Negative Nonconservative Forces: Optical "Tractor Beams" for Arbitrary Objects

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Based on the conservation of linear momentum on scattering from arbitrary objects, we demonstrate the generation of nonconservative optical forces that act in a direction opposite to the propagation of the incident beam. The concept can be applied to tailor the force fields produced on nonabsorbing bodies regardless of their sizes and shapes.

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It is well known that light can exert both conservative and nonconservative forces. Full optical manipulation relying on conservative forces and torques has been demonstrated in a number of optical tweezing experiments [1].

Nonconservative torques can be created by breaking the symmetry of a circularly polarized light field. As a result, one can induce rotations of asymmetric, absorbing, and birefringent objects [2] or even complex revolutions of bodies that interact electromagnetically [3].

Nonconservative optical forces, on the other hand, are traditionally associated only with the radiation pressure proportional to the Poynting vector determining the flow of momentum. Because they act only in the direction of beam propagation, nonconservative optical forces have been of limited interest for manipulation purposes. Nevertheless, having full control on their direction and strength could open new avenues for manipulation with reduced optical power. Using only nonconservative forces can eliminate the need for highly nonuniform fields with rather high intensities in the focusing region, a fact that has been recognized early on [4]. In addition, the range of manipulation can be extended to significantly more than a typical focal region which also means that optical forces can be used to influence and control the dynamics of larger objects. Moreover, besides the appealing flexibility for mechanical manipulation, the nonconservative transfer of energy may also permit unique examination of irreversible dissipative phenomena at micro and mesoscopic scales.

We have recently shown that the action of nonconservative forces can be reversed in certain places within regions of interfering Bessel beams [5]. In this case, the locations where such "negative optical forces" exist can be associated with superoscillations associated with local energy flows in the excitation field [6]. Because of the nature of field superoscillations, these regions are positioned in the vicinity of intensity minima and have spatial dimensions smaller than the excitation wavelength [5]. Therefore, to extend the flexibility and control the action of nonconservative forces on larger objects, one must resort to a different principle [7].

To understand why nonconservative forces are usually pressure, not drag, forces, let us consider the situation where a plane wave propagating along z direction is scattered by an object. Part of the wave's momentum is transferred to the scattered radiation that is deflected away and, therefore, some of the initial momentum along z direction is lost even for nonabsorbing objects. Because of momentum conservation, the lack of wave momentum along z after scattering should be compensated by the momentum transferred to the scattering object. This means that during any scattering event, the wave imparts a certain amount of momentum to the scattering body creating a force acting in the positive z direction, i.e., along the wave's propagation. This radiation pressure has been known to Maxwell and it depends on the light intensity I and the objects' reflectivity R as $p = (I/c)(1 + R)$ where c is the speed of light. For objects of specific shapes, the redistribution of light momentum can lead to unexpected movements of the object such as the so-called "optical lift" [8]. Depending on the scattering phase function, the radiation pressure can act in a direction different than the incidence, a fact that is also known from the theory of anisotropic scatterers [9]. However, in all these situations, some amount of momentum along the direction of light incidence is lost and this makes it impossible for a scattering object to move in a direction opposite to that of the incident beam.

To increase the "forward" momentum resultant from the scattering or, in other words, to create a negative force on the object, one can use an active medium as suggested in Ref. [10]. However, this approach is of limited practicality as most objects do not manifest optical gain. Alternatively, instead of artificially adding extra forward momentum to the scattered field, one could try from the beginning to input less momentum along z. The simplest way to achieve this is by illuminating with waves that propagate at some angle θ with respect to the z axis and have the nonabsorbing scattering object (the "black box" depicted in Fig. 1) redirect these partial waves along the z axis. Assuming that this "black box" does not produce additional scattered waves, the momentum conservation law is simply $k_{\text{wave}} \cos\theta = k_{\text{wave}} + k_{\text{box}}$ and the force acting on

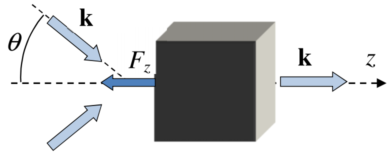

FIG. 1 (color online). A generic optical device (''black box'') converts the incident waves into waves propagating along the z axis. As a result of the momentum conservation, a negative optical force F_z will act on the ''black box.''

the object becomes $F_z = (P/c)(\cos\theta - 1)$, where P is the total optical power incident on the object. The resultant force on the object is negative for any $\theta \neq 0°$.

One of the simplest situations that can illustrate this concept is the case of two interfering plane waves

$$E = 2E_0 \exp(ik_z z)\cos(k_x x), \tag{1}$$

with $k_z = k\cos\theta$, $k_x = k\sin\theta$, and $k = 2\pi/\lambda$ being the wave number. The triangular prism shown in Fig. 2 serves as the ''black box'': when the prism parameters are chosen in such a way that, in the geometrical optics limit, all the rays transmitted through the prism are deflected parallel to the z axis, the final linear momentum of light along z increases and, consequently, the prism will experience a negative force F_z. Straightforward calculations based on Snell's law show that, in order to fulfill the forward scattering condition for a prism of refractive index n, the angle at the prism's base and the angle of incidence of incoming partial waves should be $\sin\varphi = (\sqrt{1+8n^2}+1)/4n$ and $\sin\theta = \sqrt{4n^2 - 1 - \sqrt{1+8n^2}}/\sqrt{8}$, respectively.

The illustration in Fig. 2 corresponds to a prism with $n = 1.5$ and $\varphi \approx 63°$ that is illuminated at an angle of incidence $\theta \approx 42°$. The distribution of the electromagnetic fields propagating through the prism was calculated numerically using the finite elements method (COMSOL MULTIPHYSICS v.3.5a). The overall optical force acting in the prism was then evaluated using the standard Maxwell's stress tensor approach [11]. For a prism with a base area of 5 μm $\times$ 5 μm which is illuminated by an electromagnetic field of wavelength $\lambda = 532$ nm, strength $E_0 = 10^6$ V/m, and polarized perpendicularly to the x-z plane, the resultant force acting on the prism along z direction is $F_z = -4.9$ pN. The absolute value of this optical force may, of course, be subject to losses due to both reflection on the interfaces and diffraction at the edges. Such effects could be minimized, for instance, using antireflection coatings and more refined shape designs.

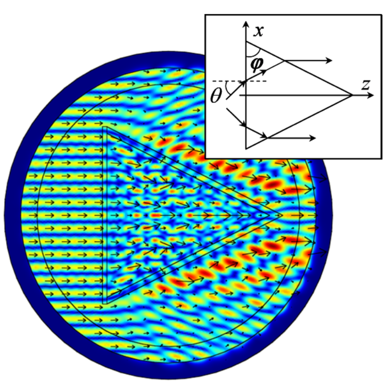

FIG. 2 (color online). Distribution of intensity during the propagation of the wave in Eq. (1) through a triangular prism. The black arrows indicate the local direction of power flow. The surfaces of the prism are antireflection coated. The inset depicts the rays' propagation in the geometrical optics limit.

For a simply shaped object, this example demonstrates that an appropriately designed field can act in such a way that the overall optical force points in a direction opposite to that of the incidence. This force is purely nonconservative and exists as long as the specified wave front is present. For a sphere, the generation of negative forces has also been theoretically investigated in the context of acoustic Bessel beams [12]. However, an even more interesting question could be asked. Can one tailor the incident field such that objects of any shape can be ''pulled'' in a nonconservative fashion? In the following, we will show that this is indeed possible.

We will consider the general case of a multiply scattering, nonabsorbing object with dimensions larger than the transport mean free path l^* such that the original direction of propagation of any incident photon is completely randomized through the scatterer. As a result, this object scatters light in all directions. In the far zone, the scattered field will consist of uncorrelated spots (speckles) with intensity and polarization randomly changing from one speckle to another.

In principle, one can shape the incident wave front and effectively convert the scattering object into a ''diffraction grating'' that accepts radiation at different angles and deflects it along the z axis. Practically, one can consider the decomposition of the incident beam into plane waves whose corresponding phases and polarizations can be adjusted in such a way that all the scattered partial waves interfere constructively along the z direction. This is somewhat similar to transforming a random medium into a useful optical component [13], such as, for instance, a focusing lens [14].

Of course, different plane waves constituting the incident beam contribute differently to the force along z. For instance, a plane wave traveling originally along the z axis can create only a pushing force ($\theta = 0$) while plane waves incident perpendicularly to the z axis ($\theta = \pi/2$) contribute the most to the negative force F_z. Obviously, there should be an angle of incidence θ_m at which the plane wave contribution to F_z is zero; negative forces can be observed only for $\theta > \theta_m$.

Although negative forces can be achieved with beams consisting of plane waves with different obliquity θ, it may sometimes be preferable to use beams having the same θ. A field that can be decomposed in plane waves having all the same obliquity θ constitutes a so-called nondiffracting beam [15]. In these conditions, the scattering object will experience the same force independently of its absolute z position, a situation that may be of particular interest in practice.

Let us now estimate the minimum angle θ_m at which F_z can still be negative. Let us assume that the random object scatters light evenly into N_s independent channels corresponding to the number of speckles in the far field. When the incident beam is composed of N_i partial plane waves, the momentum of light scattered along z direction due to constructive interference of these partial waves is $k_{sz} = (P_s/c)N_i^2/N_s$, where P_s is the scattered power for one particular plane wave. On the other hand, the total initial momentum along z is $k_{iz} = (P_s/c)N_I \cos\theta$. Thus, in order for the electromagnetic momentum to increase due to scattering, the condition $\cos\theta \leq N_i/N_s$ should be fulfilled.

Assuming that the correlation areas of intensity, phase, and polarization in the far field have similar values, the number of speckles N_s can be estimated as follows. From the far field, the scattering object is seen as a spatially incoherent field distributed within a localized aperture of finite area A. At a distance Z from this aperture, the field correlation area (the speckle size) is of the order of $A_c = (\lambda Z)^2/A$ [16] and the total number of speckles can be estimated to be

$$N_s = 4\pi Z^2/A_c = 4\pi A/\lambda^2. \quad (2)$$

Because one such speckle subtends a solid angle $\Delta\Omega = 4\pi/N_s$, the partial plane waves of the excitation field should also be separated by no less than $\Delta\Omega$. Having the same total excitation power distributed over more closely angularly spaced partial waves will not help in enhancing the scattering along a particular direction because all the waves propagating within $\Delta\Omega$ are almost in phase anyway and behave effectively as one single plane wave.

The total number of excitation plane waves can now be evaluated to be $N_i = 2\pi\sqrt{A}\sin\theta/\lambda$ which represents the number of waves distributed on the surface of a cone with an apex angle 2θ as illustrated in Fig. 3. Using these estimates, one finds that the angle of illumination is limited to $\cot\theta_m = \lambda/(2\sqrt{A})$.

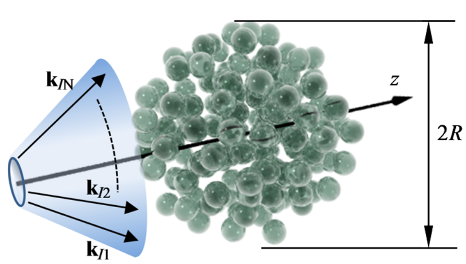

FIG. 3 (color online). Cluster of 160 spheres used in the scattering calculations and the considered geometry of illumination.

In general, because the scattering from inhomogeneous objects is complicated and not isotropic [17], one has to use numerical methods to describe the details of the process. We performed such calculations and considered aggregates consisting of spherical elements having all the same radius r as illustrated in Fig. 3. The scattering from such aggregates was calculated using a multiple sphere T-matrix FORTRAN code [18]. For the example discussed in this Letter, 160 spheres with size parameter $kr = 1$ and refractive index $n = 3$ were randomly distributed within the spherical volume with size parameter $kR = 10$ ($R \approx 7l^*$).

Wave vectors of 24 incident plane waves having the same k_z component (angle to the z axis $\theta = 84°$) were uniformly distributed in the k space as shown in Fig. 3. The plane waves had all the same amplitude $E_0/\sqrt{N_i}$ such that the total power was independent of their number; E_0 was chosen to be 10^6 V/m. The amplitude scattering matrices were calculated along 2562 scattering directions for each of the 24 incident plane waves. The angular distribution of the scattered field in the forward scattering hemisphere is shown in Fig. 4(a) for the case where all the plane waves constituting the illuminating beam are in phase. The ring of high field amplitude (close to the outer rim in Fig. 4) corresponds to the forward scattering direction for each partial plane wave [17]. To maximize the scattering along

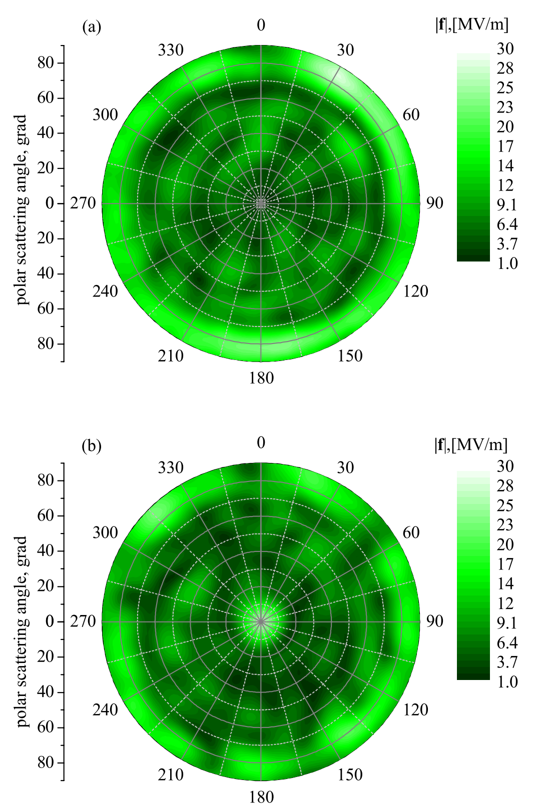

FIG. 4 (color online). Angular distribution of scattered electric field amplitude $|\mathbf{f}|$ in the forward scattering hemisphere of the cluster in Fig. 3 when illuminated by a nonoptimized (a) and optimized (b) beam as described in the text.

the z axis, we adjusted the polarization and phase of each plane wave in such a way that the x components of all partial fields scattered along z direction are maximized and in phase. The resulting scattering pattern is shown in Fig. 4(b). One can clearly see the spot of enhanced scattering located at zero polar and azimuthal angles, i.e., along the direction of the beam's propagation.

To calculate the optical forces based on the amplitude of the scattering matrix, we generalized the concept described in Ref. [9] to the case of illumination with arbitrary beams. The final expression for the optical force is

$$\mathbf{F}_{\text{opt}} = -\frac{\varepsilon_0}{2k^2}\int_\Omega |\mathbf{f}(\hat{k}_s = \hat{r}, \{\hat{k}_{Ii}\})|^2 \hat{r} d\Omega + \frac{2\pi\varepsilon_0}{k^2}\sum_{i=1}^{N_I} \hat{k}_{Ii}\,\text{Im}[\mathbf{f}(\hat{k}_s = \hat{k}_{Ii}, \{\hat{k}_{Ii}\}) \cdot \mathbf{E}_{Ii}^*], \quad (3)$$

where $\mathbf{f}(\hat{k}_s, \{\hat{k}_{Ii}\})$ denotes the scattered field in the far zone $\mathbf{E}_s = [\exp(ikr)/(kr)]\mathbf{f}(\hat{k}_s, \{\hat{k}_{Ii}\})$. The integration in Eq. (3) is performed over 4π, and $\hat{r}$ is a unit vector corresponding to the angle of integration. The scattering amplitude $\mathbf{f}(\hat{k}_s, \{\hat{k}_{Ii}\})$ depends both on the direction of scattering $\hat{k}_s$ and the directions of propagation of each partial wave $\hat{k}_{Ii}$.

The integral in Eq. (3) was calculated numerically as a summation over all the 2562 scattering directions. Because, according to formula (2), the total number of speckles in the far field of the aggregate is $N_s \approx (kR)^2 = 100$, a number of 2562 sampling points provides a good estimation of the integral. In these conditions, the calculated optical force for the nonoptimized beam is $F_z = 1.5$ pN. When illuminating with the optimized beam, the optical force along the z direction becomes negative and equals to $F_z = -0.24$ pN.

In conclusion, based on the principle of conservation of linear momentum, we have demonstrated that the nonconservative optical forces acting on a scattering object can be manipulated at will. In particular, we have shown that a negative, nongradient force can be generated not only in simple cases where the interaction obeys geometrical optics laws but also in the case of arbitrary scattering bodies without limitation of their shapes or structural morphology.

The structured illumination beam discussed in this Letter consists of the plane waves having the same longitudinal component of the wave vector. This makes the beam nondiffracting [15] and ensures that the negative optical force is maintained along the entire extent of the beam. The structured illumination concept introduced here can be further improved. More elaborate optimization procedures that not only maximize scattering in the desired direction, but also minimize scattering along other directions can be used to control the angle of convergence for partial plane waves and the entire structure of the beam.

Negative forces were also observed in solenoid type beams [19]. However, in that case the negative force appears due to the gradient part of the optical force.

Even though we have discussed only the effect of axial forces, it is straightforward to show that transversal optical forces can be similarly manipulated by changing the amplitudes of partial plane waves. This should allow access to a complete range of translations and rotations which are not discussed in this Letter. Moreover, structured illumination can augment the scattering in any direction, not only forward. Enhancing scattering in the direction opposite to the beam's propagation, for example, leads to increasing the radiation pressure force, while enhancing the $\pi/2$ scattering would create a transversal effect. Thus, our general approach based solely on the action of nonconservative optical forces permits arbitrary manipulation of objects by optimizing the shape of the excitation beam to fit the structure of the manipulated body. As the shape of the beam is specific to the object's orientation, manipulating a complex structure will require monitoring its current orientation and adjusting the phases of the partial waves.

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Note added.—An account of various phenomena leading to optical forces pointing against the main stream of photons can be found in [5]. After submission of this Letter, additional publications have been brought to our attention, which discuss physical situations leading to anomalous behavior of optical forces [20].

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