Experimental Observation of Optical Bound States in the Continuum

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We present the experimental observation of bound states in the continuum. Our experiments are carried out in an optical waveguide array structure, where the bound state (guided mode) is decoupled from the continuum by virtue of symmetry only. We demonstrate that breaking the symmetry of the system couples this special bound state to continuum states, leading to radiative losses. These experiments demonstrate ideas initially proposed by von Neumann and Wigner in 1929 and offer new possibilities for integrated optical elements and analogous realizations with cold atoms and optical trapping of particles.

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A general quantum system with a finite potential well naturally has bound states and unbound continuum states, whose energies lie below or above the continuum threshold (the asymptotic value of the potential at infinity), respectively. It can have, however, another kind of state: a bound state in the continuum (BIC). Historically, BICs were first proposed in 1929 [1], soon after the emergence of quantum mechanics. A BIC is a counterintuitive eigenmode of the system with energy above the continuum threshold, but which is nevertheless localized and square integrable. BICs were identified for specific potential structures, and can generally be described as resonance states of zero width, i.e., leaky modes (localized states with finite lifetime due to coupling to continuum states) with zero leakage [2]. von Neumann's and Wigner's method for finding a potential capable of supporting a BIC was to assume a wave function with an envelope that decays in space and is square integrable, and then tailor a suitable potential whose bound eigenmode is this wave function [1]. Such ‘‘custom made'' potentials are oscillatory in space while decaying as a power law (as their bound state does). Almost 50 years passed until the issue of BIC-supporting potentials was addressed again, this time by Stillinger and Herrick [3], who implemented Wigner's idea in a large variety of potentials, and extended it to a two-electron wave function. Another approach for designing potentials that can support BICs was proposed following the discovery of the so-called ‘‘resonance states'' in quantum systems. Resonance states are localized states with a finite lifetime, whose energies lie in the continuum [4]. These states, albeit localized, are not bound states; rather, they are constructed from continuum states; hence, they eventually decay. Under some specific parameters, interference between resonance states can yield a zero-width resonance [2,5], resulting in a state with no decay at all: a BIC. More recently, it has been proposed to construct BICs that are decoupled from all continuum states by virtue of symmetry solely, as suggested for ring structures carrying ballistic current [6] and for quantum billiards [7]. Interestingly, even though the BIC ideas date back to 1929, BICs in quantum systems have never been demonstrated. The experiment closest to BIC in quantum systems was the work of Capasso's group in 1992 [8], demonstrating an electronic bound state with energy far above the barrier height in a one-dimensional semiconductor superlattice (an array of quantum wells). However, that work, albeit pioneering, did not demonstrate a true BIC, because the state was actually a defect mode residing in a minigap formed by Bragg reflections and not in a continuum of states. Accordingly, the title of that paper is "Observation of an Electronic Bound State above a Potential Well," not claiming a BIC [8]. In a similar vein, we note another observation of a state in the continuum which was localized in one dimension but not in the other [9]. That two-dimensional scheme of ‘‘grating-mediated waveguiding'' supports a guided mode immersed in the continuum (Bloch modes of the grating). However, that mode is localized, fundamentally, in one dimension only, while in the other dimension it must be an extended state. As such, it carries infinite power, and is not a true bound state in the continuum. Altogether, to this day, BICs have never been observed in any system, quantum or classical.

The concept of BICs is based on interference, and hence not restricted to quantum systems. Rather, the idea of a BIC applies in principle to any type of wave system. Notably, in the past decade, the analogy between optical systems in the paraxial regime and general (one-dimensional or two-dimensional) quantum systems has gained much attention. This analogy has facilitated the experimental observation of a variety of phenomena that are very difficult to observe in quantum solids [10]. Examples range from Bloch oscillations [11] and Zener tunneling [12] to Shockley states [13], the Zeno effect [14], dynamic localization [15] and Anderson localization [16]. One of the nicest features of carrying out such experiments in the classical domain of spatial optics is the ability to directly and continuously image the wave function as it evolves [10–16]—a feature which is notoriously difficult to achieve in quantum systems, where one typically measures integrated quantities such as conductance through a potential or scattering from it. These arguments suggest employing optics for the observation of bound states in the continuum.

Here, we present the experimental observation of bound states in the continuum: BICs. Noticing the elegancy of the notion "BIC by virtue of symmetry," we have chosen this path, and employ an optical waveguide array structure, where we use symmetry to decouple the BIC from all continuum states. Namely, our BIC is an antisymmetric localized mode inside a continuum of symmetric extended states. To show that the BIC is indeed immersed in a continuum, we gradually break the symmetry of the structure by adding a refractive index gradient. Breaking the structural symmetry allows coupling to the continuum, manifested in light escaping to neighboring waveguides and spreading into the array. Such concepts can be carried over to other systems, such as matter waves, acoustic waves and other systems governed by wave interference. Moreover, such ideas can be used as a means to manipulate optical traps for small particles.

The structure we use to implement these ideas (Fig. 1) is a one-dimensional array of single-mode optical waveguides, with two additional waveguides above and below the central waveguide of the array. The refractive index profile of the waveguides is a sixth-order super-Gaussian, with a width of 4 μm in the horizontal direction and 11 μm in the vertical direction [17]. The maximal change in the refractive index is 8×10^{-4} with a bulk refractive index of 1.45 (fused silica). The horizontal distance between the waveguides' centers in the array is 20 μm, and the vertical distance is 15 μm. The BIC supported by this structure is localized in the two waveguides above and below the array, and its electric field has a phase difference of π between the two waveguides. The BIC is antisymmetric in the vertical direction, whereas all the eigenmodes

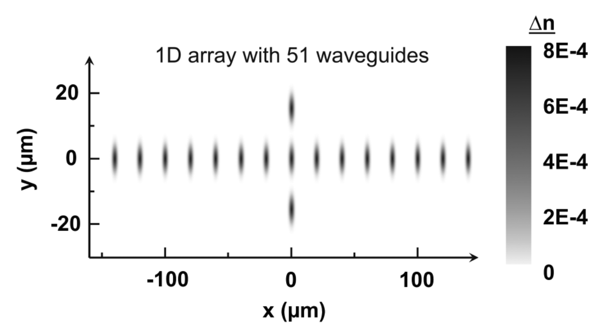

FIG. 1. Refractive index profile of the structure used for the demonstration of a BIC. The structure comprises a one-dimensional array of 51 equidistant waveguides in the x direction and two additional waveguides above and below the array. The size of each waveguide is 4 μm in the horizontal direction and 11 μm in the vertical direction.

of the array are symmetric in that direction. As such, the BIC is decoupled from all the array modes by virtue of symmetry only.

The slowly varying amplitude $\psi = \psi(x, y, z)$ of the field evolving in this structure is governed by [10]

$$\left(i\partial_z + \frac{1}{2k}(\partial_x^2 + \partial_y^2) + \frac{k\Delta n}{n_0}\right)\psi = 0 \qquad (1)$$

where z is the propagation axis, x, y are the transverse dimensions, k is the wave number, n_0 is the bulk refractive index and Δn is the spatial variation of the refractive index defining the structure. Let us analyze the eigenmodes of this structure, consider first the homogeneous array without the two additional waveguides. The array is periodic in x; hence, its eigenmodes are Bloch modes grouped into bands and separated by gaps. The calculated continuum of Bloch modes is shown in Fig. 2. Note that these modes are all y symmetric.

Next, consider the vertical structure of the three central waveguides only (without the horizontal array) and find their eigenmodes. This structure supports three bound modes: the first and the third are y symmetric, where a proper choice of parameters (spacing between waveguides) shifts their propagation constants into the gaps above and below the band, making them ordinary bound states (defect modes). On the other hand, the second mode is y antisymmetric. Importantly, the propagation constant of this second mode resides inside the continuous band of the array modes. However, this antisymmetric mode

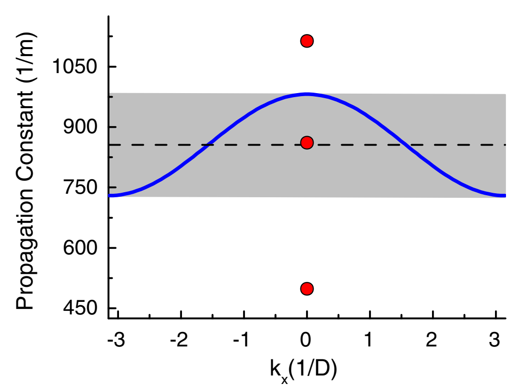

FIG. 2 (color online). Calculated dispersion relation of the waveguide array (solid curve) indicating a continuum of extended states (Bloch modes) in the x direction, comprising the first band of the array (gray). All of these modes are y symmetric. The circles indicate the propagation constants of the three discrete bound states of the three vertical waveguides in the center of the array. The two bound states above and below the band are y symmetric, but they are disconnected from the band because they reside in the gaps above and below the band, thereby forming ordinary bound states (defect modes). The central discrete state is immersed in the continuous band, but because it is antisymmetric in the vertical direction it is decoupled from the band, thereby forming a BIC.

cannot couple to the symmetric modes of the array since the refractive index profile is also y symmetric. Hence, the second mode is a true BIC. In fact, this second mode remains within the band for a wide range of parameters: the splitting between the bound states of these three identical central waveguides due to their close proximity mainly affects the separation of the symmetric modes, whereas the propagation constant of the antisymmetric mode stays close to that of an unperturbed waveguide, which under the tight binding approximation is in the middle of the band [18]. The coupling between the antisymmetric second mode $\psi_2(x, y, z) = -\psi_2(x, -y, z)$ and the band of symmetric continuum states $\phi(x, y, z) = \phi(x, -y, z)$ can be calculated using the overlap integral, which for the given symmetric index profile $\Delta n(x, y, z) = \Delta n(x, -y, z)$ yields:

$$C \propto \langle \psi_2 | \Delta n | \phi \rangle = 0. \qquad (2)$$

Consequently, the antisymmetric mode is decoupled from all Bloch modes of the array. It is therefore a BIC.

Our experiments are conducted in a waveguide system fabricated in fused silica by femtosecond direct laser writing [19]. Our sample is constituted of a one-dimensional array of 51 waveguides, plus the two additional waveguides (one above and one below), as shown in Fig. 1. Since the array is finite, the number of modes in the band is equal to the number of waveguides in the array, namely, 51. However, this discretization does not affect our experiment, because for the propagation length (of 10 cm) in our sample, two adjacent modes will acquire a relative phase of less than $\pi/10$. Hence, for all practical purposes the modes are organized in a continuous band. The propagation constant of the BIC is designed to reside in the middle of the band. Under the parameters of our sample, the fabrication accuracy, and the optical wavelength, we find that the BIC is situated near the middle of the band—far from the band edges. Likewise, we find that the maximum possible difference between the propagation constants of the BIC and of the closest Bloch mode is such that the maximum relative phase these modes can acquire in our sample is $\pi/20$. Thus, the finite extent of the array and the worst case scenario in terms of fabrication accuracies still hold that the propagation constant of the BIC effectively coincides with one of the mid-band Bloch modes of the array.

We prepare the antisymmetric BIC mode by passing half of a Gaussian laser beam through a microscope slide coverslip, tilted at an angle giving rise to a phase difference of an odd multiple of π, with respect to the phase acquired by the other half of the beam propagating in air. The tilt angle is kept close to the Brewster angle, so that reflections would not cause an imbalance in light intensity between the two halves of the beam. We launch the antisymmetric mode into the waveguide structure by overlapping the two lobes with the waveguides above and below the array, thus preventing the excitation of the central waveguide and the associated symmetric modes. Observing the optical intensity distribution at the output facet of the structure [Fig. 3(a)] clearly shows the bound (guided) nature of the antisymmetric state: light does not escape from the central waveguides into the array, despite the fact that the propagation constant of this antisymmetric mode is immersed in the continuous band of Bloch modes of the array.

Since the decoupling of the BIC from the continuum is solely based on symmetry, breaking this symmetry should yield coupling of the antisymmetric state to the continuum. In order to gradually break the vertical symmetry, we introduce a vertical refractive index gradient by heating the top side of the sample while cooling the bottom. The thermal response of the host material translates the thermal profile into a refractive index profile [20]. Upon activation of the thermal gradient, we observe light escaping from the three central waveguides [Fig. 3(b)] [21]. The amount of light coupled into the array increases monotonically with the strength of the gradient. The light escaping into the array forms two lobes with their maxima at ~11 waveguides away from the center. Comparing with simulations proves that this occurs only when the initial state (zero gradient) is in the mid-band region. Figure 4 shows the optical power emerging from the array (after escaping from the three vertical waveguides), measured at the output facet, as a function of the temperature gradient. This demonstrates conclusively that the BIC in our structure is mediated solely by virtue of symmetry.

Numerical calculations (using a standard split-step Fourier transform) reveal that, for small temperature gradients, the power in the array grows in a parabolic fashion (solid curve in Fig. 4). This result can also be obtained analytically by calculating the coupling constant perturbatively. The refractive index change induced by the thermal gradient is $\Delta n(y) = K\Delta T y$, when K is a constant

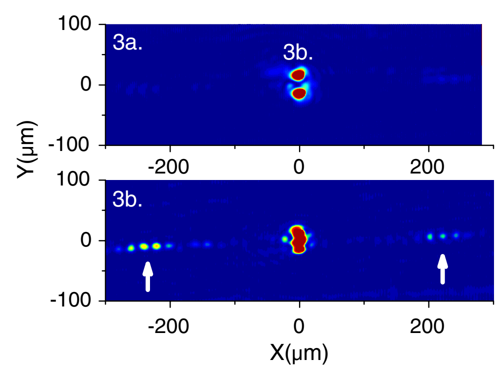

FIG. 3 (color online). Light intensity at the output plane of the structure, following proper excitation of the BIC. (a) Light intensity at the output plane when the structure is fully y symmetric. Clearly, no light is leaking into the array. (b) Light intensity at the output plane when the refractive index gradient is introduced by a 30 K temperature gradient across 1 mm (distance between upper and lower thermal contacts). Clearly, the broken symmetry mediates leakage of light into the array.

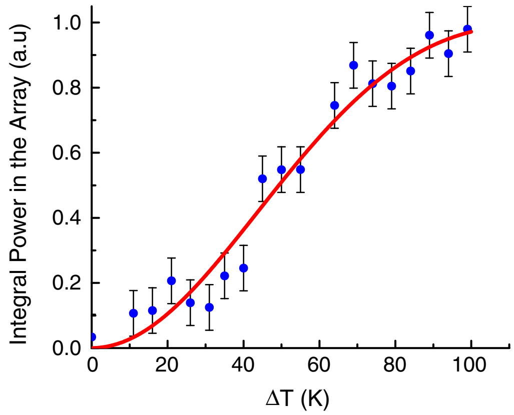

FIG. 4 (color online). Power coupled into the continuum states (array modes) as a function of the applied thermal gradient across the structure. The blue dots represent the experimental data, while the red line shows the simulation results.

representing the response of the refractive index to the temperature change. The optical power P escaping from the antisymmetric state into the array is given by

$$P(\Delta T) \propto |C|^2 \propto |\langle \psi_2 | n(x, y) | \phi \rangle|^2 \propto \Delta T^2. \qquad (3)$$

This trend is nicely reflected at the lower part of the curve in Fig. 4, as long as the perturbative approach is valid.

In conclusion, we proposed and realized a two-dimensional waveguide array structure supporting a bound state immersed in a continuum of states. The BIC is decoupled from the continuum by virtue of symmetry only. Indeed, we demonstrated that breaking the symmetry results in coupling to the continuum, which drains the power out of that mode. Our experiments constitute the first experimental demonstration of a bound state in the continuum, the two-dimensional realization of the concept envisioned by von Neumann and Wigner in 1929. This work has implications on a variety of areas in optics and beyond, arising from the fact that symmetry breaking immediately transforms the BIC into a leaky mode whose lifetime can be controlled with great sensitivity. Examples range from efficient optical modulators to quantum-confined structures offering sensitive control over injection of charge carriers and experiments with cold atoms and matter waves. In a similar vein, using symmetry to control a BIC offers sensitive control over optical traps, where altering the symmetry acts as a means to manipulate the strength of interaction between the particles and external surrounding.

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