Giant Charge Relaxation Resistance in the Anderson Model

Michele Filippone,[1] Karyn Le Hur,[2] and Christophe Mora[1]

[1]*Laboratoire Pierre Aigrain, École Normale Supérieure, Université Paris 7 Diderot, CNRS; 24 rue Lhomond, 75005 Paris, France*
[2]*Departments of Physics and Applied Physics, Yale University, New Haven, Connecticut 06520, USA*

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We investigate the dynamical charge response of the Anderson model viewed as a quantum *RC* circuit. Applying a low-energy effective Fermi liquid theory, a generalized Korringa-Shiba formula is derived at zero temperature, and the charge relaxation resistance is expressed solely in terms of static susceptibilities which are accessible by Bethe ansatz. We identify a giant charge relaxation resistance at intermediate magnetic fields related to the destruction of the Kondo singlet. The scaling properties of this peak are computed analytically in the Kondo regime. We also show that the resistance peak fades away at the particle-hole symmetric point.

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In recent years, experimental effort has been devoted to manipulate and measure electrons in nanoconductors in real time [1]. At frequencies in the GHz range and cryogenic temperatures, current and noise measurements provide information on the quantum dynamics of charge carriers. Experiments have followed essentially two directions, by either using on-chip quantum detectors [2] or by directly measuring the current using low noise amplifiers [3]. In an original experiment, Gabelli *et al.* [4] have realized the quantum equivalent of an *RC* circuit with a quantum dot connected to a spin-polarized single-lead reservoir and capacitively coupled to a metallic top gate, this setup being later used as a single-electron source [5]. By applying an ac modulation to the gate voltage, they measured the admittance of the dot at low frequency. A comparison with the classical *RC* circuit allows us to extract a capacitance and a charge relaxation resistance. Their measurements have confirmed the prediction [6,7] of a quantized charge relaxation resistance $R_q = h/2e^2$ where e is the electron charge and h the Planck constant. This prediction was recently shown [8,9] to be valid for all interaction strength. Reference [8] also predicted the emergence of an additional universal resistance $R_q = h/e^2$ in the case of a large dot.

The quantum *RC* circuit is described by the Anderson model [10] when the level spacing is sufficiently large and electron transport is not spin polarized in contrast to Refs. [6–9]. In that case, the gate voltage controls the dot single-particle energy $\varepsilon_d(<0)$. In addition to being experimentally relevant, for the transport through short nanotubes [11], small quantum dots or even molecules [12], the Anderson model is fascinating because it displays features of strong correlation with the emergence of Kondo physics at low energy. The question of how these correlations affect the quantization of the charge relaxation resistance is a fundamental issue.

The linear charge response of the quantum dot to a gate voltage oscillation defines the capacitance C_0 and the resistance R_q via the low frequency expansion [8]

$$\frac{e^2\langle\hat{n}(\omega)\rangle}{-\varepsilon_d(\omega)} = C_0 + i\omega C_0^2 R_q + \mathcal{O}(\omega^2) \tag{1}$$

where $\hat{n}$ is the number of electrons on the dot. The capacitance is thus the static response of the dot. The product $\omega C_0^2 R_q$ describes relaxation towards the changing ground state that implies energy dissipation [8,13]. In this Letter, we investigate the dynamical charge response of the Anderson model at zero temperature and finite magnetic fields and we evidence a giant charge relaxation resistance phenomenon associated with the destruction of the Kondo effect at intermediate fields.

More precisely, by applying a low-energy effective Fermi liquid theory [14,15], we derive a generalized Korringa-Shiba formula [16] for the charge susceptibility that extends to finite magnetic fields. The charge relaxation resistance then depends only on *static* susceptibilities that are computed analytically resorting to the Bethe ansatz solution [10,17,18] in the Kondo regime. At zero magnetic field, the original Korringa-Shiba [16] formula predicts the quantized value $R_q = h/4e^2$ independent of interactions. This result agrees with the noninteracting scattering approach with two (spin) conducting channels [6,7]. At large magnetic fields, the dot becomes spin polarized, reducing electron transfer in both spin channels, and the quantized value $R_q = h/4e^2$ is recovered perturbatively. In the crossover regime between these two limits, a peak was observed in the numerical renormalization group (NRG) calculations of Ref. [19], where it is attributed to spin fluctuations in the dot. Hereafter, we derive analytically the emergence of this peak in the Kondo regime and derive its scaling properties. In particular, the peak is found to disappear completely at the particle-hole symmetric point.

The origin of the peak in the resistance is related to the destruction of the Kondo singlet by the magnetic field which gives more flexibility to the spin configuration, while the charge remains frozen by interactions. As a result, a change in the gate voltage significantly modifies the magnetization, with an increase of dissipation by particle-hole excitations. The charge, however, is relatively insensitive to the gate voltage and the capacitance remains small. An increasing dissipation $\propto C_0^2 R_q$ with an almost constant capacitance C_0 thus leads to an increasing charge relaxation resistance R_q. A further increase of the magnetic field eventually polarizes the spin on the dot, reduces spin flexibility and thereby energy dissipation. Hence, the charge relaxation resistance R_q passes through a maximum when the Zeeman energy is comparable to the Kondo energy.

The Hamiltonian of the Anderson model is given by

$$H = \sum_{\sigma,k} \varepsilon_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma} + \sum_{\sigma} \varepsilon_{d\sigma} \hat{n}_\sigma + U \hat{n}_\uparrow \hat{n}_\downarrow + t \sum_{k,\sigma} (c^\dagger_{k\sigma} d_\sigma + d^\dagger_\sigma c_{k\sigma}), \tag{2}$$

with $\hat{n}_\sigma = d^\dagger_\sigma d_\sigma$ the number of spin σ electrons on the dot, $\hat{n} = \hat{n}_\uparrow + \hat{n}_\downarrow$, the linear spectrum $\varepsilon_{k\sigma} = \varepsilon_k - g\sigma\mu_B B/2$ of conduction electrons characterized by the constant density of states ν_0, and the energy levels $\varepsilon_{d\sigma} = \varepsilon_d - g\sigma\mu_B B/2$ of the dot. μ_B is the Bohr magneton, g the Lande factor, B the external magnetic field and $\sigma = \pm$ refers to $\uparrow, \downarrow$ states, respectively. The two terms in the second line of Eq. (2) describe, respectively, Coulomb interaction and tunneling from the dot to the lead with the hybridization constant $\Gamma = \pi\nu_0 t^2$. In the presence of an ac drive of very small amplitude, $\varepsilon_d = \varepsilon_d^0 + \varepsilon_\omega \cos\omega t$ with $\varepsilon_\omega \to 0$, the system relaxes towards the evolving ground state of the Hamiltonian and the dissipated power

$$\mathcal{P} = \frac{1}{2} \varepsilon_\omega^2 \omega \, \mathrm{Im}\chi_c(\omega), \tag{3}$$

which is given by linear response theory, is proportional to the imaginary part of the dynamical charge susceptibility $\chi_c(t-t') = i\theta(t-t')\langle[\hat{n}(t), \hat{n}(t')]\rangle$.

NRG calculations [15] and RG arguments [20] have shown that the low-energy properties of the Anderson model (2) are always those of a Fermi liquid. The effective Fermi liquid Hamiltonian takes the form

$$H = \sum_{\sigma,k} \varepsilon_{k\sigma} a^\dagger_{k\sigma} a_{k\sigma} + \sum_{k,k',\sigma} K_\sigma(\varepsilon_d) a^\dagger_{k\sigma} a_{k'\sigma}. \tag{4}$$

The free quasiparticles of the first term are related to the original fermions $c_{k\sigma}$ by a phase shift of $\pi/2$. The second term is a marginal perturbation corresponding to a potential scattering at the impurity site. It defines a line of fixed points parametrized by ε_d connecting the Kondo regime (for $\varepsilon_d \simeq -U/2$) to the mixed valence regimes (for $\varepsilon_d \simeq 0$ or $\varepsilon_d \simeq -U$). The potential is related to the mean occupation of the dot via the Friedel sum rule $\langle \hat{n}_\sigma \rangle = 1/2 - \frac{1}{\pi}\arctan[\pi\nu_0 K_\sigma(\varepsilon_d)]$. Note that the potentials $K_\sigma(\varepsilon_d)$ also formally depend on U, Γ and B. Again we study the response to the ac drive $\varepsilon_d = \varepsilon_d^0 + \varepsilon_\omega \cos\omega t$ with $\varepsilon_\omega, \omega \to 0$. Expanding the potentials as $K_\sigma(\varepsilon_d) = K_\sigma^0 + K'_\sigma(\varepsilon_d^0)\varepsilon_\omega \cos\omega t$, we change the basis to the one-particle states [15] that diagonalize the potential scattering terms $K_\sigma^0 = K_\sigma(\varepsilon_d^0)$. The remaining scattering term in the Hamiltonian is given by

$$\varepsilon_\omega \cos\omega t \sum_\sigma \frac{K'_\sigma(\varepsilon_d^0)}{1 + (\pi\nu_0 K_\sigma^0)^2} \sum_{k,k'} \tilde{a}^\dagger_{k\sigma} \tilde{a}_{k'\sigma}, \tag{5}$$

with the new quasiparticles $\tilde{a}_{k'\sigma}$. The derivative of the occupation numbers with respect to ε_d in the Friedel sum rule formula above introduces the static spin-dependent susceptibilities $\chi_{c\sigma} = -\partial\langle \hat{n}_\sigma \rangle / \partial\varepsilon_d$. Once inserted into Eq. (5), we obtain

$$H = \sum_{\sigma,k} \varepsilon_{k\sigma} \tilde{a}^\dagger_{k\sigma} \tilde{a}_{k\sigma} + \varepsilon_\omega \cos\omega t \sum_\sigma \frac{\chi_{c\sigma}}{\nu_0} \sum_{k,k'} \tilde{a}^\dagger_{k\sigma} \tilde{a}_{k'\sigma}. \tag{6}$$

In the static case $\omega = 0$, the second term in Eq. (6) adds the phase shift $\delta_\sigma = -\pi\nu_0 \varepsilon_\omega \chi_{c\sigma}/\nu_0$. The Friedel sum rule translates it into a shift in the occupations $\delta\langle \hat{n}_\sigma \rangle = -\chi_{c\sigma}\varepsilon_\omega$ in agreement with the definition of the charge susceptibilities. The Hamiltonian in Eq. (6) is extremely general and it only assumes a low-energy Fermi liquid fixed point. A similar model can be found in Ref. [21] where the spin susceptibility is discussed.

Interestingly, the low-energy model Eq. (6) provides an alternative to compute the dissipated power Eq. (3). Following standard linear response theory, it involves the operators $\hat{A}_\sigma = (\chi_{c\sigma}/\nu_0)\sum_{k,k'} \tilde{a}^\dagger_{k\sigma} \tilde{a}_{k'\sigma}$, coupled to the ac excitation in Eq. (6), namely

$$\mathcal{P} = \frac{1}{2} \varepsilon_\omega^2 \omega \sum_\sigma \mathrm{Im}\chi_{\hat{A}\sigma}(\omega), \tag{7}$$

with $\chi_{\hat{A}\sigma}(t-t') = i\theta(t-t')\langle[\hat{A}_\sigma(t), \hat{A}_\sigma(t')]\rangle$. The operators $\hat{A}_\sigma$ create particle-hole pairs that are responsible for energy dissipation. The calculation is straightforward and gives, at zero temperature, $\mathrm{Im}\chi_{\hat{A}\sigma}(\omega) = \pi\chi_{c\sigma}^2 \omega$, i.e., proportional to the density of available particle-hole pairs with energy ω. An identification of Eqs. (3) and (7) finally results in our generalized Korringa-Shiba formula

$$\mathrm{Im}\,\chi_c(\omega) = \pi\omega(\chi_{c\uparrow}^2 + \chi_{c\downarrow}^2), \tag{8}$$

obtained to lowest order [22] in ω. The physical meaning of this expression is explicit. In the presence of the ac driving applied to the gate voltage, relaxation is necessary to adjust the occupation numbers to the instant ground state of the Hamiltonian. This relaxation is realized by particle-hole excitations, in each spin sector independently, with amplitudes [see Eq. (6)] that are determined by the static charge susceptibilities $\chi_{c\sigma}$ controlling the variations of the spin populations with the gate voltage. Equation (8) simply states that the energy dissipated in the relaxation mechanism increases quadratically with these amplitudes as a result of the Fermi golden rule.

At a general level, the low frequency properties of the quantum RC circuit characterized by Eq. (1) derive from the knowledge of the dynamical charge susceptibility since $\langle \hat{n}(\omega)\rangle = -\chi_c(\omega)\varepsilon_d(\omega)$ in the linear regime. The capacitance $C_0 = e^2\chi_c$ is solely determined by the static charge susceptibility $\chi_c = \chi_c(\omega \to 0) = \chi_{c\uparrow} + \chi_{c\downarrow}$ that is calculated using Bethe ansatz. Hence, measuring the capacitance realizes a charge spectroscopy [24]. At zero magnetic field and large enough interaction, $U > \Gamma$, χ_c develops a double-peak structure as a function of ε_d: maximum in the valence regimes with a valley in the intermediate Kondo regime around $\varepsilon_d = -U/2$ where $\chi_c = 8\Gamma/\pi U^2$ for $U \gg \Gamma$. This strong reduction of capacitance, or charge sensitivity, characterizes the Kondo limit where the charge is frozen. It contradicts the noninteracting scattering theory [4,6] where the capacitance is proportional to the density of state and would therefore reveal the Kondo resonance [19]. The two approaches are nonetheless reconciled by noting that, the Kondo resonance is mostly exhausted by spin fluctuations, and the density of states of charge excitations of the Anderson model, the holons, reproduces [25] the exact value of the charge susceptibility and thus of the capacitance.

The Korringa-Shiba Eq. (8) substituted in the expansion Eq. (1) expresses the resistance R_q in terms of static susceptibilities, computable by Bethe ansatz. At zero magnetic field, $\chi_{c\uparrow} = \chi_{c\downarrow} = \chi_c/2$, Eq. (8) reproduces the standard Korringa-Shiba relation and the charge relaxation resistance is found to be quantized and universal,

$$R_q = \frac{h}{4e^2}, \tag{9}$$

in agreement with the scattering approach involving two equivalent spin channels [6,7]. In the general case, we introduce the charge magnetosusceptibility $\chi_m = \chi_{c\uparrow} - \chi_{c\downarrow}$ which measures the sensitivity of the magnetization to a change in the gate voltage. The resistance reads

$$R_q = \frac{h}{4e^2}\frac{\chi_c^2 + \chi_m^2}{\chi_c^2}. \tag{10}$$

For $\varepsilon_d = -U/2$, particle-hole symmetry implies that the magnetization is extremal with respect to the gate voltage and χ_m identically vanishes. Equation (9) is thus obtained for all ratios of U/Γ.

In the rest of this Letter, we focus on the Kondo regime $U \gg \Gamma$ where the gate voltage explores the valley between the Coulomb peaks located around $\varepsilon_d \simeq 0$ and $\varepsilon_d \simeq -U$. Far enough from these Coulomb peaks, $|\varepsilon_d|/\Gamma \gg \ln(U/\Gamma)$, the charge on the dot remains of order one and the renormalization [20] of the peak positions is negligible. The charge susceptibility is computed perturbatively at zero magnetic field,

$$\chi_c = \frac{\Gamma}{\pi}\left(\frac{1}{(\varepsilon_d + U)^2} + \frac{1}{\varepsilon_d^2}\right), \tag{11}$$

and remains constant as the magnetic field is increased with $g\mu_B B \ll \sqrt{|\varepsilon_d|\Gamma}$.

For moderate magnetic fields $g\mu_B B \ll \sqrt{|\varepsilon_d|\Gamma}$, the magnetization of the dot is known [10,17] from the Bethe ansatz solution of the Anderson model. In the Kondo limit, it exhibits the scaling form,

$$m = \frac{\langle \hat{n}_\uparrow\rangle - \langle \hat{n}_\downarrow\rangle}{2} = f\left(\frac{g\mu_B B}{k_B T_K}\right), \tag{12}$$

where $T_K = 2\sqrt{U\Gamma/\pi e}\exp[\pi\varepsilon_d(\varepsilon_d + U)/2U\Gamma]$ is the Kondo temperature and the scaling function $f(x)$ connects the asymptotes $f(x) = x/\sqrt{2\pi e}$ for $x \ll 1$, and $f(x) = 1/2 - 1/(4\ln x)$ for $x \gg 1$, i.e., for low and large magnetic fields, e referring to Euler's number. The dependence of the magnetization Eq. (12) on the gate voltage, or ε_d, is via the Kondo temperature. Computing the derivative of the Kondo temperature with respect to ε_d, then one finds

$$\chi_m = \frac{\pi}{\Gamma}\frac{2\varepsilon_d + U}{U}\Phi\left(\frac{g\mu_B B}{k_B T_K}\right). \tag{13}$$

The charge magnetosusceptibility χ_m is an odd function of $\varepsilon_d + U/2$ that vanishes at the particle-hole symmetric point. The scaling function $\Phi(x) = xf'(x)$ is represented in Fig. 1(a), in good agreement with Ref. [19]. It exhibits a peak at $x_0 = 1.0697$ with $\Phi(x_0) = 0.1257$. Inserting the results of Eqs. (11) and (13) into Eq. (10), the scaling form of the charge relaxation resistance is obtained,

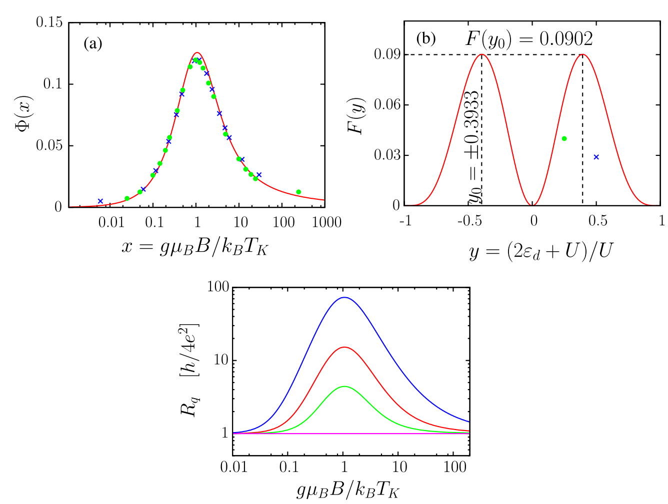

FIG. 1 (color online). (a) Scaling function $\Phi(x)$ (solid line) and (b) envelope function $F(y)$ (see main text). $F(0) = 0$ at the particle-hole symmetric point $y = 0$ where $\partial T_K/\partial\varepsilon_d = 0$. $y = \pm 1$ correspond to the two Coulomb peaks in the transport or valence regime. Green circles ($\varepsilon_d = -0.15$) and blue crosses ($\varepsilon_d = -0.1$) are extracted from Fig. 3a of Ref. [19], with $U = 0.4$ and $\Gamma = 0.02$, by implementing Eq. (14) and rescaling the x axis. (c) Charge relaxation resistance for $\varepsilon_d/U = -1/2 \pm 0.1967$ and various ratios of $U/\Gamma = 15$, 10, 7.

$$R_q = \frac{h}{4e^2}\left[1 + \left(\frac{U}{\Gamma}\right)^4 F(y)(\Phi(x))^2\right], \tag{14}$$

with $y = (2\varepsilon_d + U)/U$, $x = g\mu_B B/k_B T_K$. The peak in the resistance as a function of the magnetic field is described by the scaling function $\Phi(x)$. It is also weighted by the envelope $F(y) = (\pi^2/8)^2 y^2 (y^2 - 1)^4/(1 + y^2)^2$, shown Fig. 1(b). The agreement with Ref. [19], where $U/\Gamma = 20$ is finite, is here only approximate. The global maximum in the resistance is thus obtained for $\varepsilon_d/U = -1/2 \pm 0.1967$, $g\mu_B B = 1.0697 k_B T_K$, with $R_q = 0.001\,42(h/4e^2)(U/\Gamma)^4$ which predicts a strong increase of the resistance maximum with the ratio U/Γ, as seen Fig. 1(c).

For large magnetic fields $g\mu_B B \gg \sqrt{|\varepsilon_d|\Gamma}$, the free orbital regime [20] is reached and straightforward perturbation theory applies. The result is

$$\langle \hat{n}_\uparrow \rangle = 1 - \frac{1}{\pi}\frac{\Gamma}{\varepsilon_M - \varepsilon_d}, \qquad \langle \hat{n}_\downarrow \rangle = \frac{1}{\pi}\frac{\Gamma}{\varepsilon_M + \varepsilon_d + U},$$

with $\varepsilon_M = g\mu_B B/2$. This leads to $\chi_m = 0$ for $\varepsilon_d = -U/2$ as expected and, for very large magnetic fields $g\mu_B B \gg (|\varepsilon_d|, \varepsilon_d + U)$, the quantized value Eq. (9) is recovered for all gate voltages. Note that the standard result [6–9] $R_q = h/2e^2$ is only recovered for a fully polarized Fermi sea in the lead.

To summarize, the peak in the charge relaxation resistance is due to the enhancement of χ_m while the total charge remains quenched and χ_c small. In the presence of a finite magnetic field, the Kondo state is a mixture of singlet and triplet spin configurations controlled by the ratio of the Zeeman energy to the Kondo energy. A change in the gate voltage modifies this ratio and, while keeping the total charge of the dot almost constant, redistributes the spin up and down occupations. This leads to a larger number of particle-hole excitations for each spin species and therefore increases dissipation. At the particle-hole symmetric point, the Kondo energy is stationary with respect to the gate voltage such that no spin redistribution occurs and the peak in the resistance is absent. It is worth mentioning that the predicted peak in the charge relaxation resistance occurring at intermediate magnetic fields can be observed using current technology [4]. This work can be extended in various directions, by considering either Zeeman effects on a large cavity characterized by several energy levels [26] or a large number of channels [27]. We finally stress that our result (10) for the resistance is valid not only in the Kondo regime but for all values of U, ε_d and B.

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