Coherence Effects on Photon Absorption in Optically Thick Plasmas

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A kinetic photon transport model that accounts for spatial coherence is applied to line radiation in optically thick plasmas. It is shown that the photon emission and absorption processes are delocalized in space, which alters the global plasma opacity to spectral lines. Based on this analysis, we demonstrate that spectral profiles and escape factors can be much larger than expected from usual formulas.

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In this Letter, a mechanism associated with light's wave-particle duality is explored, which leads to the alteration of opacity effects in plasmas. Opacity is a fundamental issue in plasma spectroscopy where line shapes are a very useful diagnostic tool [1,2], in inertial and magnetic fusion plasmas [3–6] as well as in astrophysics [7] or in technical applications such as lighting [8]. Opacity models for radiative transfer are widely used in high-energy density physics, e.g., to characterize the warm and hot dense matter present in imploding inertial confinement capsules or in stars [3,9,10]. Another important application of opacity models is provided by radiation transport simulations in magnetic fusion, in particular, with the advent of large-scale devices such as the ITER facility (currently under construction in France) and the development of integrated modeling codes supporting its operation [11–14]. As a rule, the reliability of the interpretation of spectra, as well as predictions, relies on the development of accurate spectroscopy models accounting for opacity effects.

The new mechanism described here applies to line radiation whose coherence length is comparable to the photons' mean free paths. This issue cannot be addressed within the conventional radiative transfer formalism where, by definition, the light is described by pointlike particles propagating along rays (''geometrical optics limit''). More explicitly, the conventional radiation transport theory uses as a fundamental quantity, the so-called specific intensity—an energy flux per unit frequency and solid angle—and assumes it to obey a Boltzmann-like transport equation, similar to that used in the kinetic theory of gases. This analogy suggests an interpretation in terms of particles referred to as ''photons'' evolving in phase space, interacting with matter through spontaneous or stimulated emission, absorption and scattering. This interpretation is not obvious because the photon as viewed from QED does not have a definite position operator due to symmetry considerations [15]. Various papers confirm this point with different theoretical approaches [16–19]. The absence of such an operator leads to ambiguity if one tries using the correspondence principle to associate the equation of transfer with a quantum transport equation for photons. A proper approach to the particle description of radiation transport is provided by the Wigner quantum phase space formalism adapted to second quantization (e.g., [20]). Whereas several papers report on theoretical developments [21–26], no explicit application to spectral lines seems to have been carried out so far, and it is the purpose of the present Letter to address this issue.

The fundamental quantities of interest in the quantum transport model are the Wigner quasiprobability distributions $W_s(\mathbf{r}_1, \mathbf{p}_1, \ldots, \mathbf{r}_s, \mathbf{p}_s, t)$, which are generalizations of the s-particle phase space distribution functions used in classical kinetic theory. They account for nonclassical features such as Heisenberg's uncertainty principle and quantum entanglement. ''Quasiprobability'' means that these functions may become negative, which occurs for physical states that have no classical equivalent. A coarse graining procedure is done to avoid such a peculiarity, with a spatial scale much larger than the photon's thermal de Broglie wavelength $\hbar/\Delta p$ (where Δp is the typical momentum dispersion). For a spectral line, the thermal wavelength denotes the coherence length $\lambda_c \sim c/\Delta\omega_{1/2}$ where $\Delta\omega_{1/2}$ is the half width at half maximum (e.g., [27]).

We consider the radiation as unpolarized and focus on the one-photon Wigner function $W_1(\mathbf{r}_1, \mathbf{p}_1, t) \equiv W(\mathbf{r}, \mathbf{p}, t)$. This quantity is defined as the average of the phase space photon number operator $N(\mathbf{r}, \mathbf{p}, t)$ (we take $\hbar = 1$):

$$W(\mathbf{r}, \mathbf{p}, t) = \mathrm{Tr}[\rho(t) N(\mathbf{r}, \mathbf{p}, t)], \tag{1}$$

$$N(\mathbf{r}, \mathbf{p}, t) = \frac{1}{\pi^3} \sum_{\varepsilon} \int d^3k\, a_\varepsilon^\dagger(\mathbf{p} - \mathbf{k}) a_\varepsilon(\mathbf{p} + \mathbf{k}) e^{2i\mathbf{k}\cdot\mathbf{r}}. \tag{2}$$

Here, $\rho(t)$ is the density operator of the total (radiation-matter) system, $a_\varepsilon^\dagger(\mathbf{k})$ and $a_\varepsilon(\mathbf{k})$ are, respectively, the creation and annihilation operators corresponding to a photon of wave vector $\mathbf{k}$ and polarization ε, Tr denotes a trace over the whole system's Hilbert space. The creation and annihilation operators obey the bosonic commutation rules $[a_\varepsilon(\mathbf{k}), a_{\varepsilon'}^\dagger(\mathbf{k}')] = \delta_{\varepsilon\varepsilon'}\delta(\mathbf{k} - \mathbf{k}')$ and $[a_\varepsilon(\mathbf{k}), a_{\varepsilon'}(\mathbf{k}')] = 0 = [a_\varepsilon^\dagger(\mathbf{k}), a_{\varepsilon'}^\dagger(\mathbf{k}')]$. Examination of Eqs. (1) and (2) shows that $W(\mathbf{r}, \mathbf{p}, t)$ is normalized to the total number of photons. In the above definitions all operators are given in the interaction picture, i.e., of the form $A(t) = \exp(iH_0 t) A^S \exp(-iH_0 t)$ for any observable A^S in the Schrödinger picture. H_0 denotes the Hamiltonian of the free (noninteracting) radiation-matter system.

A closed transport equation for the one-photon Wigner function is obtained by applying the differential operator $\partial/\partial t + c(\mathbf{p}/p) \cdot \nabla \equiv D/Dt$ on each side of Eq. (1) and by using appropriate assumptions. The Lagrangian derivative involves two terms of different physical origins. (i) DN/Dt denotes the time variation of the phase space number operator along a trajectory. It is identically zero in the short wavelength limit, which corresponds to propagation along defined straight rays. This limit is well satisfied for spectral lines in the optical range, and we will focus on this case in the following. However, it should be noted that a closed expression in terms of the Wigner distribution can still be obtained in the general case, albeit more complicated. (ii) $D\rho/Dt \equiv d\rho/dt$ denotes evolution of the Wigner distribution due to the interactions between the radiation field and the plasma. These interactions correspond to photon emission and absorption. These processes are altered if the light's coherence length is significant with respect to the other space scales of interest. In order to obtain a closed expression for $\mathrm{Tr}[(d\rho/dt)N]$ (which is identical to DW/Dt), we assume that the radiation and matter are weakly coupled, so that the evolution of the density matrix can be described using a quantum master equation. Following standard approaches [28–30], we write

$$\frac{d\rho}{dt}(t) = -\int_0^\infty d\tau [V(t), [V(t-\tau), \rho(t)]], \tag{3}$$

where $V(t)$ describes the interaction between the charged particles and the radiation. This term is linear in the creation and annihilation operators. Within the dipolar approximation, it is given for a gas of neutral atoms by

$$V(t) = \sum_a -\mathbf{d}_a(t) \cdot \mathbf{E}(\mathbf{r}_a, t), \tag{4}$$

$$\mathbf{E}(\mathbf{r}, t) = \int \frac{d^3k}{(2\pi)^{3/2}} \sum_\varepsilon i \sqrt{\frac{\omega}{2\varepsilon_0}} a_\varepsilon(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} \boldsymbol{\varepsilon} + \mathrm{H.c.}, \tag{5}$$

where $\mathbf{d}_a$ and $\mathbf{r}_a$ are, respectively, the dipole operator and position operator of the center of mass of the ath atom and H.c. stands for Hermitian conjugate. We assume that the radiation always has a definite number of photons; i.e., quantum averages such as $\langle a_\varepsilon(\mathbf{k}) a_{\varepsilon'}(\mathbf{k}')\rangle$ and $\langle a_\varepsilon^\dagger(\mathbf{k}) a_{\varepsilon'}^\dagger(\mathbf{k}')\rangle$ are neglected. Algebraic manipulations lead to the following transport equation [26]:

$$\frac{DW}{Dt}(\mathbf{r}, \mathbf{p}, t) = S(\mathbf{r}, \mathbf{p}, t) - \int d^3r' \int d^3p' K(\mathbf{r}, \mathbf{p}, t; \mathbf{r}', \mathbf{p}') W(\mathbf{r}', \mathbf{p}', t). \tag{6}$$

Here, S denotes a source corresponding to photon creation through spontaneous emission and the kernel K represents absorption and stimulated emission. The explicit form of these terms depends on the atomic system under consideration. In the case of statistically independent atoms, they are given by

$$S(\mathbf{r}, \mathbf{p}, t) = \frac{1}{\pi^3 p^3} \mathrm{Re} \int d^3r' \int d^3p' \eta_c(\mathbf{r}', \mathbf{p}', t) \times \exp[-2i(\mathbf{p} - \mathbf{p}') \cdot (\mathbf{r} - \mathbf{r}')], \tag{7}$$

$$K(\mathbf{r}, \mathbf{p}, t; \mathbf{r}', \mathbf{p}') = \frac{c}{\pi^6} \mathrm{Re} \int d^3r'' \int d^3p'' \chi_c(\mathbf{r}'', \mathbf{p}'', t) \times \exp\{2i[(\mathbf{p}' - \mathbf{p}'') \cdot (\mathbf{r} - \mathbf{r}') - (\mathbf{p} - \mathbf{p}') \cdot (\mathbf{r} - \mathbf{r}'')]\}, \tag{8}$$

where η_c and χ_c are complex generalizations of the emission and absorption coefficients used in the radiative transfer theory. Considering the atomic transition $u \to d$, one has

$$\eta_c(\mathbf{r}, \mathbf{p}, t) = \frac{\omega_{ud}}{4\pi} N_u(\mathbf{r}, t) A_{ud} \phi_c(\omega, \hat{\mathbf{n}}, \mathbf{r}, t), \tag{9}$$

$$\chi_c(\mathbf{r}, \mathbf{p}, t) = \frac{\omega_{ud}}{4\pi} [N_d(\mathbf{r}, t) B_{du} - N_u(\mathbf{r}, t) B_{ud}] \phi_c(\omega, \hat{\mathbf{n}}, \mathbf{r}, t). \tag{10}$$

Here ω_{ud} is the Bohr frequency of the transition; A_{ud}, B_{du} and B_{ud} are the Einstein coefficients for spontaneous emission, absorption and stimulated emission, respectively; N_u and N_d are the densities of the species in the upper and lower levels; and ϕ_c is the complex spectral line profile normalized such that $\int d\omega \int d\Omega \phi_c(\omega, \hat{\mathbf{n}}, \mathbf{r}, t) = 4\pi$, with $\mathbf{p} = \omega\hat{\mathbf{n}}/c$, $\hat{\mathbf{n}} = \mathbf{p}/p$, and defined by

$$\phi_c(\omega, \hat{\mathbf{n}}, \mathbf{r}, t) = \int d^3v f(\mathbf{v}; \mathbf{r}, t) \phi_{0c}(\omega(1 - \hat{\mathbf{n}} \cdot \mathbf{v}/c), \hat{\mathbf{n}}, \mathbf{r}, t), \tag{11}$$

$$\phi_{0c}(\omega, \hat{\mathbf{n}}, \mathbf{r}, t) = \frac{1}{\pi} \int_0^\infty d\tau C(\tau; \hat{\mathbf{n}}, \mathbf{r}, t) e^{-i\omega\tau}, \tag{12}$$

where f is the atoms' velocity distribution function that accounts for thermal Doppler broadening and C is the autocorrelation function of the atomic dipole projected onto the polarization plane.

Equation (6), with the expressions of S and K given in Eqs. (7) and (8), is a generalization of the standard radiative transfer equation, which accounts for spatial coherence. The integrals in space involve a volume of typical

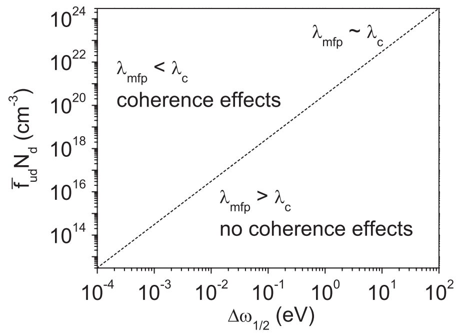

FIG. 1. Plot of the relation $\lambda_c/\lambda_{\mathrm{mfp}} = 1$. The oscillator strength $\bar{f}_{ud}$ depends on the transition under consideration. Coherence effects, expected when the ratio is larger than the unit (upper side), can occur both in low- and high-density plasmas.

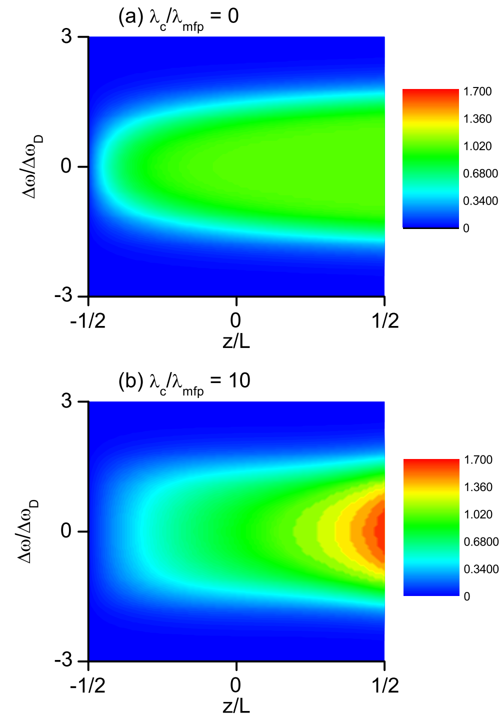

FIG. 2 (color online). Phase space map of the one-photon Wigner distribution in the $(z,\, p_z)$ plane for a slab, (a) without spatial coherence and (b) assuming $\lambda_c/\lambda_{\mathrm{mfp}} = 10$. In both cases, $p_x = 0 = p_y$ and $p_z > 0$. The spatial coherence results in a distortion of the Wigner distribution.

extent λ_c^3. It denotes an effective spatial range for delocalization of the photon-atom interaction processes. This extent is a feature of the wave-particle duality of light—in other words, it corresponds to the Heisenberg uncertainty relation. Equation (6) reduces to the radiative transfer equation in the limiting case $\lambda_c \rightarrow 0$.

The coherence length is comparable to the photon mean free path at the line center, λ_{mfp}, for a sufficiently high density of absorbers or sufficiently small line width. This stems from an estimate using Eq. (10): taking $\Delta\omega_{1/2}^{-1}$ as a typical value for the line shape function ϕ_c at the line center and neglecting the stimulated emission term, one obtains $\lambda_c/\lambda_{\mathrm{mfp}} \propto \bar{f}_{ud} N_d \Delta\omega_{1/2}^{-2}$ where $\bar{f}_{ud}$ is the oscillator strength of the transition. This ratio can be larger than unity both in low- and high-density plasmas, even for broad lines ($\Delta\omega_{1/2} \sim 10$ eV) such as those emitted in x rays by multicharged ions (Fig. 1). This suggests that accurate radiation transport simulations should rely on a quantum transport model along the lines of the theory just presented above. As an illustration of the coherence effects we show in Fig. 2 a phase space map of the Wigner function corresponding to hydrogen Lyman-α in a one-dimensional slab of size $L = 10\ \lambda_{\mathrm{mfp}}$ with perfectly absorbing walls, assuming (a) $\lambda_c/\lambda_{\mathrm{mfp}} = 0$ and (b) $\lambda_c/\lambda_{\mathrm{mfp}} = 10$. The atomic density (in both the upper and lower states) is assumed homogeneous. On the y axis, $\Delta\omega/\Delta\omega_D$ stands for the frequency detuning normalized to the Doppler width $\Delta\omega_D = \omega_{ud} v_0/c$, with $v_0 = \sqrt{2k_B T_{\mathrm{at}}/m_p}$ being the thermal velocity for atomic temperature T_{at}. We assume $T_{\mathrm{at}} = 1$ eV here. The coherence length is evaluated as $\lambda_c \equiv c/\Delta\omega_D$. In both cases, the map corresponds to $p_x = 0 = p_y$ and $p_z = (\omega_{ud} + \Delta\omega)/c > 0$; i.e., only photons propagating toward the $z > 0$ direction are considered. The coherence effects clearly modify the phase space distribution. A noticeable distortion of the Wigner function can be observed close to the slab's boundaries, with a decrease and an increase (see Fig. 3) of the function on the left ($z = -L/2$) and right ($z = L/2$) sides, respectively. This distortion results from the large value of $\lambda_c/\lambda_{\mathrm{mfp}}$ and is also a consequence of the gradients in the Wigner function arising from the presence of the boundary within one coherence length. The increase of the Wigner function could be observable by passive spectroscopy. In a diagnostic context, this means that a spectrum observed in an optically thick medium could be misinterpreted if coherence effects are not well accounted for.

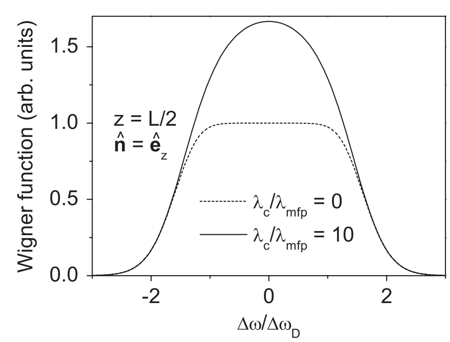

FIG. 3. Spectral profile of Lyman-α calculated at the slab's boundary $z = L/2$, neglecting (dashed line) and retaining (solid line) coherence effects. A strong increase of the Wigner function is present when the coherence effects are retained.

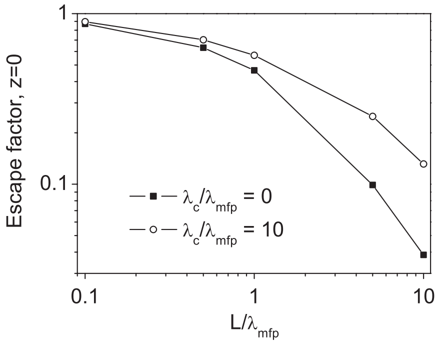

FIG. 4. Plot of the escape factor at the slab's center ($z = 0$) in terms of the size L. The spatial coherence leads to a strong increase of the escape factor.

An additional consequence of the spatial coherence is the modification of the coupled radiation—atomic populations kinetics. In optically thick media, the photon absorption process is a source of excited atoms, which contribute to ionization. A common approach to retain absorption in collisional-radiative models is provided by the so-called escape factors [31]. For a given line $u \rightarrow d$, we define the escape factor θ_{ud} (sometimes referred to as "net radiative bracket") in such a way that the net radiative deexcitation rate $N_u(A_{ud} + B_{ud}I_{ud}) - N_d B_{du} I_{ud}$ is identical to $A_{ud}\theta_{ud}$, where I_{ud} is the radiation specific intensity averaged with the spectral line shape function. Figure 4 shows plots of the escape factor at the slab's center for various values of the size L, again assuming $\lambda_c/\lambda_{\mathrm{mfp}} = 0$ and 10. The spatial coherence leads to a strong increase of the escape factor, by up to a factor 3 when $L = 10\ \lambda_{\mathrm{mfp}}$. Accordingly, the plasma is less opaque to the Ly-α radiation on average, which means that there are fewer excited atoms, hence less ionization induced by photon absorption.

In summary, the present Letter shows the possibility for an alteration of the radiation trapping mechanisms provided by the coherence properties of light. The photon emission and absorption processes are delocalized in a volume of size λ_c^3. This delocalization affects the photon phase space distribution. By applying a quantum transport model to a slab, we have shown that a spectral profile can be strongly higher than expected from usual radiative transfer models, indicating a reduction of the global plasma opacity. The magnitude of this reduction is governed by the ratio $\lambda_c/\lambda_{\mathrm{mfp}}$. In collisional-radiative models, the coherence effects can be accounted for through escape factors. When applied either to low- or high-density conditions, the ratio $\lambda_c/\lambda_{\mathrm{mfp}}$ can be as large as 10, which means that coherence effects are realistic. Although the present analysis is limited to a one-dimensional system, it puts forward important physics to consider in plasma spectroscopy.

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