dimensions, we measure the dependence of  $g^{(3)}$  on the gas parameter in a BEC, finding good agreement with the theoretical prediction accounting for beyond-mean-field effects. In one dimension, we observe a reduction of  $g^{(3)}$  by several orders of magnitude upon increasing interactions from the weakly interacting BEC to the strongly interacting Tonks-Girardeau regime, in good agreement with predictions from the Lieb-Liniger model for all strengths of interaction.

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Correlation functions reflect the nonclassical nature of quantum many-body systems. They may be used to characterize the latter when quantities such as temperature, density, dimensionality, and particle statistics are varied in experiments. It is particularly instructive to monitor a system's correlation functions as the strength of particle interactions is tuned from weak to strong. A paradigm is given by an ensemble of bosons in one-dimensional (1D) geometry with contact interactions [1]: For weak repulsive interactions, in the zero-temperature limit, the system is a quasicondensate with essentially flat particle correlation functions in position space to all orders. For strong repulsive interactions, the bosons avoid each other, leading to loss of coherence and strong increase of local correlations. In the context of ultracold atomic gases, with exquisite control over temperature, density, and dimensionality [2], tuning of interactions is enabled by Feshbach resonances [3]. Local two- and three-body correlations in atomic many-body systems can be probed, e.g., in measurements of photoassociation rates [4] and of three-body recombination processes [5,6], respectively. Nonlocal two-body correlations for atomic matter waves have been measured in atom counting [7–10], noise-correlation [11–13], and *in situ* imaging [14] experiments. Recently, also nonlocal three-body correlations have become accessible in experiments [15,16].

Recombination processes are sensitive to the properties of the many-body wave function at short distances. In particular, the process of three-body recombination, in which three particles collide inelastically to form a dimer, is directly connected to the local three-particle correlation function  $g^{(3)} \equiv \langle \hat{\psi}^\dagger(x)^3 \hat{\psi}(x)^3 \rangle / n^3$ , which compares the probabilities of having three particles at the same position for a correlated and an uncorrelated system. Here,  $\hat{\psi}^\dagger$  and  $\hat{\psi}$  are atomic field operators and  $n$  is the density.

The function  $g^{(3)}$  depends strongly on quantum statistics [5,16] and temperature  $T$  [17–19]. For example, in 3D geometry, statistics change the value of  $g^{(3)}$  from zero for identical fermions to one for noninteracting classical particles and to six for thermal (noncondensed) bosons. For noninteracting bosons statistical bunching is suppressed in a Bose-Einstein condensate (BEC), for which  $g^{(3)} = 1$ . In addition, interactions also have a pronounced effect on  $g^{(3)}$ : In a 3D BEC, quantum depletion due to quantum fluctuations reduces the condensate fraction by increasing the number of occupied single-particle modes. In this case, beyond-mean-field calculations [20] predict an increase of  $g^{(3)}$  proportional to the square root of the gas parameter  $(na_{3D}^3)^{1/2}$ , where  $a_{3D}$  is the 3D *s*-wave scattering length. This increase of  $g^{(3)}$  has never been seen experimentally and is in stark contrast to the behavior of 1D systems. In 1D geometry, bosons with repulsive interactions minimize their interaction energy by avoiding spatial overlap. For very strong repulsive interactions in the Tonks-Girardeau (TG) limit [1,21–24] a strong reduction of  $g^{(3)}$  with a  $\gamma^{-6}$  scaling is predicted [25]. Here,  $\gamma$  is the dimensionless Lieb-Liniger parameter, which characterizes interactions in a homogeneous 1D system [1,26]. Recently,  $g^{(3)}$  has been calculated all the way from the weakly to the strongly interacting 1D regime for  $T = 0$  [27,28] and also for  $T \geq 0$  [18,19]. Experimentally, Laburthe Tolra *et al.* [6] have observed a reduction of  $g^{(3)}$  by a factor of about 7(5) for a weakly interacting gas of Rb atoms with  $\gamma = 0.45$ .

In this work we experimentally determine  $g^{(3)}$  in 3D and in 1D geometry using a trapped ultracold gas of Cs atoms with tunable (repulsive) interactions. For a BEC in 3D geometry we find clear evidence for an increase of  $g^{(3)}$  with increasing interaction strength, in good agreement with the prediction of Ref. [20]. In 1D, for which we can

tune  $\gamma$  from zero to above 100 [24], we determine  $g^{(3)}$  in the crossover regime from weak (1D BEC regime) to strong interactions (TG regime). Here our data agree well with the prediction of Ref. [27]. For strong interactions in the TG regime, our measurements show that  $g^{(3)}$  is suppressed by at least 3 orders of magnitude. For high densities and strong interactions, we observe a rather sudden increase of three-body losses after long hold times in the trap. Understanding the behavior of  $g^{(3)}$  at short and long times is an important step towards understanding integrability and thermalization in 1D systems [29,30].

A three-body loss process [3,31] consists of the collision of three particles, the formation of a dimer, and the release of the dimer's binding energy typically sufficient to allow both, the dimer and the remaining particle, to escape from the trap. The loss, assuming negligible one- and two-body loss, is modeled by the rate equation  $\dot{n} = -\alpha K^{(3)} g^{(3)} n^3$ . Here, we have explicitly split the loss rate coefficient  $\alpha K^{(3)} g^{(3)}$  into its three contributions. The parameter  $\alpha = 3$  describes a situation where exactly three particles are lost in each recombination event. In principle, secondary losses [32] could modify its value. However, in the following we will be interested in *relative* measurements of  $\alpha K^{(3)} g^{(3)}$ , which are only weakly dependent on the precise value of  $\alpha$  [26], allowing us to neglect a possible deviation of  $\alpha$  from the value of 3. The parameter  $K^{(3)}$  contains the effect of few-body physics on the loss process [3]. It depends on the probability of dimer formation (a process that can be strongly enhanced near Efimov resonances [33]) and generally varies strongly with  $a_{3D}$  [31,34–37]. For  $a_{3D}$  much larger than the range of the scattering potential,  $K^{(3)}$  shows a generic  $a_{3D}^4$  scaling. Contributions of many-body physics are contained in the three-particle distribution function  $g^{(3)} n^3$ . In what follows, we aim to measure  $g^{(3)}$  as a function of  $a_{3D}$  both in 3D and 1D geometry.

We determine  $K^{(3)} g^{(3)}$  from measurements of the decay of the total number of atoms  $N(t)$  in our trap [5,37], which obeys the loss equation  $\dot{N} = -3K^{(3)} g^{(3)} \int n^3(\mathbf{r}) d^3r$ . Figures 1(a) and 2(c) show typical atom number measurements for 3D and 1D geometry. The data in 3D geometry is well fitted by solutions to the loss equation. The determination of  $K^{(3)} g^{(3)}$  depends critically on an exact knowledge of the atomic density profile  $n(\mathbf{r})$ . In particular, particle loss and loss-induced heating of the sample [37] can modify the density profile in a nontrivial way. Also, on long time scales evaporative losses might start to play a role. To avoid these complications we restrict ourselves to short time intervals, during which not more than 15% of the atoms are lost, and we determine the slope  $\dot{N}(0)$  from a linear fit to the data. We determine  $\int n^3(\mathbf{r}) d^3r$  from a measurement of the total atom number  $N$  and the trap frequencies  $\omega_{x,y,z}$  using interaction dependent models for  $n(\mathbf{r})$  [26]. We find that the linear approximation underestimates  $K^{(3)} g^{(3)}$  by approximately 12%; however, the

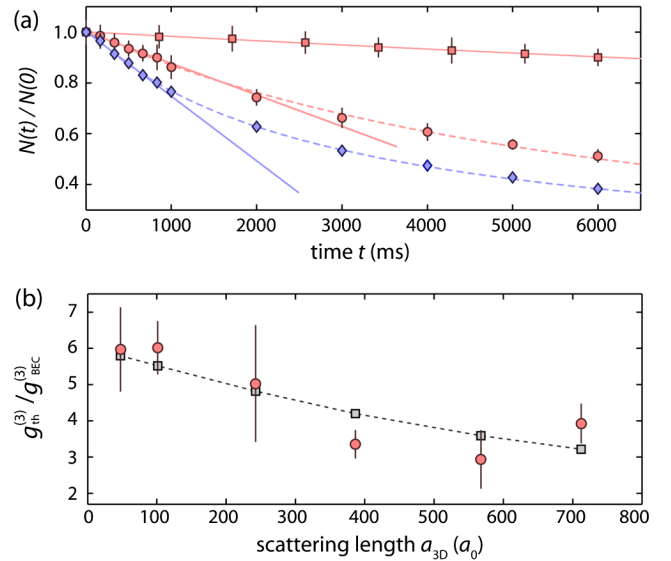


FIG. 1 (color online). (a) Relative atom number  $N(t)/N(0)$  vs hold time  $t$  in 3D geometry: BEC (squares and circles) and thermal gas (diamonds) for  $a_{3D} = 101(2)a_0$ ,  $386(3)a_0$ , and  $386(3)a_0$ , respectively. The dashed lines are fits to the data based on the loss equation (see text). The solid lines are linear fits that include the data from 100% to 85%. (b) The ratio of correlation functions  $g_{th}^{(3)}/g_{BEC}^{(3)}$  as a function of  $a_{3D}$  (experimental data: circles; prediction [20]: squares). All error bars reflect the 1 $\sigma$  statistical uncertainty.

data analysis is greatly simplified, especially in 1D. Finally, a comparative measurement of  $K^{(3)} g^{(3)}$  allows us to eliminate  $K^{(3)}$ , as explained below, and to determine  $g^{(3)}$  in 3D and 1D geometry.

*Correlation function in 3D.*—We measure  $K^{(3)} g^{(3)}$  for both a noncondensed thermal sample and a BEC as a function of  $a_{3D}$ . For the thermal sample we start with typically  $3.5 \times 10^5$  Cs atoms at a temperature of  $T \approx 200$  nK. The peak density is about  $n_0 = 1 \times 10^{14}$  cm $^{-3}$ . In the BEC [37,38] we have about  $9 \times 10^4$  Cs atoms without any detectable noncondensed fraction at about  $n_0 = 5 \times 10^{13}$  cm $^{-3}$ . We tune  $a_{3D}$  in the range from 50  $a_0$  to 800  $a_0$  by means of a broad magnetic Feshbach resonance [37,39] ( $a_0$  is Bohr's radius). The magnetic field gradient needed to levitate the atoms against gravity [37] introduces a slight (less than 5  $a_0$ ) variation of  $a_{3D}$  across the samples. We determine  $N$  by means of absorption imaging after a variable hold time  $t$  and 50 ms of expansion in the presence of the levitation field. We note that we do not observe the appearance of any noncondensed fraction in all measurements using the BEC. Figure 1(b) displays the ratio  $K_{th}^{(3)} g_{th}^{(3)} / (K_{BEC}^{(3)} g_{BEC}^{(3)}) = g_{th}^{(3)} / g_{BEC}^{(3)}$  determined from the thermal sample and the BEC as a function of  $a_{3D}$ . Here we have made the reasonable assumption that  $K^{(3)}$  is independent of the system's phase in 3D geometry, i.e.  $K_{th}^{(3)} = K_{BEC}^{(3)}$ . Our measurement shows that the ratio  $g_{th}^{(3)} / g_{BEC}^{(3)}$  attains the expected value of 6 for weak

interactions [5], but then exhibits a pronounced decrease as  $a_{3D}$  is increased. For comparison, we plot the prediction of Ref. [20]

$$g_{\text{th}}^{(3)}/g_{\text{BEC}}^{(3)} = 6/\left(1 + \frac{64}{\sqrt{\pi}}\sqrt{n_0 a_{3D}^3}\right). \quad (1)$$

We note that the density  $n_0$  enters into this equation as a measured quantity. In general, we find good agreement between the experimental and the theoretical result, establishing our measurement as a clear demonstration of beyond mean-field effects on  $g^{(3)}$  in 3D bosonic quantum gases.

**Correlation function in 1D.**—Figure 2(a) illustrates our experimental setup to generate an array of 1D systems. We load a BEC of typically  $8 \times 10^4$  atoms within 400 ms into approximately 5000 vertically ( $z$  direction) oriented tubes that are formed by two horizontally propagating, retro-reflected lattice laser beams. Each tube with index  $(i, j)$  in the  $x$ - $y$  plane has a transversal trapping frequency of  $\omega_{\perp} = 2\pi \times 12.2(5)$  kHz and an aspect ratio  $\omega_{\perp}/\omega_z$  of approximately 800. The transversal motion of the atoms in the tubes is effectively frozen out as kinetic and interaction energy are much smaller than  $\hbar\omega_{\perp}$ . We adjust  $a_{3D}$  in 100 ms to its final value. After time  $t$  we turn off the lattice potential and determine the total atom number  $N(t)$  by absorption imaging in a time-of-flight measurement.

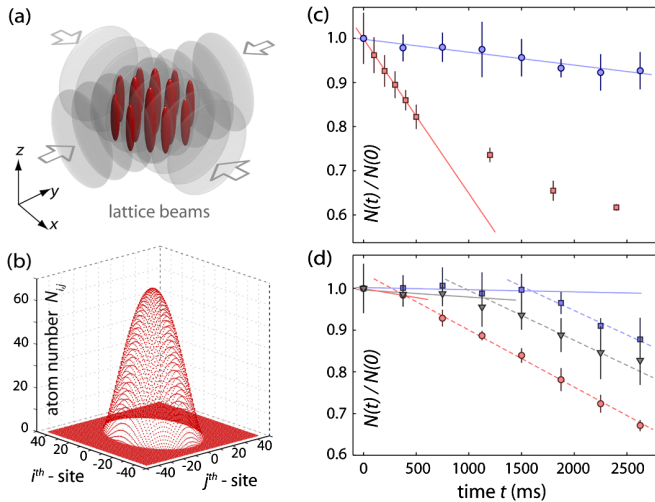


FIG. 2 (color online). (a) Sketch of the experimental setup: A 2D optical lattice traps atoms in an array of 1D tubes. (b) Example of a computed atom number distribution  $N_{i,j}$  (see text). (c) The relative atom number  $N(t)/N(0)$  as a function of time  $t$  in 1D geometry: squares and circles correspond to  $a_{3D} = 23(1)a_0$  and  $568(3)a_0$  with initial densities of  $4.5 \mu\text{m}^{-3}$  and  $1.7 \mu\text{m}^{-3}$  at the center of the center tube, respectively. The solid lines are linear fits to the initial slopes. (d) The relative atom number  $N(t)/N(0)$  in 1D for fixed  $a_{3D} = 568(3)a_0$  and for various values of  $\gamma$  as the 1D density is changed:  $\gamma = 12$  (circles),  $\gamma = 13$  (triangles), and  $\gamma = 14$  (squares). The solid (dashed) lines are linear fits to the data points for short (large) times to guide the eye.

In order to determine  $g_{1D}^{(3)}$  we calculate the ratio  $K_{1D}^{(3)}g_{1D}^{(3)}/(K_{3D}^{(3)}g_{3D}^{(3)}) = g_{1D}^{(3)}/g_{3D}^{(3)}$ . Here, it is not obvious that few-body physics is not affected by the confinement and that hence  $K_{1D}^{(3)}$  and  $K_{3D}^{(3)}$  cancel each other. Nevertheless, it is reasonable to assume that  $K^{(3)}$  is not significantly changed by the confinement as long as the confinement length  $a_{\perp} = \sqrt{\hbar/(m\omega_{\perp})}$  is larger than the extent of the dimer produced in the recombination event and the range of the scattering process, which are both of order of  $a_{3D}$ . Here,  $m$  is the atom mass. We choose a moderately deep lattice potential with  $a_{\perp} \approx 1500a_0$  and restrict  $a_{3D}$  to  $a_{3D} \lesssim 800a_0$ . In particular, we avoid the confinement-induced resonance condition  $a_{3D} \approx a_{\perp}$  [24,40].

The main difficulty in the determination of  $K_{1D}^{(3)}g_{1D}^{(3)}$  comes from the fact that the initial atom number of the tubes varies across the lattice as a result of the harmonic confinement. We choose to always load the lattice in a regime of weak repulsive interactions such that almost all 1D samples are initially in the 1D Thomas-Fermi (TF) regime [41]. The local chemical potentials  $\mu_{i,j}$ , the total atom number  $N$ , and the chemical potential  $\mu$  are then unambiguously related, and we can directly calculate the initial occupation number  $N_{i,j}$  for each tube  $(i, j)$  [[26] and Fig. 2(b)]. The variation in  $N_{i,j}$  results in a considerable variation in the type of density profile for each of the 1D systems after the strength of interactions is increased to the desired value: Some tubes remain in the 1D TF regime, while others are now in the TG regime. For tubes that are in the weakly interacting regime we determine the 1D density  $n_{1D}$  numerically by solving the 1D Gross-Pitaevskii equation. For the TG regime the density profiles are determined following Ref. [41]. In general, we find good agreement when we compare the numerical results to integrated density distributions from *in situ* absorption images. For the interaction parameter  $\gamma$  we take a mean value that is calculated as an average over all local  $\gamma_{i,j}$  at the center of each tube  $(i, j)$  weighted by  $N_{i,j}$  [26].

As before we determine  $K_{1D}^{(3)}g_{1D}^{(3)}$  from the initial slope of the loss curve as shown in Fig. 2(c). In Fig. 3(a) we compare the data that we obtain in 1D geometry to our data for  $K_{3D}^{(3)}g_{3D}^{(3)}$  for a 3D BEC as we vary  $a_{3D}$ . We note that the BEC data is in good agreement with previous three-body loss data on thermal samples when one takes into account the combinatorial factor  $3! = 6$  [33,37]. In particular, the 3D data follows the universal scaling law  $K^{(3)} \sim a_{3D}^4$  for sufficiently large  $a_{3D}$  [31,34–37]. We exclude data points affected by the presence of a narrow Feshbach resonance in the vicinity of  $a_{3D} = 150a_0$  [42]. Note that in the range from  $a_{3D} \approx 10a_0$  to  $a_{3D} \approx 850a_0$  three-body losses in 3D increase by nearly 3 orders of magnitude. This behavior is in stark contrast to the measurements in 1D. In 1D, we observe a *reduction* of  $K^{(3)}g^{(3)}$

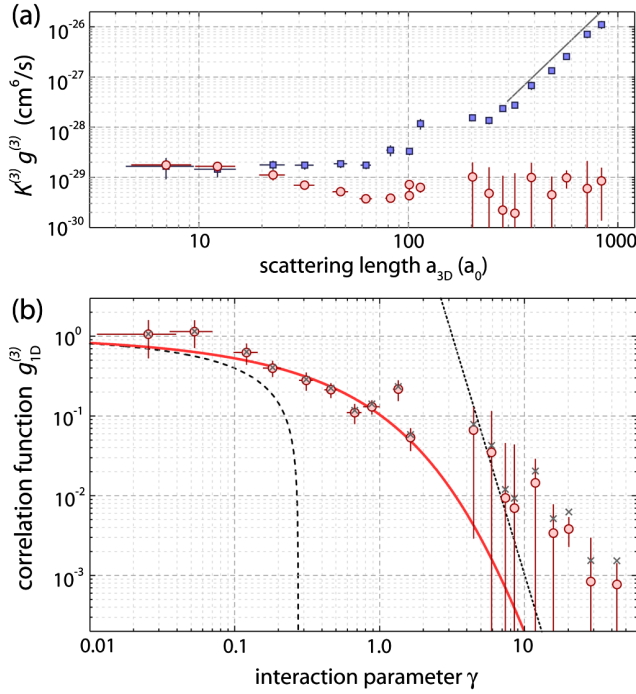


FIG. 3 (color online). (a) Three-body loss coefficient  $K^{(3)}g^{(3)}$  vs  $a_{3D}$  for a quantum degenerate gas in 3D (squares) and in 1D (circles). The line gives the  $K^{(3)} = C\hbar a_{3D}^4/m$  scaling in the universal regime in 3D with  $C = 67.9$  [31,34–37]. The error bars of  $K^{(3)}g^{(3)}$  reflect the  $1\sigma$  statistical uncertainty of the linear fit. The increased relative error for the larger values of  $a_{3D}$  is the result of greatly reduced atom loss, giving smaller slopes for the decay curves at roughly the same shot-to-shot particle variations, primarily due to the behavior of  $g^{(3)}$ , but also due to a reduction of the density (for details, see text). (b) The measured correlation function  $g_{1D}^{(3)}$  vs  $\gamma$  in 1D geometry (circles). The crosses indicate the values for  $g_{1D}^{(3)}$  corrected for the variations of  $g_{3D}^{(3)}$  with  $a_{3D}$  as in Eq. (1) (see text). The dashed and dotted lines are approximate analytical solutions for  $\gamma \ll 1$  and  $\gamma \gg 1$  from Ref. [25]. The solid line is the prediction from Refs. [27,28].

by approximately a factor of 2 upon increasing  $a_{3D}$  over the same range of values. In fact, for  $a_{3D} \geq 200a_0$  our measurement only gives an upper bound on  $K_{1D}^{(3)}g_{1D}^{(3)}$  as losses become so small that we have difficulty in determining  $\dot{N}(0)$  in view of shot-to-shot particle number variations. We note that not only the behavior of  $g_{1D}^{(3)}$  reduces the atom loss, but also the fact that the density is decreased as repulsive interactions are increased. In addition, on a more technical side, our loading procedure for  $a_{3D} \geq 200a_0$ , aimed at avoiding the Feshbach resonance near  $150a_0$ , leads to a lower density. Also note that tunneling between tubes (on a time scale of 1 s for the parameters of our lattice) sets an upper bound for the time scale for which the tubes can be considered to be independent and hence fully in the 1D regime.

In Fig. 3(b) we plot  $K_{1D}^{(3)}g_{1D}^{(3)}/(K_{3D}^{(3)}g_{3D}^{(3)}) \approx g_{1D}^{(3)}$  as a function of  $\gamma$ . A striking decrease by 3 orders of magnitude from the value 1 at  $\gamma \approx 0.03$  to  $10^{-3}$  at  $\gamma \approx 50$  can be

seen. We compare this result to the predictions based on the Lieb-Liniger model of interacting bosons in 1D: In the weakly interacting Gross-Pitaevskii regime ( $\gamma \ll 1$ ) the Bogoliubov approach yields  $g^{(3)}(\gamma) \approx 1 - 6\sqrt{\gamma}/\pi$ , while in the TG regime,  $\gamma \gg 1$ ,  $g^{(3)}$  can be expressed through derivatives of the three-body correlation function of free fermions, giving  $g^{(3)} = 16\pi^6/(15\gamma^6)$  [25]. Cheianov *et al.* [27] have recently calculated numerically  $g^{(3)}$  for all strengths of interactions within the Lieb-Liniger model, providing an interpolation between the weakly and strongly interacting limits [red continuous line in Fig. 3(b)]. We find very good agreement between the result of our experiment and the theory that is valid for all strengths of interactions. This is the central result of this work.

Finally, for large values of  $a_{3D}$  and  $n_0$ , and for long hold times in 1D geometry, we find a surprisingly sudden increase of losses as shown in Fig. 2(d), accompanied by a rapid increase for the expansion energy in the longitudinal direction (data not shown). The onset of increased losses shifts to later times with decreased density in the tubes, i.e., increased  $\gamma$ , and it is rather sensitive to the precise value of  $\gamma$ . We believe that the 1D tubes suffer from a recombination-heating induced breakdown of correlations: For sufficiently large values of  $a_{3D}$  the binding energy of the weakly bound dimer produced in the recombination process becomes comparable to the trap depth (here  $h \times 45$  kHz). This leads to a positive feedback cycle in the many-body system in which three-body losses lead to an increase of temperature [37] and thus of  $g^{(3)}$  [17–19], which in turn increases three-body losses.

In summary, we have measured the local value  $g^{(3)}$  for the three-particle correlation function for quantum degenerate gases in 3D and 1D. In 3D, increasing interactions deplete the condensate and increase the value of  $g^{(3)}$  in accordance with beyond mean-field calculations. In 1D, we observe a strong suppression for  $g^{(3)}$  by 3 orders of magnitude as the TG regime is entered. The accompanying suppression of three-body losses is crucial to the study of strongly interacting matter in and out of equilibrium in 1D [24,29,30,43].

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